Reconstruction of the Earth Surface Potential Distribution using Radial Basis Functions

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Abstract **–** A credible reconstruction of a continuous two-dimensional earth surface potential distribution from a discrete set of noisy samples is essential for correct identification of regions with high values of touch and step voltages, as a part of grounding grid safety assessment procedure. This research provides some new and useful insights on the earth surface potential distribution reconstruction quality, focused mainly on the most commonly used radial basis function (RBF) interpolation kernels. The RBF approach to reconstruction was chosen because of the underlaying algorithm simplicity and the fact that it provides natural mathematical framework for smooth interpolation and approximation of potential fields from irregular sampling sets. Since the quality of potential distribution reconstruction is heavily influenced by the choice of interpolation method, sampling layout and *a priori* knowledge of the grounding grid structure under test, this research also elaborates some heuristic guidelines for planning proper sampling procedure and illustrates some examples of reconstructed functional shape distortions due to poor sampling layouts.

I. INTRODUCTION

Substation grounding system design must meet requirements for personnel safety in the vicinity of grounded facilities by keeping potential differences on the earth surface below critical levels. Potential differences appear under fault conditions as the result of a short-circuit current flowing into earth through a grid of connected grounding conductors, while the grounded structure rises to some potential with respect to a remote reference ground (grounding potential rise, GPR). Two principal mechanisms affecting the personnel safety under ground fault conditions are described by touch and step voltage safety parameters ([1] - [4]). Touch voltage U_t represents a potential difference between grounded metallic structure (on a potential approximately equal to GPR) and a nearby point on the earth surface, where the person is standing while at the same time touching the grounded structure. Step voltage U_s is the earth surface potential difference subjected to person’s feet, separated by the distance of 1 m, with no contact to any grounded structure. The critical values of touch and step voltages depend on several factors, such as current path through a human body, duration of shock current etc. (standard recommendations may be found in [1], [2]).

Maximum values of touch and step voltages must be checked periodically by field measurements under test conditions, as a part of the grounding grid safety assessment procedure. Heavy current injection U-I measuring method [2] is routinely employed for periodic safety inspections to simulate potential distribution under realistic fault conditions by injecting a test current into the grounding grid. However, in order to acquire enough data for a reliable grounding grid safety assessment, touch and step voltages must be densely sampled throughout a relatively large area (typ. 900 – 20000 m^2, see [4]), thus making the measuring process very time consuming and expensive. Moreover, incorrect probes orientation during step voltage measurements may lead to false safety conclusions, due to the fact that maximum step voltage is measured only in the direction perpendicular to local equipotential lines.

The knowledge of the actual earth surface potential distribution shape $\varphi(x,y)$ may significantly improve the safety parameters measuring process and the credibility of results interpretation. Therefore, the relevant standards recommend improved two-stage measuring approach (see [2]). In the first phase earth surface potential contour survey is conducted by measuring potential values in a relatively large number of sampling points, from which the approximate earth surface potential distribution is estimated by means of some standard interpolation technique. In the second stage, touch and step voltages may be either estimated from the interpolated potential distribution $\varphi(x,y)$ or measured directly, but in a smaller number of measuring points, limited only to the areas characterized by large surface potential deviations from GPR near grounded metallic structures and high potential gradients, respectively.

Authors' previous work encompasses the development and implementation of a complex measuring system complying to the described two-stage measuring methodology (see [5] - [8]). Practical research in that field lead to the conclusion that under certain conditions estimated distribution may differ significantly from the actual shape of the earth surface potential distribution, thus introducing errors in the grounding grid safety assessment process. The cause of the problem lays in the fact that standard general-purpose gridding techniques have no connection with the underlaying physical model that describes the generation of earth surface potential distribution. If the potential contour survey sampling density is high enough, virtually any standard interpolation technique will perform well. However, as the sampling density decreases, the estimated potential distribution functional shape begins to degrade considerably. In practice, potential contour survey sampling density is determined as a trade-off between available resources (time, personnel, measuring equipment etc.) and overall inspection costs (including significant costs of putting the facility under test temporarily out of service). On the other hand, theoretical criteria and guidelines for critical sampling densities for different scattered data interpolation schemes are not well defined (unlike e.g. Shannon theorem for regular grid sampling of band-limited functions and reconstruction using *sinc* kernel).

