Accuracy measures of the four arm bridge of broadly variable resistances
Part 1 Backgrounds, general case of four variable arms

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Abstract. After short introduction the open circuit output voltage of four-arm resistance (4R) bridge supplied by current or voltage is given. Normalized forms of its current to voltage and voltage to voltage open circuit transmittances are defined. The simplified description of accuracy of the broadly variable resistance by instantaneous, limited and random errors of its initial value and of increment is introduced. Borders (the worst cases) of the field of its possible error values are found for constant values of both this limited errors. Accuracy measures of current to voltage transmittance, i.e. instantaneous and limited errors and mean square random measures (probabilistic errors or uncertainties) are discussed in detail for general case of arbitrary variable all bridge arms. Basic formulas together with their equivalents for voltages transmittance are completed in the table 1. Literature is included.
Accuracy description of other common in practice particular cases of the 4R bridge and example of its application for temperature sensors Pt 100 are described in the part 2. Final conclusions are also there.

I. Introduction

The generalized description of the accuracy of the unbalanced 4R bridge of arbitrary variable arm resistances was not given in the literature even as the first such balanced bridge had been developed 174 years ago. Such description is especially needed now in indirect measurements of not only single (1D) but also of *n* variables (nD) by many sensors of broadly variable immitance, e.g. thermo-resistors, semiconductor strain gages, magneto-resistors, potentiometers, etc., and should be developed for:

- analogue circuits of initial conditioning of measurement signals of resistance sensor sets,
- identification the changes of equivalent internal parameters of the arbitrary circuit working as twoport X from its terminals for testing, monitoring and diagnostic purposes.

In relation to particular application bridge could be considered as:

- single integrated element of metrological data obtained from producer or from calibration,
- circuit, for which accuracy measures of their parameters in analysis or synthesis process are estimated from values and measures of its equivalent scheme and variable elements could be considered as concentrated or as distributed on sensor set dimensions.

The last case is typical for the setting of sensor bridges in production or in practice and also if bridge has few remote sensors or the equivalent circuit of the space distributed object is tested from its terminals.

Terminal bridge circuit parameters and their accuracy measures are described as multivariable functions. They differently depend on bridge arm resistances and their accuracy measures due to the type of supply: by current (the more linear case) or voltage source, single or double ones; ratios of initial arm resistances and number (1-4), values and signs of their increments - independent or joined differently. Then some generalization and simplification is necessary. The analysis of the metrological properties of arbitrary supplied 4T (four terminal) circuit and its particular case: the 4R bridge as twoport, are given in the monograph [1] and in publications [6, 7]. The development of author's ideas about accuracy measures of the 4R bridge are presented below in the part 1 and continued in the part 2 of this paper for the most commonly used particular cases of this bridge.

II. Basic equations of the 4R bridge open-circuit transmittance r_{21}

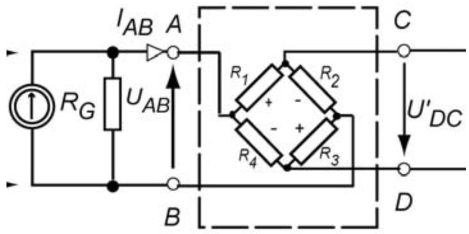


Fig 1 Four arms bridge as the unloaded twoport of type X with the supply source branch

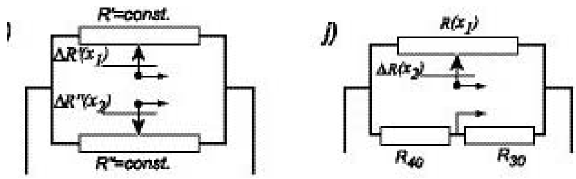


Fig 2 Two examples of 4R bridge of jointed arm increments and dependent on two variables x_1, x_2.

Four resistance (4R) bridge circuit of terminals ABCD working as passive twoport of variable parameters is shown on Fig 1, some its examples of jointed changes of arm resistances are on Fig 2. Voltages and currents of terminals are described by set of two linear equations of three different parameters only, e.g.: open circuit current to voltage transmittance $r_{21}=r_{12}$ or transmittance $k_{21}=k_{12}$ as ratio of output and input bridge voltages and two open-circuited resistances: input one R_{AB}^{∞} (when load resistance $R_L \to \infty$) and output one R_{CD}^{∞} for the source resistance $R_G \to \infty$. They depend on four bridge arm resistances R_i. For single variable it is enough to measure changes of one terminal parameter and the output circuit voltage U_{DC} is the most commonly used as it may change its sign for some set of arm resistances. With notations of fig 1 it is

$$U_{DC}^{\infty} \equiv U'_{DC} = I_{AB} \frac{R_1 R_3 - R_2 R_4}{\sum R_i} \equiv I_{AB}\, r_{21} \qquad \text{or} \qquad U'_{DC} = U_{AB} \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} \equiv U_{AB} k_{21} \tag{1a, b}$$

where: I_{AB}, U_{AB} - current or voltage on bridge supply terminals A B,

$R_i \equiv R_{i0} + \Delta R_i \equiv R_{i0}(1+\varepsilon_i)$ - arm resistance of initial value R_{i0} and absolute ΔR_i and relative ε_i increments (1c)

r_{21}, k_{21}– current to voltage and voltages bridge transmittance of the open-circuited output (transfer resistance, voltage ratio coefficient)

If r_{21}=0 or k_{12}=0, bridge is in balance and its conditions of both supply cases are the same: $R_1R_3=R_2R_4$. It could happen for many different combinations of ε_i. The basic balance state is defined for all ε_i=0, i.e. when:

$$R_{10} R_{30} = R_{20} R_{40} = R_{AB0} R_{CD0} \tag{2}$$

To the commonly known form the third term - product of two terminal initial resistances is added as useful tool of parameter identification of the equivalent circuit of 4T twoport of unknown internal structure [1], [2-chapter 127]. In measurements commonly is used the ideal supply, by current i.e.: $R_G \to \infty$ and $I_{AB} \to J$, or by voltage: U_{AB}=const. All formulas of the unbalanced bridge are simplified if terminal parameters are referenced to their initial values in the balance, i.e. $R_i = R_{i0}(1+\varepsilon_i)$. If also $R_{20} \equiv mR_{10}$, $R_{40} \equiv nR_{10}$ and from (2) $R_{30} = mnR_{10}$, then normalized form of r_{21} is:

$$r_{21} \equiv t_0\, f(\varepsilon_i) = t_0 \frac{\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4 + \varepsilon_1 \varepsilon_3 - \varepsilon_2 \varepsilon_4}{1 + \dfrac{\varepsilon_1 + m\,\varepsilon_2 + n\,(m\,\varepsilon_3 + \varepsilon_4)}{(1+m)\,(1+n)}} \tag{3}$$

where: $t_0 \equiv \partial r_{21} / \partial f(\varepsilon_i) \big|_{\text{all } \varepsilon_i \to 0} = \dfrac{R_{10} R_{30}}{\sum R_{i0}} = \dfrac{m\,n}{(1+m)(1+n)} R_{10}$ - initial open circuit sensitivity of r_{21} (3a)

$\sum R_{i0}$ - initial value of the sum of arm resistances $\sum R_i$

$f(\varepsilon_i) \equiv t_0 \dfrac{\Delta L(\varepsilon_i)}{1 + \varepsilon_{\Sigma R}(\varepsilon_i)}$ - normalized unbalance function of r_{21} (3b)

$\Delta L(\varepsilon_i)$; $\varepsilon_{\Sigma R}(\varepsilon_i)$ - increments of numerator and denominator of the function $f(\varepsilon_i)$.

Transmittance r_{21} and unbalance function $f(\varepsilon_i)$ could theoretically take values of the range (-∞, +∞). In practice there are some limitations due to extremely possible values of increments ε_i, limited dissipated powers of arms and limited voltage of the real current source or limited current of the voltage source. Initial sensitivity $t_0 \to 1$, when $m \to \infty$ and $n \to \infty$, maximum initial sensitivity k_0 (see table 1a) of k_{12} is ¼ for m=1 and n arbitrary. Then bridges supply by current could be more sensitive then by voltage ones and they are more linear. Function $f(\varepsilon_i)$ is generally nonlinear, but it becomes practically linear if increments ε_i are very small, i.e. pairs of their products in nominator ΔL of (3) and $\varepsilon_{\Sigma R}$ in denominator of are negligible. Exact linearization could also happen for such relations between increments of at least two variable arms that ratio of resistances of parallel bridge branches (R_1+R_2/R_3+R_4 or R_1+R_4/R_2+R_3) become constant [1-3].

The complete set of the normalized formulas of terminal parameter of the 4R bridge are given in tables in [1, 2, 5]. They include also working terminal parameters of the bridge if values of equivalent source R_G and load R_L resistances are arbitrary. They allow describing terminal parameters of any 4R loop and in any mode of 4R loop operation as four-terminal circuit 4T in single (1D) and multi-variable (nD) measurements [1, 5-7]. The computer program for simulation of 4R bridge parameters including their accuracy measures was also developed.

III. Accuracy description of broadly variable resistances

Accuracy measures (errors, uncertainty) of the single value of circuit parameter are expressed by numbers, of variable parameter - by functions of its values. In both cases they depend on equivalent scheme of the circuit, on environmental conditions and parameters of instrumentation used or have to be use in the experiment. Accuracy of instrumentation is described by absolute or related systematic and random errors, instantaneous and limited ones. For the accuracy estimation of measurements uncertainties recommended by international guide GUM are also used.

The broadly variable resistance R_i, e.g. of the temperature sensor, could be expressed by two components: its initial value and increment as in (1c) and by their measures. Relative instantaneous error δ_{Ri} of R_i is

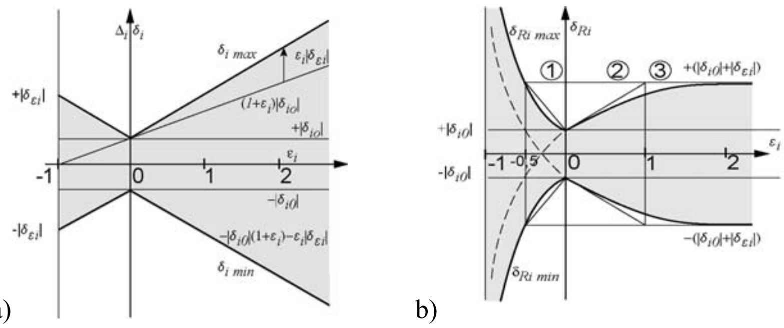
$$\delta_{Ri} \equiv \frac{\Delta_i}{R_i} = \delta_{i0} + \frac{\Delta_{\varepsilon i}}{1+\varepsilon_i} = \delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i} \tag{4}$$

where: Δ_i, Δ_{i0}, $\Delta_{\varepsilon i}$; δ_{Ri}, $\delta_{i0} = \frac{\Delta_{i0}}{R_{i0}}$, $\delta_{\varepsilon i} = \frac{\Delta_{\varepsilon i}}{\varepsilon_i}$ – absolute and relative errors of R_i, its initial value R_{i0} and increment ε_i.

