METHOD FOR FAST AND ACCURATE FREQUENCY MEASUREMENT

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Abstract- An fast and accurate alternative frequency measurement method based on the coincidence of pulses between two regular independent pulse trains and rational approximations of the number theory is presented. Based on the model of measurement process, it is shown that measurement result is a rational approximation to the true value and its approximation has a higher accuracy than approximations by systematic fractions. Results obtained from simulation of the measurement model are presented. This method can be implemented using low cost hardware.

I. Introduction

Many technologies and applications depend on precise measurements of time and frequency including financial markets, the wired and wireless telephone networks, radio broadband stations, the electrical power grid, radionavigation systems [1], GPS-based timekeeping [2], radar warning receiver design [3], physical parameter measurement using surface acoustic waves (SAW) [4], distance meter design based on phase difference measure [5], etc. Fast and precise frequency measurement method can be useful for many practical applications because many physical parameters are possible to convert to frequency. In this way can be measured: acceleration in any automatic control system; gravitation force in airplane navigation; fast mass variations, etc. Therefore, some digital measurement techniques of time and its reciprocal value, frequency, have been proposed [6-14].

In basic digital measurement method of frequency, the input signal is first amplified and its zero crossings are detected and converted into a series of uniform pulses. The pulses are them applied to a main gate, which is opened for a known time interval controlled by the system time base. During the gating interval, the signal pulses pass though the main gate and the pulses are counted. The mean frequency is determined by the number of whole cycles occurring during that fixed time interval. This is the direct frequency measurement method or classic method. The principal sources of error for this method are analysed satisfactorily in [15-16]. In reciprocal counting methods [14] or period measurement methods [15] the input signal controls the main gate, and the set measuring time is not an exactly defined gate time. The desired measuring time or reference measurement time is external electronically set and the actual measurement time is synchronized to the input signal and the time base. Then, the gate time can be defined by two consecutives electronics events as: the electronic detection of two kinds of the same phase difference situation between two signals using a phase coincidence detection circuit with high distinguishability [9] or the electronic detection of two coincidence pulses in two regular independent pulse trains [6-7]. In this way, the gate time is synchronized by both measured signal and standard frequency signal. Therefore, ±1 word measuring error is eliminated. Some of this methods are mathematic described in articles of Tyrsa, and Wei [6,8,9,11,13], but not defined the general criterion to select the optimum star and stop event for exact frequency estimation.

In the frequency measurement method based on continuos time stamping the standard frequency and the input signal are continuously counted, without reset. In a one-second frequency measurement hundreds or thousands of passed time-stamped events are stored in memory and linear regression using least-square line fitting is used to improve accuracy. The post-processing of hundreds of samples data is time consuming, even if the raw data collection is fast. That means that the measuring speed is reduced [14].

In this paper is presented a fast and accuracy frequency measurement method based on the pulse coincidence of two regular independent pulse trains and the rational approximations by the number theory formalism. In our new method the stop event to measure the frequency of an electrical signal, is not an electronic detected event as other methods, is a numeric condition derived from the number theory. This stop condition is easy to implement with basics digital circuits.

II. Pulse coincidence principle

Pulse coincidence principle has been used to frequency measurement of electrical signals [6-8]. In this method, a desired frequency is measured by comparing it with a standard frequency. The zero crossings of both frequencies are detected, and a narrow pulse is generated at each crossing. Two regular independent pulse trains are generated. The desired and standard trains of narrow pulses are compared for coincidence. This is made with an AND-gate. A coincidence pulse train is generated. The coincident pulses can be used as triggers to start and stop a pair of digital counters (start and stop events). The standard and desired pulse trains are applied to the counters and a measure of the desired frequency is obtained by multiplying the known standard frequency by the ratio between the desired count and the standard count obtained in the two digital counters [6,8].

The coincidence of pulses has been investigated over the last sixty years [16-21]. In these works, is shown the connections of the coincidence of pulse of regular independent pulse trains with the number theory, particularly with linear congruence theory [16-17] and Diophantine approximations [20-21].

Consider f_x as the desired or unknown frequency and f_o as the standard frequency. In figure 1, S_x and S_o are the unknown and standard trains of narrows pulses, with pulse width τ respectively

Consider ΔT as the greatest common divisor (g.c.d) of both periods $T_x = 1/f_x$ and $T_o = 1/f_o$. ΔT represents a minimally distinguishable time interval and indirectly a quantum, which as shown below, is defined by the stability of the standard frequency. τ and ΔT are independent parameters.

Let us admit that there exists a pair of narrow pulses in the pulse trains which exactly coincide on the time axis. Completely coincident pair of pulses (Figure 1) is designated as count reference. This pair of pulses is a command to start the frequency measurement. P_n and Q_n are the numbers of counted pulses from the S_x and S_0 sequences that occur between adjacent coincidences. Between two completely coincident pairs also exist some partial coincidences (Figure 1). Adjacent coincidences may be either partial or total. The index n refers to both partial or total coincidences. From [6-8], each fractions P_0/Q_0 , P_1/Q_1 , P_2/Q_2 in Figure 1 can be used independently to calculate an estimation of the unknown frequency, as shown in [7]. According to [22-24], if we have any two fractions P_0/Q_0 , P_1/Q_1 they can form a mediant fraction with the property

$$\frac{P_1}{Q_1} < \frac{P_1+P_0}{Q_1+Q_0} < \frac{P_0}{Q_0}. \qquad (1)$$

The mediant fraction of two fractions always lies between them in value. If the actual unknown frequency value lies between rational approximations, formed by experimental results Pi/Qi , the mediant formed by results number i and i+1 always is better approximation to true value that each of both [22]. So, the best approximation we have to search between mediants.