The principal motivation for this research was to investigate the earth surface potential distribution reconstruction errors, due to deficiencies of some widely used general-purpose interpolation techniques, in conjunction with different parameterizations of sampling process. Such a research is important as neither relevant standards nor available scientific literature address this practical problem explicitly and point out possible misinterpretations of reconstructed potential distribution. This research focuses on a particular case of radial basis function (RBF) interpolation [9], as RBF kernels make convenient candidates for reconstruction because of their advantageous mathematical properties (meshless scattered data interpolation method), suitability for reconstruction of smooth potential fields and simplicity of the underlaying algorithm implementation.

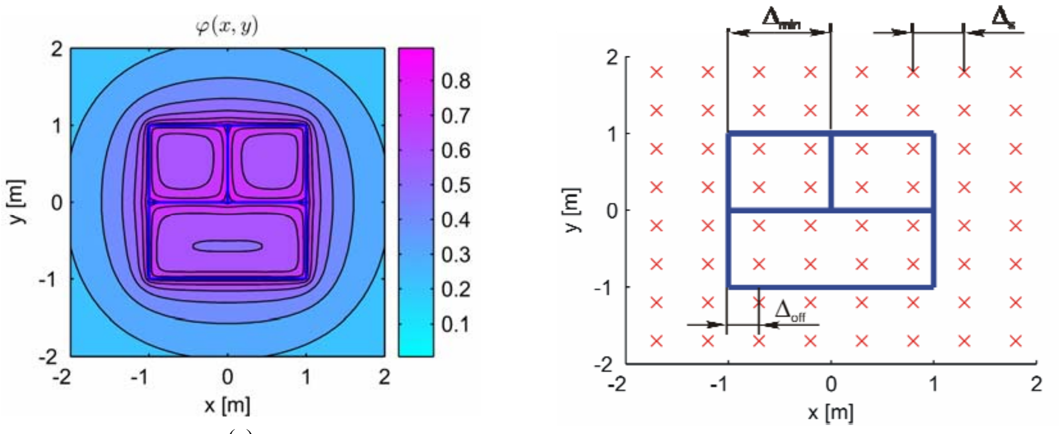
II. METHODOLOGY

Performance and characteristics of the RBF reconstruction approach were tested on a simple example of a grounding grid model, which structure is depicted in Fig. 1. Reference earth surface potential distribution $\varphi(x,y)$ (Fig. 1a) was calculated using BEM numerical model [10], under the assumptions of equipotential grounding grid, single-layer uniform soil and parabolic approximation of current line densities over horizontally buried grounding conductors (GPR is normalized to value 1.0). Sharp spikes in the potential profile over positions of grounding conductors introduce high-frequency components in the spectrum of potential distribution $\varphi(x,y)$ (see e.g. Fig. 2c). Although the analysis of smoothness of the potential field leads to the possibility of reconstruction by a class of band-limited functions (by means of trigonometric polynomials for irregular sampling case, see e.g. [11] for the case of smooth geophysical potential fields), high-frequency spectral components lead to the requirement for high density sampling in order to prevent aliasing, thus making such an approach unacceptable for practice of grounding grid potential surveys. RBF interpolants make a much more favourable choice as they are not bound to band-limited assumption and provide natural means for meshless scattered data interpolation and approximation.

However, this research has shown that potential distribution reconstruction quality, obtained by RBF interpolation, is highly dependant on the sampling density and the sample set arrangement relative to the grounding grid structure. The most abrupt changes in the surface potential distribution are present in close proximity of grounding conductors, while potential field is the smoothest in the midpoints of grounding grid mesh. Since the areas with highest local variations in the potential distribution contain the majority of useful informative content, the sampling procedure must cover these regions well. As there is no analytic counterpart of Shannon sampling theorem for the case of earth surface potential RBF interpolation from irregularly arranged samples, some heuristic sampling strategies were examined and mutually compared by means of empirical simulation. For the sake of simplicity of parametric comparison among different sampling strategies, only rectangular regular sampling grids are considered, without loss of generality (see sampling set arrangement parameters definitions in Fig. 1b). However, the interpolation algorithm itself must be able to handle more general case of non-uniform sampling, due to practical restrictions on rectangular uniform grid sampling in realistic conditions.