Errors Δ_{i0}, δ_{i0} of the initial resistance are of single values, which may vary with the resistance R_i by various functions. For simplicity they may be linearizated, if possible for the whole range of R_i or of its few segments. Even in this case their actual values vary for various reasons and are unknown. Then more useful in practice is to estimate the possible area of error values, i.e. to find borders of this area for any value of R_i known as ***limited errors***, notated by $|\Delta_{i0}|$, $|\delta_{i0}|$ and $|\Delta_{\varepsilon i}|$, $|\delta_{\varepsilon i}|$. If they are independent, then on the poorest case of (4) limited relative error of R_i is:

$$|\delta_{Ri}| \le |\delta_{i0}| + \frac{|\Delta_{\varepsilon i}|}{1+\varepsilon_i} = |\delta_{i0}| + \frac{|\varepsilon_i|}{1+\varepsilon_i}|\delta_{\varepsilon i}| \tag{5}$$

Borders $\pm|\delta_{Ri\max}|$ of possible values of δ_{Ri} area nonlinearly dependent on ε_i even if $|\delta_{i0}|$ and $|\delta_{\varepsilon i}|$ are constant – fig 2b.



a) b)

Fig 2. Areas of absolute Δ_i and relative $\delta_i = \frac{\Delta_i}{R_{i0}}$ a), or $\delta_{Ri} = \frac{\Delta_i}{R_i}$ b) errors of the variable resistance $R_i \equiv R_{i0}(1+\varepsilon_i)$ for constant values of limited relative errors $|\delta_{i0}|$, $|\delta_{\varepsilon i}|$ of its components R_{i0}, ε_i.

From (4) or (5) it is also easy to find formula of the standard random error or the standard uncertainty.

$$\bar{\delta}_{Ri} \equiv \frac{\bar{\Delta}_i}{R_i} = \sqrt{\bar{\delta}_{i0}^2 + \left(\frac{\varepsilon_i}{1+\varepsilon_i}\right)^2 \bar{\delta}_{\varepsilon i}^2 + 2k_i \frac{\varepsilon_i}{1+\varepsilon_i}\bar{\delta}_{i0}\bar{\delta}_{\varepsilon i}} \tag{6}$$

where: $\bar{\delta}_{i0}$, $\bar{\delta}_{\varepsilon i}$ –standard random measures of initial value and increment of R_i,

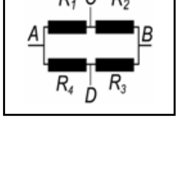
$k_i \subset (-1...0..+1)$ – their normalized correlation coefficient.

Distribution of the initial values of sensor resistances depends on their production process, actual values of its relative increment ε_i – mainly on environmental influences. Exacte correlation coefficient could be find only experimentally. If errors of increment and initial value of resistance are statistically independent then $k_i = 0$.

IV. Accuracy measures of transmittance r_{21} in general case - arbitrary resistance increments

Accuracy of measurements applying bridges depends on errors of supply and of terminal bridge parameters. Last ones arise when real values of resistances R_i are different then nominal ones. If supply is constant accuracy of the bridge depends only on accuracy of its transmittances. Accuracy of transmittance r_{21} is analyzed in details below and main formulas for it and for k_{21} are given in table 1.

A. Instantaneous error of r_{21}



Near the bridge balance stage the absolute error of r_{21} (or of k_{21}) and its relation to initial sensitivity t_0 or to range $r_{21\max}$ - $r_{21\min}$, should be only used (as ratio of initial error Δ_{210} and r_{21}=0 is $\pm\infty$). Measurement errors result from the total differential of analytical equations of bridge parameters. **Absolute error of r_{21}** is:

$$\Delta_{r21} = \frac{R_1 R_3(\delta_{R1}+\delta_{R3}) - R_2 R_4(\delta_{R2}+\delta_{R4})}{\sum R_i} - r_{21}\delta_{\Sigma R} \tag{7}$$

where: $\delta_{Ri} \equiv \frac{\Delta_i}{R_i}$ – relative errors of bridge resistance, $\delta_{\Sigma R} = \frac{\sum \Delta_i}{\sum R_i} = \frac{\sum (R_i \delta_{Ri})}{\sum R_i}$ - relative error of ΣR_i (7a)

Error Δ_{r21} could be expressed in two generalized forms:

$$\Delta_{r21} = \frac{1}{\sum R_i} \sum_{i=1}^{4} \left[(-1)^{i+1} R_j - r_{21} \right] \Delta_i = \sum w_{Ri} \delta_{Ri} \tag{8}$$

where: $w_{Ri} \equiv R_i \frac{(-1)^{i+1} R_j - r_{21}}{\sum R_i}$ - weight coefficients of relative errors δ_{Ri}: (8a)

- subscript i = 1,2,3,4 when j=3,4,1,2; - multiplier $(-1)^{i+1}$=+1 if i is 1, 3 or –1 if i is 2, 4.

Expanded forms of absolute errors Δ_{r21} and Δ_{k21} of both transmittances r_{21} and k_{21} are in the line 1 of table 1. Influences of neighboring arms are of opposite signs and partly compensate each other.
If errors δ_{Ri} of broadly variable resistances R_i are expressed by δ_{i0} and $\Delta_{i\varepsilon}$ then:

$$\Delta_{r21} = \sum w_i \left[(1+\varepsilon_i)\, \delta_{i0} + \Delta_{\varepsilon i} \right] \tag{9}$$

where: $w_i = \frac{t_0}{1+\varepsilon_{\Sigma R}} [(-1)^{i+1}(1+\varepsilon_j) - \frac{r_{21}}{R_{j0}}]$ (9a)

For constant arm resistance R_i is ε_i=0, δ_{Ri}=δ_i=δ_{i0} and w_{Ri}=w_i, but its component in Δ_{r21} depends on others increments ε_j. When r_{21}=0:

$$w_{Ri0} = w_{i0} = (-1)^{i+1} t_0 \tag{9b}$$

If (8) has be referenced to t_0, the relative error defined as $\delta_{r21} \equiv \frac{\Delta_{r21}}{t_0}$ is obtained. It could be presented as sum:

$$\delta_{r21} = \delta_{210} + \delta_{r21\varepsilon}(\varepsilon_i) \tag{10a}$$

where: $\delta_{210} = \delta_{10} - \delta_{210} + \delta_{30} - \delta_{40}$ – initial (or zero) relative error of r_{21}=0 referenced to the sensitivity t_0;
$\delta_{r21\varepsilon}(\varepsilon_i)$ –relative error of $f(\varepsilon_i)$ when $r_{21} \neq 0$.

Error δ_{210} may be corrected on different ways: by adjustment of the bridge resistances, by the opposite voltage to output or by the digital correction of converted output signal. In such cases it is

$$\delta_{r21\varepsilon} = \frac{1}{t_0} \sum \left[w_i (1+\varepsilon_i) - (-1)^{i+1} \right] \delta_{i0} + \frac{1}{t_0} \sum w_i \Delta_{\varepsilon i} \tag{10b}$$

From (10b) follows that transmittance relative error $\delta_{r21\varepsilon}$ depends not only on increment errors $\Delta_{\varepsilon i}$, but also on δ_{i0} errors of initial resistances even when δ_{210}=0 because weight coefficients w'_i=w_i/t_0 of δ_{i0} errors depends on ε_i. The component of particular error δ_{i0} despairs in (10b) only when δ_{i0}=0. Relations become simpler if functions of $\Delta_{\varepsilon i}$ or $\delta_{\varepsilon i}$ are approximated for some ε_i intervals by constant values.
The actual errors of r_{21} could be calculated very rare, only if signs and values of instantaneous errors of all resistances are known. In reality more frequently are given their limited systematic and random errors. Then measures of r_{21} could be obtained from general formulas of table 1a or from proper simplified ones, as in table 2 and table 3 of part 2.

B. Limited errors of transmittance r_{21}

From (8) limited error $|\Delta_{r21}|_m$ has such general forms:

$$|\Delta_{r21}|_m = \frac{1}{\sum R_i} \sum_{i=1}^{4} \left| (-1)^{i+1} R_j - r_{21} \right| |\Delta_i| = \sum |w_{Ri}| |\delta_{Ri}| \tag{11}$$

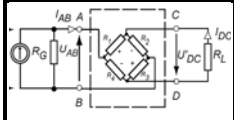
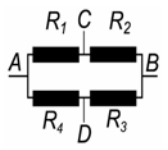
where: $|w_{Ri}| = R_i \frac{R_j + (-1)^i r_{21}}{\sum R_i}$ (11a)

If limited errors of increments ε_i are negligible, i.e. $|\Delta_{\varepsilon i}| << |\delta_{i0}|$ (e.g. for temperature sensors of given material composition) or in opposite situation $|\delta_{i0}| << |\Delta_{\varepsilon i}|$, then it is

$|\delta_{r21}|_m = \sum |w'_{Ri}| |\delta_{i0}|$ (12a) or $|\delta_{r21}|_m = \sum |w'_i| |\Delta_{\varepsilon i}|$ (12b)

where: w'_{Ri}=w_{Ri}/t_0, w'_i=w_i/t_0 – weight coefficients

Table 1 Accuracy measures of transmittances of arbitrary variable resistances of four arm bridge – general case [6, 7]

		Open circuit current to voltage transmittance r_{21}	**Open circuit voltages transmittance k_{21}**
	 $U_{DC}=r_{21}I_{AB}-R^{\infty}_{CD}I_{DC}$	$r_{21}\equiv\frac{U'_{DC}}{I_{AB}}=\frac{R_1R_3-R_2R_4}{\sum R_i}\equiv t_0\,f(\varepsilon_i)=t_0\,\frac{\Delta L(\varepsilon_i)}{1+\varepsilon_{\Sigma R}(\varepsilon_i)}$ where: $R_i=R_{i0}(1+\varepsilon_i)$ $t_0\equiv\frac{m\,nR_{10}}{(1+m)(1+n)}$ $\Delta L(\varepsilon_i)=\varepsilon_1-\varepsilon_2+\varepsilon_3-\varepsilon_4+\varepsilon_1\varepsilon_3-\varepsilon_2\varepsilon_4$ $\varepsilon_{\Sigma R}=\frac{\varepsilon_1+m\varepsilon_2+n(\varepsilon_4+m\varepsilon_3)}{(1+m)(1+n)}$	$k_{21}\equiv\frac{U'_{DC}}{U_{AB}}=\frac{R_1R_3-R_2R_4}{(R_1+R_2)(R_3+R_4)}\equiv k_0\,f_E(\varepsilon_i)$ where: $k_0\equiv\frac{m}{(1+m)^2}$ $f_E(\varepsilon_i)\equiv\frac{\Delta L(\varepsilon_i)}{(1+\varepsilon_{12})(1+\varepsilon_{34})}$ $\varepsilon_{12}\equiv\frac{\varepsilon_1+m\varepsilon_2}{1+m}$ $\varepsilon_{43}\equiv\frac{\varepsilon_4+m\varepsilon_3}{1+m}$
	$R_L\rightarrow\infty$, $U'_{DC}=r_{21}I_{AB}=k_{21}U_{AB}$	**Accuracy measures of R_i:** **systematic error** $\delta_{Ri}\equiv\frac{\Delta_i}{R_i}=\delta_{i0}+\frac{\Delta_{\varepsilon i}}{1+\varepsilon_i}$ **limited error** $\lvert\delta_{Ri}\rvert\equiv\frac{\lvert\Delta_i\rvert}{R_i}=\lvert\delta_{i0}\rvert+\frac{\lvert\Delta_{\varepsilon i}\rvert}{1+\varepsilon_i}$	**random error or uncertainty type B** $\bar{\delta}_{Ri}\equiv\frac{\bar{\Delta}_i}{R_i}=\sqrt{\bar{\delta}^2_{i0}+\frac{\bar{\Delta}^2_{\varepsilon i}}{(1+\varepsilon_i)^2}}$
	Variable 4R_i 	**a) Measures δ_{r21}, $\lvert\delta_{r21}\rvert$, $\bar{\delta}_{r21}$ of r_{21} transmittance**	**b) Measures δ_{k21}, $\lvert\delta_{k21}\rvert$, $\bar{\delta}_{k21}$ of k_{21}**
1		$\Delta_{r21}=\frac{t_0}{1+\varepsilon_{\Sigma R}}\left\{\begin{array}{l}(1+\varepsilon_3-\frac{r_{12}}{R_{30}})[(1+\varepsilon_1)\delta_{10}+\Delta_{\varepsilon 1}]-(1+\varepsilon_4+\frac{r_{12}}{R_{40}})[(1+\varepsilon_2)\delta_{20}+\Delta_{\varepsilon 2}]+\\+(1+\varepsilon_1-\frac{r_{12}}{R_{10}})[(1+\varepsilon_3)\delta_{30}+\Delta_{\varepsilon 3}]-(1+\varepsilon_2+\frac{r_{12}}{R_{20}})[(1+\varepsilon_4)\delta_{40}+\Delta_{\varepsilon 4}]\end{array}\right\}$	$\Delta_{k21}=k_0\left[\frac{(1+\varepsilon_1)(1+\varepsilon_2)}{(1+\varepsilon_{12})^2}(\delta_{R1}-\delta_{R2})+\frac{(1+\varepsilon_3)(1+\varepsilon_4)}{(1+\varepsilon_{34})^2}(\delta_{R3}-\delta_{R4})\right]$
2		$\delta_{r21}\equiv\frac{\Delta_{r21}}{t_0}=\sum_{i=1}^{4}w'_{Ri}\delta_{Ri}$ $\lvert\delta_{r21}\rvert=\sum_{i=1}^{4}\lvert w'_{Ri}\rvert\lvert\delta_{Ri}\rvert$ $\bar{\delta}_{r21}=\sqrt{\sum_{i=1}^{4}w'^2_{Ri}\bar{\delta}^2_{Ri}}$	$\delta_{k21}\equiv\frac{\Delta_{k21}}{k_0}=\sum_{i=1}^{4}w'_{ki}\delta_{Ri}$ $\lvert\delta_{k21}\rvert=\sum_{i=1}^{4}\lvert w'_{ki}\rvert\lvert\delta_{Ri}\rvert$ $\bar{\delta}_{r21}=\sqrt{\sum_{i=1}^{4}w'^2_{ki}\bar{\delta}^2_{Ri}}$
3		where: $w'_{Ri}=\frac{1}{1+\varepsilon_{\Sigma R}}[(-1)^{i-1}(1+\varepsilon_j)-\frac{r_{21}}{R_{j0}}](1+\varepsilon_i)$ $j=(i+2)$ mode 4, i.e.: if $i=1, 2, 3, 4$ then $j=3, 4, 1, 2$ correlation coefficients $k_{ij}=0$	where:; $w'_{k1}=-w'_{k2}=\frac{(1+\varepsilon_1)(1+\varepsilon_2)}{1+\varepsilon_{12}}$; $w'_{k3}=-w'_{k4}=\frac{(1+\varepsilon_3)(1+\varepsilon_4)}{1+\varepsilon_{34}}$ $k_{ij}=0$ n - arbitrary
	$R_1=R_{10}(1+\varepsilon_1)$	**Particular cases**	
4	**$R_2=mR_{10}(1+\varepsilon_2)$ $R_3=mnR_{10}(1+\varepsilon_3)$ $R_4=nR_{10}(1+\varepsilon_4)$**	$\varepsilon_i\,\varepsilon_j<<1+\varepsilon_i+\varepsilon_j$ $>>\varepsilon_i r_{12}/R_{j0}$ $\delta_{r21}\approx\frac{1}{1+\varepsilon_{\Sigma R}}\sum\left[(-1)^{i+1}(1+\varepsilon_i+\varepsilon_j)-\frac{r_{12}}{R_{j0}}\right]\delta_{Ri}$	$\varepsilon_i\,\varepsilon_j<<1+\varepsilon_i+\varepsilon_j$ $\delta_{k21}\approx\frac{1+\varepsilon_1+\varepsilon_2}{1+2\frac{\varepsilon_1+m\varepsilon_2}{1+m}}(\delta_{R1}-\delta_{R2})+\frac{1+\varepsilon_3+\varepsilon_4}{1+2\frac{\varepsilon_4+m\varepsilon_3}{1+m}}(\delta_{R3}-\delta_{R4})$
5	Bridge balance: $r_{21}=0$ $R_{10}R_{30}=R_{20}R_{40}$	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$, $\lvert\Delta_{\varepsilon i}\rvert=\lvert\Delta_\varepsilon\rvert$ $\lvert\delta_{r21}\rvert=\left(\sum\lvert w'_{Ri}\rvert\right)\lvert\delta_0\rvert+\frac{1}{1+\varepsilon_{\Sigma R}}\left[4+\sum\varepsilon_i-a\,f(\varepsilon_i)\right]\lvert\Delta_\varepsilon\rvert$ where: $\sum\lvert w'_{Ri}\rvert=\frac{1}{1+\varepsilon_{\Sigma R}}\left[4+2\sum\varepsilon_i+2(\varepsilon_1\varepsilon_3+\varepsilon_2\varepsilon_4)-\frac{\Delta L(\varepsilon_i)}{1+\varepsilon_{\Sigma R}}\left(a+\frac{\varepsilon_1-m\varepsilon_2+n(m\varepsilon_3-\varepsilon_4)}{(1+m)(1+n)}\right)\right]$	$2\frac{(1+\varepsilon_1)(1+\varepsilon_2)\lvert\delta_0\rvert+[1+0{,}5(\varepsilon_1+\varepsilon_2)]\lvert\Delta_\varepsilon\rvert}{(1+\varepsilon_{12})^2}+2\frac{(1+\varepsilon_3)(1+\varepsilon_4)\lvert\delta_0\rvert+[1+0{,}5(\varepsilon_3+\varepsilon_4)]\lvert\Delta_\varepsilon\rvert}{(1+\varepsilon_{34})^2}$
6		$\bar{\delta}_{i0}=\bar{\delta}_0$ $\bar{\Delta}_{\varepsilon i}=\bar{\Delta}_\varepsilon$ $\sqrt{\bar{\delta}^2_0\sum_{i=1}^{4}w'^2_{Ri}+\bar{\Delta}^2_\varepsilon\sum_{i=1}^{4}\frac{w'^2_i}{(1+\varepsilon_i)^2}}$ where: $a\equiv\frac{(1-m)(1-n)}{(1+m)(1+n)}$	$2(1-\varepsilon^2)\frac{\sqrt{(1+b^2\varepsilon^2)^2+4b^2\varepsilon^2}}{(1-b^2\varepsilon^2)^2}\sqrt{\bar{\delta}^2_0+\frac{1+\varepsilon^2}{(1-\varepsilon^2)^2}\bar{\Delta}^2_\varepsilon}$ where: $b\equiv\frac{1-m}{1+m}$ n arbitrary
7	**Accuracy of balance:** (r_{21}=0)	**Instantaneous initial error** $\delta_{210}=\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}$ **Limited initial error** $\lvert\delta_{210}\rvert_m=\sum\lvert\delta_{i0}\rvert$	**Initial uncertainty** $\bar{\delta}_{210}=\sqrt{\sum\bar{\delta}^2_{i0}}$

C. Random measures of r_{21} accuracy

Accuracy measure of the circuit output parameter are treated as random for

- calculate the random component of its error dependent on random components of resistance errors, e.g. noise and interferences, or estimate the uncertainty of measurements of this parameter,
- averaging the limited error for set of similar circuits if it is needed in their production or application.

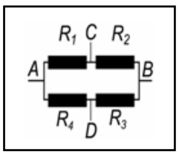
In both types of these cases random errors are estimated from commonly known equations of the square root form. If random errors $\bar{\delta}_{Ri}$ of all resistances and $\bar{\delta}_{i0}$, $\bar{\Delta}_{\varepsilon i}$ of their components are not correlated then it is

$$\bar{\Delta}_{r21}\equiv\sqrt{\sum w^2_{Ri}\left(\bar{\delta}_{Ri}\right)^2}=\sqrt{\sum_{i=1}^{4}w^2_i\left[(1+\varepsilon_i)^2\left(\bar{\delta}_{i0}\right)^2+\left(\bar{\Delta}_{\varepsilon i}\right)^2\right]} \tag{13}$$

where:- $\bar{\Delta}_{r21}$, $\bar{\delta}_{Ri}$, $\bar{\delta}_{i0}$, $\bar{\Delta}_{\varepsilon i}$ - standard mean value measurement errors or uncertainties.

Random error $\bar{\Delta}_{r21}$ of the electrical circuit is commonly smaller then its limited one $\lvert\Delta_{r21}\rvert_m$. In the accuracy evaluation for series of transmitters in production or in measurements made by set of instruments may be assumed that systematic errors of particular examples are random and formula (13) could be used in calculations with limited level of confidence.