Test sampling sets are algorithmically generated according to the two key parameters: the regular sampling grid density Δ_s and the sampling grid offset Δ_{off}, relative to the grounding grid structure origin (see Fig. 1b). Sampling grid density Δ_s is expressed in terms of a minimum grounding grid mesh dimension Δ_{min}, while the sampling grid offset Δ_{off} represents the phase shift expressed in percents of sampling step Δ_s. The sampling grid density Δ_s and the sampling grid offset Δ_{off} have the same values in both x and y directions, in order to simplify sampling strategy parameterization and reconstruction error analysis. Minimum grounding grid mesh dimension is $\Delta_{min} = 1.0$ in normalized reference coordinate system for the case of grounding grid model shown in Fig. 1. Noise margin u_0, expressed in percents of reference GPR value, is also included in the parameterization of simulated measuring conditions as it affects the reconstruction quality considerably.

The two most commonly used RBF interpolation kernels were chosen to test earth surface potential distribution reconstruction quality, namely thin-plate splines and multiquadratics [9]. Examples of potential distribution reconstruction are depicted in Fig. 2 (for the case of thin-plate splines) and in Fig. 3 (for multiquadratics).



(a) (b)

Fig. 1: Calculated reference potential distribution (a) and description of sampling set parameters (b)

III. RESULTS, DISCUSSION AND APPLICATION

The example of thin-plate spline reconstruction for the highest density sampling case, sampling raster aligned exactly with the grounding grid structure and noiseless measurements is shown in Fig. 2a (sampling step $\Delta_s = 0.25\ \Delta_{min}$, sampling grid offset $\Delta_{off} = 0$ % and the measurement noise $u_0 = 0$ %). By comparing potential distributions in Fig. 1a and Fig. 2a it can be concluded that sampling at $\Delta_s = 0.25\ \Delta_{min}$ under such conditions yields fairly good reconstruction results. However, if the sampling raster is not well aligned with the grounding grid conductors and some noise is present in measurements, reconstructed functional shape may differ significantly from the reference earth surface potential distribution (compare Fig. 1a with Fig. 2b, for the case of $\Delta_s = 0.25\ \Delta_{min}$, $\Delta_{off} = 50$ % and $u_0 = 5$ %). Surface potential profiles over the reference ray $y_0 = 0.5$ for cases shown in Fig. 2a and Fig. 2b are depicted in Fig. 2c and Fig. 2d, respectively, along with the reference potential profile. Although functional shapes of reference and reconstructed potential distributions in Fig. 1a and Fig. 2b differ considerably, the reconstruction results may still be useful for the identification of areas with high expected values of touch and step voltages, because the local extremes of these functions are situated relatively close together (see the potential profile shown in Fig. 2d).

Decrease of the sampling density inevitably deteriorates potential distribution reconstruction quality. For the best case of a low density sampling procedure simulation shown in Fig. 2e (under conditions $\Delta_s = 0.50\ \Delta_{min}$, $\Delta_{off} = 0$ % and $u_0 = 0$ %) the resulting functional shape still resembles reference distribution well (in terms of correct local extremes positions and similar gradients distribution, essential for identification of areas with high values of touch and step voltages). However, the misalignment of a lower density sampling raster with respect to grounding grid elements may lead to highly unreliable reconstruction, even when the noise is not taken into account. For example, in the case shown in Fig. 2f ($\Delta_s = 0.50\ \Delta_{min}$, $\Delta_{off} = 50$ % and $u_0 = 0$ %) the functional shape and local extremes of reconstructed distribution are not correlated with reference distribution, thus leading to incorrect conclusions about grounding grid safety status. The results for lower density sampling case $\Delta_s = 0.50\ \Delta_{min}$ are even worse in the presence of measurement noise (see Tab. 1).

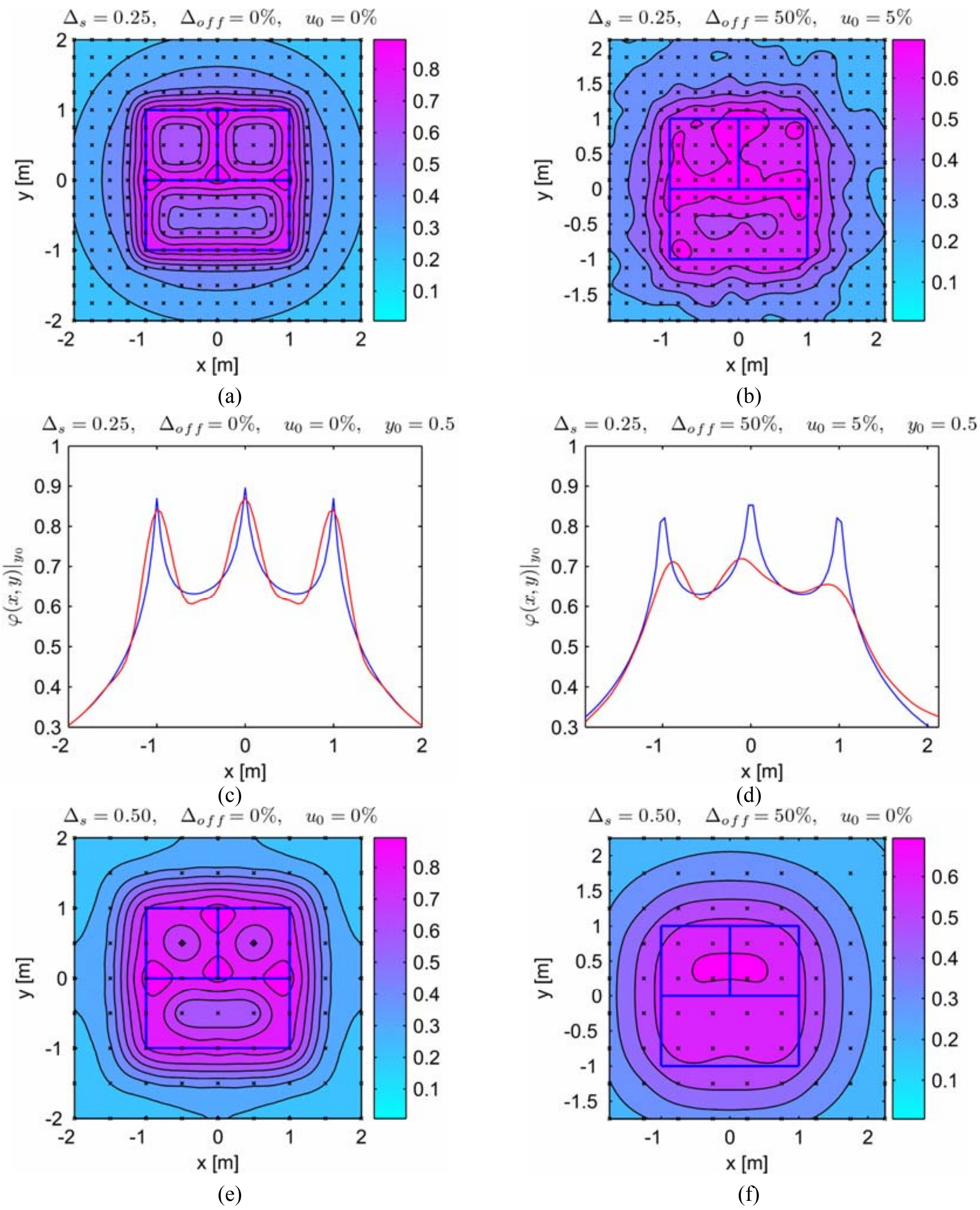


Fig. 2: Potential distribution reconstruction results obtained by using thin-plate spline RBF interpolation under different sampling conditions, described by parameters Δ_s, Δ_{off} and u_0

Reconstruction results for the multiquadratic RBF interpolation are shown in Fig. 3. Sampling sets and measuring conditions correspond to examples shown in Fig. 2, with the exactly same realization of the random noise vector for Fig. 2b and Fig. 3b. By comparing results shown in Fig. 2 and Fig. 3 it can be concluded that the choice of RBF interpolation kernel does not affect reconstruction quality significantly because both interpolation methods yield similar results under the same measuring conditions, regardless of minor differences in the actual reconstructed functional shapes (compare also results of numerical error analysis for both methods in Tab. 1).