D. Some specific cases of four arm resistance measures



In the case of testing the arbitrary circuit as twoport by measuring changes of its terminal parameters, initial values, increments and measures of its equivalent circuit elements could be arbitrary and initial stage is not necessary the balanced one.

On other hand bridges used in measurement instrumentation have specially chosen nominal values of resistances in manner to obtain require balance state and simpler output characteristic. Even if their equations are simpler that general one, this one has to be used to find proper relations of bridge accuracy measures. Limited errors or uncertainties of resistances of the same type, nominal value and accuracy should be generally considered as independent; otherwise obtained formulas could be not proper. Any constant resistance, i.e. of ε_i=0, has only initial error δ_0 in general smaller then for sensors.

1[0] Equal instantaneous errors $\delta_{R\,i}$ of all resistances
All terminal parameters have also the same error.

2[0] Negligible increment errors $\Delta_{\varepsilon i}\rightarrow 0$, equal all limited ones $|\delta_{i0}|=|\delta_0|$

Errors of resistance R_i are equal to initial ones, i.e. $\delta_{Ri}=\delta_{i0}$ and if $\delta_{i0}\neq 0$ it is

$$\Delta_{r21}=\sum w_{R\,i}\,\delta_{i0} \tag{14}$$

For equal limited errors of all arm resistances $|\delta_{i0}|=|\delta_0|$ absolute limited error of r_{21} is:

$$\left|\Delta_{r21}\right|_m=\left(\sum\left|w_{R\,i}\right|\right)\left|\delta_0\right| \tag{15}$$

and from (11)

$$\left|\Delta_{r21}\right|_m=\frac{\left|2\left(R_1R_3+R_2R_4\right)+r_{21}\left(R_2+R_4-R_1-R_3\right)\right|}{\sum R_i}\left|\delta_0\right| \tag{16}$$

or after inserting r_{21} from (1a):

$$\left|\Delta_{r21}\right|_m=\frac{1}{\sum R_i}\left|\left(3-2\frac{R_1+R_3}{\sum R_i}\right)R_1R_3+\left(3-2\frac{R_2+R_4}{\sum R_i}\right)R_2R_4\right|\left|\delta_0\right| \tag{17}$$

Initial error of the balanced bridge is

$$\left|\Delta_{r21}\right|_{m0}=4\left|t_0\right|\left|\delta_0\right| \tag{18}$$

3[0] Equal limited errors of all initial values and of all increments of arm resistances

If increments ε_i are of arbitrary values and $\left|\delta_{i0}\right|=\left|\delta_0\right|$, $\left|\Delta_{\varepsilon i}\right|=\left|\Delta_\varepsilon\right|$ then from (5), (11) and (11a, b):

$$\Delta_{21}=\left(\sum\left|w_{Ri}\right|\right)\left|\delta_0\right|+\left(\sum w_i\right)\left|\Delta_\varepsilon\right| \tag{19}$$

and when these increments are not to high, i.e. if $\varepsilon_i\varepsilon_j\leq 0{,}2$, then with accuracy satisfied for error estimation:

$$\left|\Delta_{r21}\right|_m\approx 4t_0\left[\frac{\left|1+\sum\varepsilon_i\right|}{\left(1+\varepsilon_{\Sigma R}\right)^2}\left|\delta_0\right|+\left|\Delta_\varepsilon\right|\right] \tag{20}$$

Measures for different cases of the 4R resistance bridge are discussed in details and tabularized in [1, 6, 7] and part 2.

to be continued in part 2

References (to part 1 and 2)

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Accuracy measures of the four arm bridge of broadly variable resistances
Part 2. Bridge of double and single sensors with Pt100 as example

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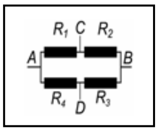
Abstract. In the part 1 the open circuit voltage of 4R bridge supplied by current or voltage are given and normalized forms of its transmittances are defined. Their accuracy measures for the general case - broadly and independently variable all bridge arm resistances have been considered. Literature to both parts is included.

In this part 2, measures of the particular cases of 4R bridge, i.e.: of joined four arm increments, different pairs of them and variable single arm only, are discussed. Some of valuable examples and corresponding to them measures of both transmittances of 4R bridges are given in table 2. As the example limited errors of the bridge including industrial Pt100 temperature sensors of class A and B tolerances are calculated for four cases of circuit zero adjustment. Some final conclusions are included.

V. Jointed increments of four arm resistances $\pm\varepsilon$



In the bridge of opposite increments $\pm\varepsilon$ in neighboring arms, i.e. $\varepsilon_i = (-1)^{i+1}\varepsilon$ and $|\varepsilon_i| \equiv \varepsilon \le 1$, instantaneous error of r_{21} transmittance could be obtained from (11) as

$$\delta_{r21} = \frac{(1+\varepsilon)^2(\delta_{R1}+\delta_{R3})-(1-\varepsilon)^2(\delta_{R2}+\delta_{R4})}{1+a\varepsilon} + -4\varepsilon\frac{(1+\varepsilon)(\delta_{R1}+mn\delta_{R3})+(1-\varepsilon)(m\delta_{R2}+n\delta_{R4})}{(1+a\varepsilon)^2(1+m)(1+n)} \quad (21)$$

where: $a \equiv \frac{(1-m)(1-n)}{(1+m)(1+n)}$, $a \in (-1,+1)$, $\varepsilon_{\Sigma R} = \varepsilon a$.

If in this bridge limited errors of both components are equal for all resistances R_i, i.e. $|\delta_{i0}| = |\delta_0|$ and $|\Delta_{\varepsilon i}| = |\Delta_\varepsilon|$ is

$$|\Delta_{r21}|_m = 4|t_0|\left[\frac{|1+a\varepsilon^3|}{(1+a\varepsilon)^2}|\delta_0| + \frac{|\Delta_\varepsilon|}{(1+a\varepsilon)^2}\right] \quad (22)$$

Coefficient a as function of arm resistance ratios m and n, is given on Fig 4a.

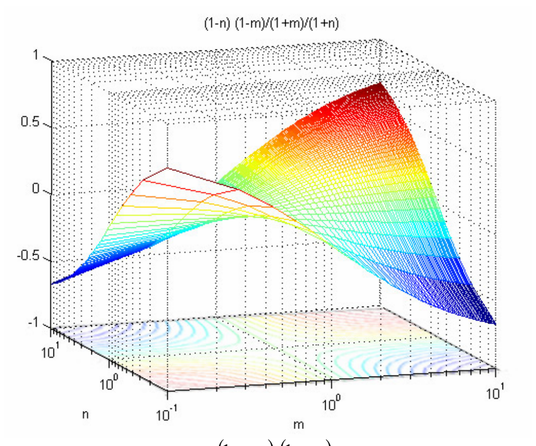


Fig 4a Coefficient $a \equiv \frac{(1-m)(1-n)}{(1+m)(1+n)}$ as function of arm initial resistance ratios m and n of the 4R bridge.

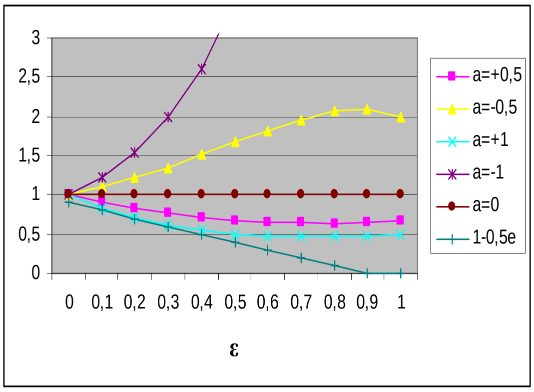


Fig 4b Coefficient $\frac{1+a\varepsilon^3}{(1+a\varepsilon)^2}$ of equation (30) of limited error of 4R bridge transmittance r_{21} for increments $\pm\varepsilon$, limited errors $|\delta_{i0}|=|\delta_0|$ and $|\Delta_{\varepsilon i}|=0$

The same value of a is possible to obtain for many pairs of m and n laying on the hyperbolic function. Sign of a is positive when m>1 and n>1 or m<1 and n<1. If m=1 or n=1 then a=0. If $|\Delta_{\varepsilon i}| \to 0$ related limited error $|\delta_{r21}|_m$ depends only on coefficient $\frac{1+a\varepsilon^3}{(1+a\varepsilon)^2}$ of initial error $|\delta_0|$ term. Its curves for different a as parameter are given on Fig 4b. If a<0 this error is strongly raising with ε. Coefficient of $|\Delta_{\varepsilon i}|$ is always positive and increasing with a and ε. Errors in ranges $a\varepsilon \le$0,4 and ε≤0,4, dependently on required accuracy could be calculated from simplified following forms

$$\left|\Delta_{r21}\right|_m \approx 4|t_0|\left[\left|1-2a\varepsilon+3a^2\varepsilon^2\right|\left|\delta_0\right|+\left|1-2a\varepsilon\right|\left|\Delta_\varepsilon\right|\right]\approx 4|t_0|\left|1-2a\varepsilon\right|\left(\left|\delta_0\right|+\left|\Delta_\varepsilon\right|\right) \tag{23}$$

It is possible to find similar curves for limited error coefficients for other particular cases of 4R bridge, e.g. $\left|\delta_{k21}\right|_m$ of the voltage bridge transmittance k_{21} - column b) of Table 2 as function of ε and single parameter m only, because n is here arbitrary.

For the bridge of $\pm\varepsilon$ increments and of one axis of initial resistance symmetry, i.e. if m=1 or n=1 is a=0, $\varepsilon_{\Sigma R}=0$ and

$$\left|\Delta_{r21}\right|_m = 4t_0\left(\left|\delta_0\right|+\left|\Delta_\varepsilon\right|\right) \tag{24}$$

where: $t_0 = R_{10}\dfrac{n}{2(1+n)}$ or $t_0 = R_{10}\dfrac{m}{2(1+m)}$

Absolute limited error is rising linearly with $4t_0$.

For bridge of two axes of symmetry in balance (m=1=n), i.e. for similar initial resistances $R_{i0}=R_{10}$ and increments $\varepsilon_i=\pm\varepsilon$, equation (22) is simplified to

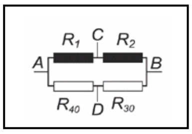
$$\left|\Delta_{r21}\right|_m = R_{10}\left(\left|\delta_0\right|+\left|\Delta_\varepsilon\right|\right) \tag{25}$$

and in balance ($\Delta_\varepsilon = 0$)

$$\left|\Delta_{r21}\right|_{m0} = R_{10}\left|\delta_0\right| \tag{26}$$

Related random measures of both transmittances r_{21} and k_{21} for 4R bridge of four variable arm $\pm\varepsilon$ increments and similar random measures of all resistances are in line 6 and 7 of Table 2. It is also possible to find its coefficients and functions of ε similar as on Fig 4a,b.

VI. Opposite increments $\pm\varepsilon$ of two neighboring resistances only



The bridge circuit of two variable resistances R_1, R_2 is applied for measurements of increment difference or sum of them if they have opposite signs. If increments are so jointed that ε_1= -$\varepsilon_2\equiv\varepsilon$ ($\varepsilon_3=\varepsilon_4=0$) from (11) it is

$$\Delta_{r21} = \frac{t_0}{1+\varepsilon_{\Sigma R}}\left[(1+\varepsilon)^2\delta_{10}+(1+\varepsilon)\Delta_{\varepsilon1}-(1-\varepsilon)^2\delta_{20}-(1-\varepsilon)\Delta_{\varepsilon2}+\delta_{30}-\delta_{40}-2\varepsilon\delta_{R\Sigma}\right] \tag{27}$$

where: $\varepsilon_{\Sigma R} = \varepsilon\dfrac{1-m}{(1+m)(1+n)}$; $\delta_{R\Sigma} = \dfrac{\delta_1+mn\delta_{30}+m\delta_2+n\delta_{40}}{(1+m)(1+n)(1+\varepsilon_{\Sigma R})}$

If m=n=1, i.e. all resistances of initial bridge balance are equal, then $\varepsilon_{\Sigma R}$=0, $\delta_{R\Sigma} = \frac{1}{4}\sum\delta_i$ and

$$\Delta_{r21} = \frac{R_{10}}{4}\left\{(1-0{,}5\varepsilon)\left[(1+\varepsilon)\delta_{10}+\Delta_{\varepsilon1}-\delta_{40}\right]+(1-0{,}5\varepsilon)\left[(1-\varepsilon)\delta_{20}+\Delta_{\varepsilon2}-\delta_{30}\right]\right\} \tag{28}$$

Conclusion: Compensation of the open circuit transmittance error of the 4R bridge of resistance increments $\pm\varepsilon$ of two its arm R_1, R_2, depends on ε even if all its resistances in balance are equal.

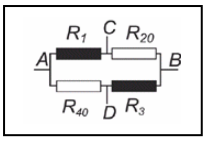
Limited error or random measure couldn't be obtained from (27) directly as error $\delta_{R\Sigma}$ of the sum of bridge resistances is not independent from arm errors. These measures have to be estimated as particular cases of general formulas (8), (11) of part 1 and Table 1 in it. If m=1, n=1, $\varepsilon_1=\varepsilon$=- ε_2 and $\left|\delta_{10}\right| = \left|\delta_{20}\right|$, $\left|\delta_{30}\right| = \left|\delta_{40}\right|$, $\left|\Delta_{\varepsilon1}\right| = \left|\Delta_{\varepsilon2}\right| \equiv \left|\Delta_\varepsilon\right|$ from (28)

$$\left|\Delta_{r21}\right|_m = \frac{R_{10}}{2}\left[\left(1-0{,}5\,\varepsilon^2\right)\left|\delta_{10}\right|+\left|\Delta_\varepsilon\right|+\left|\delta_{30}\right|\right] \tag{29}$$

and random measure is

$$\overline{\Delta}_{r21} = \frac{\sqrt{2}R_{10}}{4}\sqrt{\left(1-\varepsilon^2\right)^2\overline{\delta}_{10}^2+\overline{\delta}_{30}^2+\left(1+\varepsilon^2\right)\overline{\Delta}_\varepsilon^2} \tag{30}$$

Accuracy measures for jointed opposite increments of arms R_1, R_4 are given in lines of Table2.



VII. Equal increments ε of opposite resistances R_1, R_3

For 4R bridge of two opposite arms variable from (9b) and (14) transmittance r_{21} and its relative error δ_{r21} are

$$r_{21} = t_0\frac{\varepsilon_1+\varepsilon_3+\varepsilon_1\varepsilon_3}{1+\varepsilon_{\Sigma R}}$$

(31a)

$$\delta_{r21} \equiv \frac{\Delta_{r21}}{t_0} = \frac{1}{1+\varepsilon_{\Sigma R}} \left\{ \left(1+\varepsilon_3 - \frac{r_{12}}{R_{30}}\right)\left[(1+\varepsilon_1)\delta_{10} + \Delta_{\varepsilon 1}\right] - \left(1+\frac{r_{12}}{R_{40}}\right)\delta_{20} + \left(1+\varepsilon_1 - \frac{r_{12}}{R_{10}}\right)\left[(1+\varepsilon_3)\delta_{30} + \Delta_{\varepsilon 3}\right] - \left(1+\frac{r_{12}}{R_{20}}\right)\delta_{40} \right\} \quad (31b)$$

where: $t_0 = \frac{mn}{(1+m)(1+n)} R_{10}$; $\varepsilon_{\Sigma R} = \frac{\varepsilon_1 + nm\,\varepsilon_3}{(1+m)(1+n)}$

This bridge is applied mainly if m=1 and $\varepsilon_1=\varepsilon_3=\varepsilon$, ($\varepsilon_2=\varepsilon_4=0$) then transmittance r_{21} linearly depends on ε [1], [2]

$$r_{21} = R_{10} \frac{n}{1+n} \varepsilon \quad (32a) \quad \text{and} \quad \varepsilon_{\Sigma R} = \frac{\varepsilon}{2} \quad (32b)$$

Positive increments of ε may be higher then 1 and are only limited by permissible dissipated power of sensors or maximum voltage of the current supply source. If also n=1 (two similar sensors: $R_{10}=R_{30}\equiv R_0$ and $\varepsilon_1=\varepsilon_3=\varepsilon$, then initial sensitivity $t_0=0{,}25R_o$ and error relations become very simply. Absolute error of bridge transmittance r_{21} is

$$\Delta_{r21} = \tfrac{1}{4} R_0 \left[(1+\varepsilon)(\delta_{10} + \delta_{30}) + \Delta_{\varepsilon 3} + \Delta_{\varepsilon 1} - \delta_{20} - \delta_{40} \right] \quad (33)$$

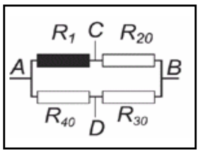
Relative error δ_{r21} with separated bridge zero error δ_{210} is

$$\delta_{r21} = \delta_{210} + \varepsilon\left(\delta_{10} + \delta_{30} + \delta_{\varepsilon 1} + \delta_{\varepsilon 3}\right) \quad (34)$$

If $\varepsilon \neq 0$ and $\delta_{10} \neq -\delta_{30}$ then initial errors δ_{10}, δ_{30} of R_1, R_3 influenced on δ_{r12} even when δ_{210}=0.
From (16) or (17) it is easy to find also limited and random errors of this bridge. If additionally $|\delta_{10}|=|\delta_{30}|$, $|\delta_{20}|=|\delta_{40}|$and $|\delta_{\varepsilon 1}| = |\delta_{\varepsilon 2}|$ then

$$|\delta_{r21}|_m = 2\left[(1+\varepsilon)|\delta_{10}| + \varepsilon|\delta_{\varepsilon 1}| + |\delta_{20}|\right] \quad (18) \quad \text{and} \quad \bar{\delta}_{r21} = \sqrt{2}\sqrt{(1+\varepsilon)\bar{\delta}_{10}^2 + \bar{\Delta}_{\varepsilon 1}^2 + \bar{\delta}_{20}^2} \quad (35)$$

VIII. Bridge of one variable resistance



In temperature measurements the most popular are bridges applying only one variable resistance, e.g. R_1. If ε_2=0, ε_3=0, ε_4=0, transmittance is hyperbole $r_{21} = t_0 \frac{\varepsilon_1}{1+\varepsilon_{\Sigma R}}$ of asymptotes: $r_{12} = m\,nR_{10}$, ε = -(1+m)(1+n), and its error equation is

$$\delta_{r21} \equiv \frac{\Delta_{r12}}{t_0} = \frac{1}{1+\varepsilon_{\Sigma R}} \left\{ \left(1-\frac{r_{12}}{R_{30}}\right)\left[(1+\varepsilon_1)\delta_{10} + \Delta_{\varepsilon 1}\right] - \left(1+\frac{r_{12}}{R_{40}}\right)\delta_{20} + \left(1+\varepsilon_1 - \frac{r_{12}}{R_{10}}\right)\delta_{30} - \left(1+\frac{r_{12}}{R_{20}}\right)\delta_{40} \right\} \quad (36)$$

or

$$\delta_{r21} = \frac{(1+\varepsilon_1)(\delta_{10}+\delta_{30}) + \varepsilon_1\delta_{\varepsilon 1} - (\delta_{20}+\delta_{40}) - \varepsilon_1\delta_{\Sigma R}}{1+\varepsilon_{\Sigma R}} \quad (37a)$$

where: $\delta_{\Sigma R} = \delta_{\Sigma R0} + \delta_{\varepsilon 1}\frac{\varepsilon_{\Sigma R}}{1+\varepsilon_{\Sigma R}}$-, $\delta_{\Sigma R0} = \frac{\sum R_{i0}\delta_{i0}}{\sum R_{i0}}$, $\varepsilon_{\Sigma R} = \frac{\varepsilon_1}{(1+m)(1+n)}$ - relative errors of ΣR_i and ΣR_{i0} and increment $\varepsilon_{\Sigma R}$

After separation of δ_{210} relation (37a) may be written in other way as

$$\delta_{r21} = \frac{1}{1+\varepsilon_{\Sigma R}}\left[\delta_{210} + \varepsilon_1\left(\frac{\delta_{\varepsilon 1}}{1+\varepsilon_{\Sigma R}} + \delta_{10} + \delta_{30} - \delta_{\Sigma R0}\right)\right] \quad (37b)$$

where: $\delta_{210} = \delta_{10} + \delta_{30} - (\delta_{20} + \delta_{40})$ - relative initial error of transmittance r_{21}=0 (bridge zero output signal).

Also initial error $\delta_{\Sigma R0}$ of ΣR_{i0} depends on δ_{i0}.
Errors of few zero adjustment cases of the measurement circuit channel are given below.