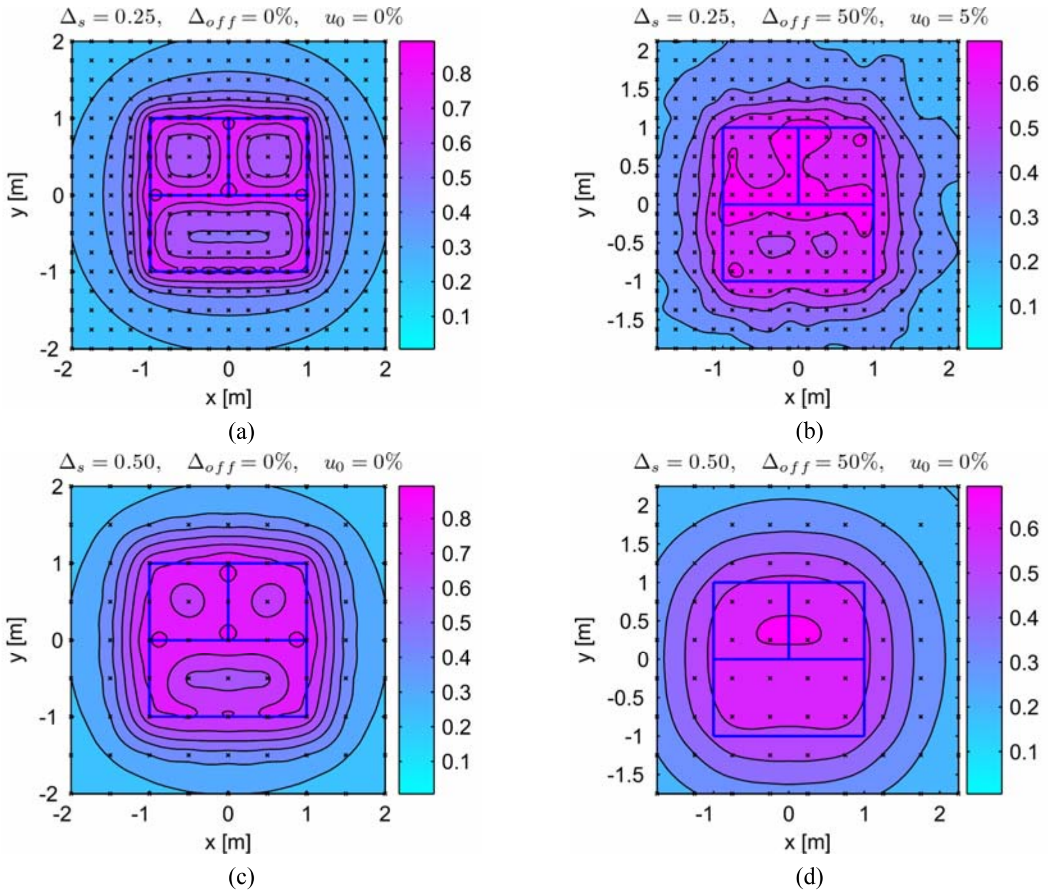


Fig. 3: Potential distribution reconstruction results obtained by using multiquadratic RBF interpolation under different sampling conditions, described by parameters Δ_s, Δ_{off} and u_0

Tab. 1 contains the results of quantitative error analysis under various measuring conditions for both thin-plate spline and multiquadratic RBF reconstruction. Error analysis includes the following metrics: maximum absolute difference between ideal and reconstructed distribution $|\varphi|_{max}$, mean absolute integral error $I_{|\varphi|}$, root-mean-square (RMS) error and functional shape correlation coefficient r_φ. Numerical values are calculated under the assumption of equipotential grounding grid with normalized value of GPR = 1.0. Error analysis is restricted to the reduced area over grounding grid elements only, defined by $\Omega_{err} = [-1,1] \times [-1,1]$. Correlation coefficient represents the most important integral quality measure, because it describes how reconstructed functional shape, essential for identifying areas with critical values of touch and step voltages, resembles ideal distribution. On the other hand, pointwise maximum absolute errors are not crucial in the process of estimation of these regions. It should be pointed out that the worst case of sampling set arrangement ($\Delta_s = 0.5\ \Delta_{min}$, $\Delta_{off} = 0.25\ \Delta_{min}$) yields even negative correlation coefficients, meaning that such potential distribution reconstruction results cannot be used for correct estimation of areas with high values of touch and step voltages.