A. Accuracy measures of the single variable arm 4R$_0$ bridge

For of equal R_{i0}, $\varepsilon_{\Sigma R} = \frac{1}{4}\varepsilon_1$, $\delta_{\Sigma R0} = \Sigma\delta_{i0}$ and from (36) or (37a) is

$$\delta_{r21} = \frac{1}{\left(1+\frac{1}{4}\varepsilon_1\right)^2}\left[(1+\varepsilon_1)\delta_{10} + \left(1+\tfrac{1}{2}\varepsilon_1\right)^2\delta_{30} - \left(1+\tfrac{1}{2}\varepsilon_1\right)(\delta_{20}+\delta_{40}) + \Delta_{\varepsilon 1}\right] \quad (38)$$

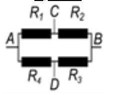
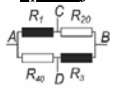
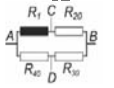
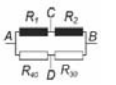
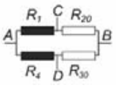
After subtraction of initial error δ_{210} by external compensation of it is

$$\delta_{r21} = \frac{\varepsilon_1}{1+\frac{1}{4}\varepsilon_1}\left[\frac{\delta_{\varepsilon 1}}{1+\frac{1}{4}\varepsilon_1} + \frac{1}{2}(\delta_{10} + \delta_{30})\right] \quad (39)$$

If the bridge zero δ_{210}=0 is adjusted by its resistances, then $\delta_{10}+\delta_{30}=\delta_{20}+\delta_{40}$ and

$$\delta_{r21}=\frac{\varepsilon_1}{1+\frac{1}{4}\varepsilon_1}\left[\frac{\delta_{\varepsilon1}}{1+\frac{1}{4}\varepsilon_1}-\frac{1}{2}\left(\delta_{10}+\delta_{30}\right)\right] \tag{40}$$

Table 2. Accuracy measures of transmittances r_{21} and k_{21} of the unbalanced four arm resistance (4R) bridge - particular cases

No	Type of bridge	Errors	a) Measures δ_{r21}, $\lvert\delta_{r21}\rvert$, $\bar\delta_{r21}$	if $R_{i0}=R_{10}$, i.e.: $m=n=1$		b) Measures δ_{k21}, $\lvert\delta_{k21}\rvert$, $\bar\delta_{k21}$	if $R_{20}=R_{10}$, i.e.: $m=1$	
3	**Variable 4R** $\varepsilon_i=\pm\varepsilon$ 	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$, $\lvert\Delta_{\varepsilon i}\rvert=\lvert\Delta_\varepsilon\rvert$	$\lvert\delta_{r21}\rvert$		$4\frac{(1+a\varepsilon^3)(\lvert\delta_0\rvert+\lvert\Delta_\varepsilon\rvert)}{(1+a\varepsilon)^2}\approx 4(1-2a\varepsilon)(\lvert\delta_0\rvert+\lvert\Delta_\varepsilon\rvert)$ where: $a\equiv\frac{(1-m)(1-n)}{(1+m)(1+n)}$	$\lvert\delta_{k21}\rvert$		$4\frac{1+b^2\varepsilon^2}{(1-b^2\varepsilon^2)^2}\left[(1-\varepsilon^2)\lvert\delta_0\rvert+\lvert\Delta_\varepsilon\rvert\right]$ where: $b\equiv\frac{1-m}{1+m}$
4				$m=1$ $n=1$	$=4\left[\lvert\delta_0\rvert+\lvert\Delta_\varepsilon\rvert\right]$		$m=1$	$4\left[(1-\varepsilon^2)\lvert\delta_0\rvert+\lvert\Delta_\varepsilon\rvert\right]$
5	$R_1=R_{10}(1+\varepsilon)$		δ_{r21}	$r_{21}=R_{10}\varepsilon$ $a=0$	$(1+\varepsilon)(\delta_{10}+\delta_{30})-(1-\varepsilon)(\delta_{20}+\delta_{40})+\Delta_{\varepsilon1}+\Delta_{\varepsilon3}-\Delta_{\varepsilon2}-\Delta_{\varepsilon4}$	δ_{k21}	$k_{21}=\varepsilon$ $b=0$	$(1-\varepsilon^2)\left(\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}+\frac{\Delta_{\varepsilon1}+\Delta_{\varepsilon3}}{1+\varepsilon}-\frac{\Delta_{\varepsilon2}+\Delta_{\varepsilon4}}{1-\varepsilon}\right)$
6	$R_2=mR_{10}(1-\varepsilon)$ $R_3=mnR_{10}(1+\varepsilon)$	$\bar\delta_{i0}=\bar\delta_0$, $\bar\Delta_{\varepsilon i}=\bar\Delta_\varepsilon$	$\bar\delta_{r21}$		$2\sqrt{(1+\varepsilon^2)\bar\delta_0^2+\bar\Delta_\varepsilon^2}$	$\bar\delta_{k21}$		$2\sqrt{(1-\varepsilon^2)^2\bar\delta_0^2+(1+\varepsilon^2)\bar\Delta_\varepsilon^2}$
7	$R_4=nR_{10}(1-\varepsilon)$ $\Delta L=4\varepsilon$, $\varepsilon_{\Sigma R}=a\varepsilon$				$\sqrt{\bar\delta_0^2\sum_{i=1}^{4}w'^2_{Ri}+\bar\Delta_\varepsilon^2\sum_{i=1}^{4}w'^2_{Ri}\frac{1}{(1+\varepsilon_i)^2}}$			$2(1-\varepsilon^2)\frac{\sqrt{(1+b^2\varepsilon^2)^2+4b^2\varepsilon^2}}{(1-b^2\varepsilon^2)^2}\sqrt{\bar\delta_0^2+\frac{1+\varepsilon^2}{(1-\varepsilon^2)^2}\bar\Delta_\varepsilon^2}$
8	**Variable R_1, R_3** $\varepsilon_1=\varepsilon=\varepsilon_3$ 	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$ $\lvert\Delta_{\varepsilon i}\rvert=\lvert\Delta_\varepsilon\rvert$	$\lvert\delta_{r21}\rvert$		$\frac{2}{(1+a''\varepsilon)^2}\left\{\left[1+\varepsilon+0{,}5a''\varepsilon^2\right]\left[(1+\varepsilon)\lvert\delta_{10}\rvert+\lvert\Delta_\varepsilon\rvert\right]+\left[1+\varepsilon+0{,}5\varepsilon^2(1-a'')\right]\lvert\delta_{20}\rvert\right\}$	$\lvert\delta_{k21}\rvert$		$\left[\left(\frac{1+m}{1+m+\varepsilon}\right)^2+\left(\frac{1+m}{1+m+m\varepsilon}\right)^2\right]\left[(1+\varepsilon)(\lvert\delta_{10}\rvert+\lvert\delta_{20}\rvert)+\lvert\Delta_\varepsilon\rvert\right]$
9				$m=1$ $n=1$	$\lvert\delta_{r21}\rvert_m=2\left[(2+\varepsilon)\lvert\delta_0\rvert+\lvert\Delta_\varepsilon\rvert\right]$		$m=1$	$\lvert\delta_{k21}\rvert_m=2\frac{1+\varepsilon}{(1+0{,}5\varepsilon)^2}\left(2\lvert\delta_0\rvert+\frac{\lvert\Delta_\varepsilon\rvert}{1+\varepsilon}\right)$
10	$R_1=R_{10}(1+\varepsilon)$		δ_{r21}	$r_{21}=\frac{1}{2}R_{10}\varepsilon$ $a''=0{,}5$	$(1+\varepsilon)(\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40})+\Delta_{\varepsilon1}+\Delta_{\varepsilon3}$	δ_{k21}	$k_{21}=\frac{\varepsilon}{2(1+\frac{1}{2}\varepsilon)}$	$\frac{(1+\varepsilon)(\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40})+\Delta_{\varepsilon1}+\Delta_{\varepsilon3}}{(1+0{,}5\varepsilon)^2}$
11	$R_{20}=mR_{10}$, $R_{40}=nR_{10}$ $R_3=mnR_{10}(1+\varepsilon)$	$\bar\delta_{i0}=\bar\delta_0$, $\bar\Delta_{\varepsilon i}=\bar\Delta_\varepsilon$	$\bar\delta_{r21}$		$\bar\delta_{r21}=2\sqrt{(1+\varepsilon)^2\bar\delta_0^2+0{,}5\bar\Delta_\varepsilon^2}$	$\bar\delta_{k21}$		$\bar\delta_{k21}=2\frac{1+\varepsilon}{(1+0{,}5\varepsilon)^2}\sqrt{2\bar\delta_0^2+\frac{\bar\Delta_\varepsilon^2}{(1+\varepsilon)^2}}$
12	$\Delta L=2\varepsilon+\varepsilon^2$ $a''\equiv\frac{1+mn}{(1+m)(1+n)}$ $\varepsilon_{\Sigma R}=a''\varepsilon$				$\frac{1}{1+a''\varepsilon}\sqrt{\left[\left(1+\varepsilon-\frac{r_{12}}{R_{30}}\right)^2+\left(1+\varepsilon-\frac{r_{12}}{R_{10}}\right)^2\right]\left[(1+\varepsilon)^2\bar\delta_{10}^2+\bar\Delta_\varepsilon^2\right]+\left[\left(1+\frac{r_{12}}{R_{40}}\right)^2+\left(1+\frac{r_{12}}{R_{20}}\right)^2\right]\bar\delta_{20}^2}$			$\left[\left(\frac{1+m}{1+m+\varepsilon}\right)^2+\left(\frac{1+m}{1+m+m\varepsilon}\right)^2\right]\sqrt{(1+\varepsilon)^2\left(\bar\delta_{10}^2+\bar\delta_{20}^2\right)+\bar\Delta_\varepsilon^2}$
13	**Variable only R_1 ε_1** 	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$, $\lvert\Delta_{\varepsilon i}\rvert=\lvert\Delta_\varepsilon\rvert$	$\lvert\delta_{r21}\rvert$		$\frac{1}{(1+\varepsilon_{\Sigma R})^2}\left\{\left[4+2\varepsilon_1+(3+m+n-mn+\varepsilon_1)\varepsilon_{\Sigma R}\right]\lvert\delta_0\rvert+\lvert\Delta_{\varepsilon1}\rvert\right\}$	$\lvert\delta_{k21}\rvert$		$\frac{(1+\varepsilon_1)(\lvert\delta_{10}\rvert+\lvert\delta_{20}\rvert)+\lvert\Delta_{\varepsilon1}\rvert}{(1+\frac{1}{1+m}\varepsilon_1)^2}+\lvert\delta_{30}\rvert+\lvert\delta_{40}\rvert$
14				$m=1$ $n=1$	$\frac{1}{(1+\frac{1}{4}\varepsilon_1)^2}\left[(4+3\varepsilon_1+\frac{1}{4}\varepsilon_1^2)\lvert\delta_0\rvert+\lvert\Delta_{\varepsilon1}\rvert\right]$		$m=1$	$\frac{(1+\varepsilon_1)(\lvert\delta_{10}\rvert+\lvert\delta_{20}\rvert)+\lvert\Delta_{\varepsilon1}\rvert}{(1+0{,}5\varepsilon_1)^2}+2\lvert\delta_{20}\rvert$
15	$R_1=R_{10}(1+\varepsilon_1)$		δ_{r21}	$r_{21}=\frac{R_{10}}{4}\frac{\varepsilon}{1+\frac{1}{4}\varepsilon_1}$	$\frac{1}{(1+\frac{1}{4}\varepsilon_1)^2}\left[(1+\varepsilon_1)\delta_{10}+\Delta_{\varepsilon1}+\left(1+\frac{1}{2}\varepsilon_1\right)^2\delta_{30}-\left(1+\frac{1}{2}\varepsilon_1\right)(\delta_{20}+\delta_{40})\right]$	δ_{k21}	$k_{21}=\frac{\varepsilon}{4(1+\frac{1}{2}\varepsilon)}$	$\frac{(1+\varepsilon_1)(\delta_{10}-\delta_{20})+\Delta_{\varepsilon1}}{(1+0{,}5\varepsilon_1)^2}+\delta_{30}-\delta_{40}$
16	$R_{20}=mR_{10}$, $R_{40}=nR_{10}$ $R_{30}=mnR_{10}$ $r_{21}=t_0\frac{\varepsilon_1}{1+\varepsilon_{\Sigma R}}$ $\varepsilon_{\Sigma R}=\frac{\varepsilon_1}{\Sigma r_{i0}}$	$\bar\delta_{i0}=\bar\delta_0$, $\bar\Delta_{\varepsilon i}=\bar\Delta_\varepsilon$	$\bar\delta_{r21}$		$\frac{1}{(1+0{,}25\varepsilon_1)^2}\sqrt{(1+\varepsilon_1)^2\bar\delta_{10}^2+\bar\Delta_{\varepsilon1}^2+\left(1+\frac{1}{2}\varepsilon_1\right)^2\left[2+\left(1+\frac{1}{2}\varepsilon_1\right)^2\right]\bar\delta_{20}^2}$	$\bar\delta_{k21}$		$\sqrt{\frac{(1+\varepsilon_1)^2\left(\bar\delta_{10}^2+\bar\delta_{20}^2\right)+\bar\Delta_{\varepsilon1}^2}{(1+0{,}5\varepsilon_1)^4}+2\bar\delta_{20}^2}$
17	$\Delta L=\varepsilon_1$ $\Sigma r_{i0}=(1+m)(1+n)$				$\frac{1}{(1+\varepsilon_{\Sigma R})^2}\sqrt{(1+\varepsilon_1)^2\bar\delta_{10}^2+\bar\Delta_{\varepsilon1}^2+\left\{\begin{matrix}[1+(1+m)\varepsilon_{\Sigma R}]^2+\\+[1+\varepsilon_1+(1-mn+\varepsilon_1)\varepsilon_{\Sigma R}]^2+[1+(1+n)\varepsilon_{\Sigma R}]^2\end{matrix}\right\}\bar\delta_{20}^2}$			$\sqrt{\frac{(1+\varepsilon_1)^2\left(\bar\delta_{10}^2+\bar\delta_{20}^2\right)+\bar\Delta_{\varepsilon1}^2}{(1+\frac{1}{1+m}\varepsilon_1)^4}+2\bar\delta_{20}^2}$
18	**Variable R_1, R_2** $\varepsilon_1=\varepsilon=-\varepsilon_2$ 	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$ $\lvert\Delta_{\varepsilon i}\rvert=\lvert\Delta_\varepsilon\rvert$	$\lvert\delta_{r21}\rvert$		$\frac{2}{(1+a'\varepsilon)^2}\left[\left(1+0{,}5a'\varepsilon-\frac{\varepsilon^2}{1+n}\right)\lvert\delta_{10}\rvert+(1-0{,}5a'\varepsilon)\lvert\Delta_\varepsilon\rvert+(1+0{,}5a'\varepsilon++0{,}5\,na'\varepsilon)\lvert\delta_{30}\rvert\right]$ where: $a'\equiv\frac{1-m}{(1+m)(1+n)}$	$\lvert\delta_{k21}\rvert$		$\frac{2}{(1+b'\varepsilon)^2}\left[(1-\varepsilon^2)\lvert\delta_{10}\rvert+\lvert\varepsilon\rvert\lvert\delta_\varepsilon\rvert\right]+2\lvert\delta_{30}\rvert$ where: $b'\equiv\frac{1-m}{1+m}$
19				$m=1$ $n=1$	$(4-\varepsilon^2)\lvert\delta_0\rvert+2\lvert\Delta_\varepsilon\rvert$		$m=1$	$4\left[(1-0{,}5\varepsilon^2)\lvert\delta_0\rvert+0{,}5\lvert\Delta_\varepsilon\rvert\right]$
20	$R_1=R_{10}(1+\varepsilon)$ $R_2=mR_{10}(1-\varepsilon)$		δ_{r21}	$r_{21}=\frac{1}{2}R_{10}\varepsilon$ $\varepsilon_{\Sigma R}=0$ $a'=0$	$(1-0{,}5\varepsilon)\left[(1+\varepsilon)\delta_{10}+\Delta_{\varepsilon1}-\delta_{40}\right]-(1+0{,}5\varepsilon)\left[(1-\varepsilon)\delta_{20}+\Delta_{\varepsilon2}-\delta_{30}\right]$	δ_{k21}	$k_{21}=\frac{\varepsilon}{2}$	$(1-\varepsilon^2)(\delta_{10}-\delta_{20})+(1-\varepsilon)\Delta_{\varepsilon1}+(1+\varepsilon)\Delta_{\varepsilon2}+\delta_{30}-\delta_{40}$
21	$R_3=mnR_{10}$, $R_4=nR_{10}$ $\Delta L=2\varepsilon$ $\varepsilon_{\Sigma R}=a'\varepsilon$	$\bar\delta_{i0}=\bar\delta_0$, $\bar\Delta_{\varepsilon i}=\bar\Delta_\varepsilon$	$\bar\delta_{r21}$		$2\sqrt{\left[1+(1-\varepsilon^2)^2\right]\bar\delta_0^2+(1+0{,}25\varepsilon^2)\bar\Delta_{\varepsilon1}^2}$	$\bar\delta_{k21}$		$\sqrt{2}\sqrt{\left[1+(1-\varepsilon^2)^2\right]\bar\delta_0^2+(1+\varepsilon^2)\bar\Delta_\varepsilon^2}$
22	**Variable R_1, R_4** $\varepsilon_1=\varepsilon=-\varepsilon_4$ 	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$ $\lvert\Delta_{\varepsilon i}\rvert=\lvert\Delta_\varepsilon\rvert$	$\lvert\delta_{r21}\rvert$		$2\frac{(1+0{,}5a'''\varepsilon-\frac{1}{1+m}\varepsilon^2)\lvert\delta_{10}\rvert+(1-0{,}5a'''\varepsilon)\lvert\Delta_\varepsilon\rvert+\left[1+0{,}5a'''\varepsilon(1+m)\right]\lvert\delta_{20}\rvert}{(1+a'''\varepsilon)^2}$	$\lvert\delta_{k21}\rvert$		$\frac{(1+\varepsilon)(\lvert\delta_{10}\rvert+\lvert\delta_{20}\rvert)+\lvert\Delta_\varepsilon\rvert}{(1+\frac{1}{1+m}\varepsilon)^2}+\frac{(1-\varepsilon)(\lvert\delta_{10}\rvert+\lvert\delta_{20}\rvert)+\lvert\Delta_\varepsilon\rvert}{(1-\frac{1}{1+m}\varepsilon)^2}$
23				$m=1$ $n=1$	$4\left[(1-0{,}25\varepsilon^2)\lvert\delta_0\rvert\right]+2\lvert\Delta_\varepsilon\rvert$ where: $a'''\equiv\frac{1-n}{(1+m)(1+n)}$		$m=1$	$\frac{4(1-0{,}75\varepsilon^2)\lvert\delta_{10}\rvert+2(1+0{,}25\varepsilon^2)\lvert\Delta_\varepsilon\rvert}{(1-0{,}25\varepsilon^2)^2}$
24	$R_1=R_{10}(1+\varepsilon)$		δ_{r21}	$r_{21}=\frac{1}{2}R_{10}\varepsilon$ $\varepsilon_{\Sigma R}=0$ $a'''=0$	$(1-0{,}5\varepsilon)\left[(1+\varepsilon)\delta_{10}+\Delta_{\varepsilon1}-\delta_{20}\right]-(1+0{,}5\varepsilon)\left[(1-\varepsilon)\delta_{40}+\Delta_{\varepsilon4}-\delta_{30}\right]$	δ_{k21}	$k_{21}=\frac{\varepsilon}{2(1-\frac{1}{4}\varepsilon^2)}$	$\frac{(1+\varepsilon)(\delta_{10}-\delta_{20})+\Delta_{\varepsilon1}}{(1+0{,}5\varepsilon)^2}+\frac{(1-\varepsilon)(\delta_{30}-\delta_{40})-\Delta_{\varepsilon1}}{(1-0{,}5\varepsilon)^2}$
25	$R_{20}=mR_{10}$, $R_{30}=mnR_{10}$ $R_4=nR_{10}(1-\varepsilon)$ $\Delta L=2\varepsilon$ $\varepsilon_{\Sigma R}=a'''\varepsilon$	$\bar\delta_{i0}=\bar\delta_0$, $\bar\Delta_{\varepsilon i}=\bar\Delta_\varepsilon$	$\bar\delta_{r21}$		$2\sqrt{\left[1-0{,}25\varepsilon^2+0{,}125\varepsilon^4\right]\bar\delta_0^2+0{,}5(1+0{,}25\varepsilon^2)\bar\Delta_\varepsilon^2}$	$\bar\delta_{k21}$		$2\sqrt{\frac{(1-0{,}75\varepsilon^2)^2\bar\delta_0^2+0{,}5(1+0{,}25\varepsilon^2)\bar\Delta_\varepsilon^2}{(1-0{,}25\varepsilon^2)^2}}$
26	**Accuracy in balance:** (r_{21}=0)	**instantaneous initial error**			$\delta_{210}=\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}$	**limited initial error** $\lvert\delta_{210}\rvert_m=\sum\lvert\delta_{i0}\rvert$	**initial uncertainty**	$\bar\delta_{210}=\sqrt{\sum\bar\delta_{i0}^2}$