Simulations showed that an acceptable earth surface potential distribution reconstruction using RBF interpolation may be obtained at minimum sampling density of 0.5 Δ_{min}, but under ideal conditions only (see Fig. 2e, Fig. 3c). If the sampling raster is not aligned perfectly with grounding grid conductors and the measurement noise is present, reconstructed functional shape may become highly inaccurate (see Fig. 2f, Fig. 3d). Although low sampling density may be in favour of decreasing overall inspection costs, it must be taken into account that the reconstruction results under such conditions are unreliable. Moreover, it is difficult to ensure exact positions of buried conductors during realistic routine field measurements, even in cases when substation project documentation is available. Reconstruction distortions due to sampling raster offset may be minimized under two conditions: (1) grounding grid structure must be known *a priori* almost perfectly and (2) surface potential measuring probe positioning must be highly accurate, in order to align sampling raster optimally. However, requirements (1) and (2) put much higher demands on the sampling process itself: reliable documentation on grounding conductor placement may not always be available and the need for a high accuracy sample positioning introduces additional practical complexities in the overall measuring process. Therefore, the minimum recommended sampling density is at least 0.25 Δ_{min}, in order to prevent significant functional shape distortions due to sampling raster offset and measurement noise. Results in Tab. 1 show high correlation coefficients for both interpolation methods in the best sampling case $\Delta_s = 0.25\ \Delta_{min}$, $\Delta_{off} = 0$ % ($r_\varphi \sim 0.85 - 0.90$). Noise level $u_0 = 5$ % does not significantly affect correlation coefficients under such sampling conditions. Cases $\Delta_s = 0.25\ \Delta_{min}$, $\Delta_{off} = 50$ % and $\Delta_s = 0.50\ \Delta_{min}$, $\Delta_{off} = 0$ % yield notably smaller correlation coefficients ($r_\varphi \sim 0.65 - 0.75$). Finally, the worst sampling case $\Delta_s = 0.50\ \Delta_{min}$, $\Delta_{off} = 50$ % has negative correlation coefficients under all conditions.

Tab. 1: Quantitative error analysis of the earth surface potential distribution reconstruction, obtained by means of thin-plate spline and multiquadratic RBF interpolation, for measuring conditions described by parameters Δ_s, Δ_{off} and u_0 and errors analyzed using $|\varphi|_{max}$, $I_{|\varphi|}$, RMS and r_φ metrics

	Thin plate spline RBF interpolation							
Δ_s [% Δ_{min}]	25				50			
Δ_{off} [% Δ_s]	0		50		0		50	
u_0 [% GPR]	0	5	0	5	0	5	0	5
$\|\varphi\|_{max}$	0.137	0.170	0.212	0.217	0.202	0.206	0.329	0.347
$I_{\|\varphi\|}$	0.047	0.053	0.036	0.040	0.110	0.096	0.085	0.090
RMS	0.059	0.067	0.057	0.060	0.122	0.110	0.118	0.125
r_φ	0.892	0.872	0.765	0.722	0.728	0.723	– 0.191	– 0.143
	Multiquadratic RBF interpolation							
Δ_s [% Δ_{min}]	25				50			
Δ_{off} [% Δ_s]	0		50		0		50	
u_0 [% GPR]	0	5	0	5	0	5	0	5
$\|\varphi\|_{max}$	0.111	0.139	0.225	0.229	0.178	0.170	0.335	0.349
$I_{\|\varphi\|}$	0.044	0.050	0.039	0.042	0.111	0.098	0.087	0.091
RMS	0.053	0.060	0.061	0.063	0.120	0.106	0.122	0.127
r_φ	0.887	0.867	0.716	0.689	0.652	0.670	– 0.204	– 0.164

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