Limited errors and random measures of the single variable arm bridge could be established as particular cases of (11) and (13) or on transformation of (38) - (40). If the sensor initial resistance is equal to other arm resistances and their limited errors are equal, i.e. $R_{i0}=R_0$, $|\delta_{i0}|=|\delta_0|$, then relative limited error of r_{21} of the circuit without zero adjustment is

$$\left|\delta_{r21}\right|_m=\frac{\left[\left(4+3\varepsilon_1+\frac{1}{4}\varepsilon_1^2\right)\left|\delta_0\right|+\left|\varepsilon_1\right|\left|\delta_{\varepsilon1}\right|\right]}{\left(1+\frac{1}{4}\varepsilon_1\right)^2} \tag{41}$$

For external zero setting

$$\left|\delta_{r21}\right| = \frac{\left|\varepsilon_1\right|}{\left(1+0{,}25\varepsilon_1\right)^2}\left[\left|\delta_{\varepsilon 1}\right| + \left|\delta_0\right|\left(1+0{,}25\varepsilon_1\right)\right] \tag{42}$$

If. resistance R_3 is adjusted, the value of its limited error $|\delta_{30}|$ could reach even $3|\delta_0|$, but $|\delta_{10}+\delta_{30}|$ doesn't exceed $2|\delta_0|$.
Limited bridge transmittance error $|\delta_{r21}|_m$ of negligible initial resistance errors $|\delta_{i0}|$ and calibrated to the nominal characteristic of the sensor results from (41) and is

$$\left|\delta_{r21}\right|_m = \frac{\left|\varepsilon_1\right|}{\left(1+\varepsilon_{\Sigma R}\right)^2}\left|\ \delta_{\varepsilon 1}\right| \tag{43}$$

Relative random measure of the $4R_0$ bridge is

$$\bar{\delta}_{r21} = \frac{1}{\left(1+\frac{1}{4}\varepsilon_1\right)^2}\sqrt{\left(1+\varepsilon_1\right)^2\bar{\delta}_{10}^{2} + \left(1+\frac{1}{2}\varepsilon_1\right)^4\bar{\delta}_{30}^{2} - \left(1+\frac{1}{2}\varepsilon_1\right)^2\left(\bar{\delta}_{20}^{2}+\bar{\delta}_{40}^{2}\right) + \bar{\Delta}_{\varepsilon 1}^{2}} \tag{44}$$

when random measures of resistances are also equal, i.e. $\bar{\delta}_{i0} = \bar{\delta}_0$, then

$$\bar{\delta}_{r21} = \frac{1}{\left(1+\frac{1}{4}\varepsilon_1\right)^2}\sqrt{\left[\left(1+\varepsilon_1\right)^2 + \left(1+\frac{1}{2}\varepsilon_1\right)^4 + 2\left(1+\frac{1}{2}\varepsilon_1\right)^2\right]\bar{\delta}_0^{2} + \bar{\Delta}_{\varepsilon 1}^{2}} \tag{45}$$

B. Bridge errors for different zero adjustments

1^0 Bridge without adjustment. Measurement circuit has been calibrated to standard characteristic and sensors are changed without any adjustment. Limited error and random measure may be fund after transformations of (36) or (37b).

2^0 Bridge zero error externally compensated. It could be done by voltage opposite to zero signal or after converting it to digital form. From (37b)

$$\delta_{r21} = \frac{1}{1+\varepsilon_{\Sigma R}}\left[\varepsilon_1\left(\frac{\delta_{\varepsilon 1}}{1+\varepsilon_{\Sigma R}} + \delta_{10} + \delta_{30} - \delta_{\Sigma R 0}\right) - \varepsilon_{\Sigma R}\,\delta_{210}\right] \tag{46}$$

3^0. Zero adjustment in the bridge: $\delta_{210} = 0$

$$\delta_{r21} = \frac{\varepsilon_1}{1+\varepsilon_{\Sigma R}}\left(\frac{\delta_{\varepsilon 1}}{1+\varepsilon_{\Sigma R}} + \delta_{10} + \delta_{30} - \delta_{\Sigma R 0}\right) \tag{47}$$

In 2^0 and 3^0 cases after zero adjustment errors $\delta_{i0} \neq 0$ effect δ_{r21}.

4^0 Negligible initial arm errors $\delta_{i0} = 0$.

In this case $\delta_{210} = 0$ and $\delta_{\Sigma R0} = 0$, then on (37b)

$$\delta_{r21} = \frac{\varepsilon_1}{\left(1+\varepsilon_{\Sigma R}\right)^2}\delta_{\varepsilon 1} \tag{48}$$

IX. Accuracy analysis of the bridge with Pt 100 temperature sensors

A. Standardized tolerances of industrial Pt100 sensors

Platinum sensors Pt 100 of A and B industrial classes are commonly used in practice of temperature measurements. Tolerated differences from their nominal characteristic are given in standard PN-EN 60751÷A2 1997 [8]. They are expressed in ^{0}C or as permissible resistance values in [ohm] - see $|\Delta|_{maxA}$ and $|\Delta|_{maxB}$ on Fig 3.
Characteristic of class A sensors is determined up to 650^0C and for less accurate class B – up to 850^0C. Initial limited errors $|\delta_{10}|$ of both classes are 0,06% and 0,12% respectively. On the base of nominal characteristic of the both class sensors the relative limited errors $|\delta_\varepsilon|_{A\ max}$ and $|\delta_\varepsilon|_{B\ max}$ of the resistance increment ε are calculated and given on Fig 2. These errors could be approximated by the single value and related to the maximum or mean value of the temperature range of each class sensor. In the full range of positive Celsius temperatures the limited error $|\delta_\varepsilon|_A$ doesn't exceed 0,2% of ε and respectively $|\delta_\varepsilon|_B \leq 0{,}5\%$.

B. Estimation of the limited accuracy of the 4R bridge with Pt 100 industrial sensors

Limited error of the 4R bridge with the single industrial of A or B class sensor in it may be calculated from above formulas. It may be assumed that limited errors of no variable arms are not higher that the sensor initial error $|\delta_{10}|$, balance is at 0^oC, and current of supply source is stable enough. If the sensor arm is $R_1(T)$ of initial resistance R_{i0}, its limited initial error and errors $|\delta_0|$ of all other resistances doesn't exit tolerances of the class A sensor then numerical formulas of limited errors $|\delta_{r21}|$ could be estimated.

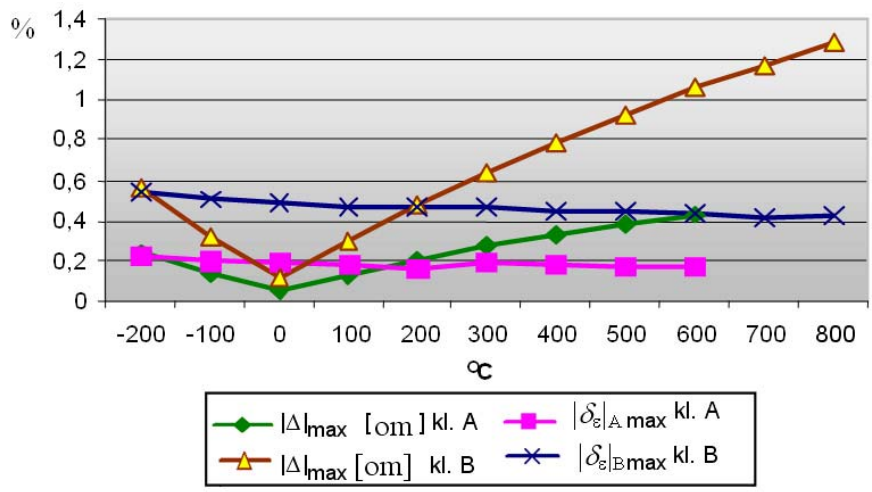


Fig. 3. Tolerances of Pt 100 industrial temperature sensors

Case 1⁰ (Bridge without adjustment)

$$\cdot\left|\delta_{r21}\right|_m = \frac{0,24+0,38\,\varepsilon_1+0,015\,\varepsilon_1^2}{\left(1+0,25\,\varepsilon_1\right)^2}\ [\%] \qquad (49)$$

Cases 2⁰ and 3⁰ (External or in the bridge null setting)

$$\cdot\left|\delta_{r21}\right| = \frac{0,26\left|\varepsilon_1\right|+0.06\varepsilon_1^2}{\left(1+0,25\varepsilon_1\right)^2}\ [\%] \qquad (50)$$

Case 4⁰ (Negligible initial arm errors)

$$\left|\delta_{r21}\right|_m = \frac{\left|\varepsilon_1\right|}{\left(1+0,25\varepsilon\right)^2}\,0,2\cdot[\%] \qquad (51)$$

For the maximum temperature range 600^0C relative increment of sensor resistance is: ε_{max}=2,137 and for the above pointed cases limited errors $|\delta_{r21}|$ are: (0,48, 0,36 i 0,19)%. Difference between first and second one is 1/3, and its ratio to the last one is 2,5.

For comparison: relative limited error of the two similar Pt 100 sensors of the class A bridge in opposite arms calculated from (33) for the same temperature range doesn't exceed 0,68% - without null correction, and 0,43% - if is corrected. These errors are calculated for the twice higher signal that for single sensor. They are slightly higher but signal linearly depends on equal resistance increments of both sensors.

Similarly have been estimate limited errors of the class B sensor bridge as: (0,59, 0,51, 0,46)%. For lower temperature ranges relative limited errors and uncertainties type B are higher.

X. Final Conclusions

Given formulas in the text of both parts of this paper and in tables 1 and 2 allow to estimate accuracy measures of 4R bridge transmittances in all its variants used in practice. It is valid for 1D and more dimensions measurements.
From table 2 it is possible to compare accuracy measure formulas of main particular cases of 4R bridges. They are different for current and voltage supply of the same bridge. Even if transmittance is linear to increment ε its measures differently depends on it.
Formulas of errors of the single variable arm bridge are more complicated then if two or four arm resistances are variable. For small values of ε it is possible to approximate them by polynomial of the first or higher order.
Systematic errors could be calculated as random ones for set of sensor bridges in production or in exploitation with proper correlation coefficients and obtained values should be smaller than for limited errors (of the worst case).
Similar formulas as presented in this and other author papers e.g. [1], [6, 7] could be formulated for other types of impedance sensor bridges, DC and AC, single and double supplied, with applying the same as above methods of the simplification of their description.
As accuracy measures are described above as functions of initial resistances and their increments, this method is independent from measured quantity and sensor characteristic. It is valuable also for accuracy evaluation of testing circuits from their terminals as two-port and for diagnostics and for impedance tomography.