The mediant fractions and its approximants have the common fundamental property [22-24]

$$P_n Q_{n+1} - P_{n+1} Q_n = \pm 1 \qquad (2)$$

But is posible to form mediants whit three o more fractions, and this mediant can be generally written in the form:

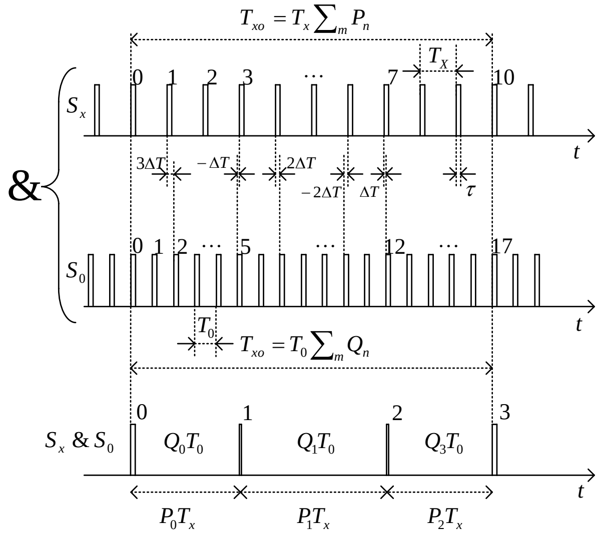

Figure 1. Process of direct frequency comparison

$$\sum_m P_n / \sum_m Q_n,\ (\ n = 1,2,...; m = 2,3,..., n-1). \tag{3}$$

where n is the number of the fraction and m is the number of the mediant. Thus, from Figure 1 and eq. (3), $\sum_2 P_3 / \sum_2 Q_3 = (P_0 + P_1 + P_2)/(Q_0 + Q_1 + Q_2)$. According to [6-7] measured frequency can expresed by

$$f_{xm} = f_0 \sum_m P_n / \sum_m Q_n \tag{4}$$

The relative value of the systematic measurement error β_{xm} (frequency offset), using (4) can be writen

$$\beta_{xm} = \frac{\Delta f_{xm}}{f_x} = \frac{1}{\sum_m P_n \times \sum_m Q_n} \tag{5}$$

It is known that, if both pulse trains with frequencies f_x, f_0, are applied to the input ports of an AND gate, an irregular pulse train is generated and average frequency of coincidences is $\overline{f}_{x0} = 2\tau f_x f_0$ with a period expresed by [6 or 8]

$$\overline{T}_{x0} = \frac{1}{2\tau f_x f_0} \tag{6}$$

If the value of frequency f_x is approximated by a particular mediant with index i, then the relative value of an instrumental error is limited by two factors: 1) the duration of the pulses participating in coincidences, and 2) the time of measurement t_m. Thus $\beta_{xi} = \Delta f_{xi} / f_x \le 2\tau / t_m$.

It is experimentally known, that the specified instrumental error is two to three orders less than in a classical frequency meter [8]. At a pulse width $\tau \approx 7 \times 10^{-9}$ s the measured values of frequency $f_x = 1 \times 10^6$ Hz of the thermostatic quartz generator had a root-mean-square (RMS) error $s_x = 1.79 \times 10^{-3}$ Hz at 50 observations during total measurement time $t_m \le 1$ s. But the estimation of relative systematic error using formula (5) for the method of coincidences used in [6,8], is $\beta_{xm} \cong 1 \times 10^{-12}$. This circumstance encourages us to more fully investigate the opportunities given by the approach using mediants.

III. Principle of best approximation choice

Numerator and denominator of every mediant in (3) corresponds to time intervals $T_x \sum_m P_n$ and $T_0 \sum_m Q_n$, see Figure 1. This time intervals can be have different duration due to existence of parcial coincidences and error of comparison of the time intervals is reduced to the maximum duration of the pulses of coincidence, 2τ [8]. Then

$$0 \le \left| T_x \sum_m P_n - T_0 \sum_m Q_n \right| < 2\tau\ . \tag{7}$$

This ecuation shows, that in the sequence of mediants $\sum_m P_n / \sum_m Q_n$, it is possible to choose one, which satisfies the expression

$$0 \le \left| T_x \sum_m P_n - T_0 \sum_m Q_n \right| < \Delta T \tag{8}$$

ΔT is the greatest common divisor of oboth periods $T_x = 1/f_x$ and $T_o = 1/f_o$.

From Archimedes axiom [24, p.7, property V] it follows that product of two numbers $ab = c$ can be considered as the sum $a + a + ... + a$, in which the number of summands is equal to b or as the sum $b + b + ... + b$, in which the number of summands is equal to a. The number $c = ab$ represents the common multiple (CM) of numbers a, b.

Using the greatest common divisor ΔT it is possible to represent the periods T_x, T_0 by numbers $a = T_x / \Delta T$, $b = T_0 / \Delta T$. Then $c = T_x / \Delta T \times T_0 / \Delta T = T_{x0} / \Delta T$ is a common multiple as described mentioned above. Thus we have:

$$T_{x0} = T_x T_0 / \Delta T \tag{9}$$

where T_{x0} is least common multiple of $T_x = 1/f_x$ and $T_o = 1/f_o$, practically it can be fixed as a period of totally overlapped coincidences.

Thus, in ecuation (8) each segments $T_x \sum_m P_n$ and $T_0 \sum_m Q_n$ within ΔT are equal to each other, and to the segment $T_{x0} = T_x T_0 / \Delta T$. Considering this, we can write: $T_x \sum_m P_n = T_{x0}$. Thus we find: $\sum_m P_n = T_{x0} / T_x = T_0 / \Delta T$.

From the statements above it follows for the accepted decimal notation that the mediant which provides the best approximation for $\alpha = f_x / f_0$ and the greatest possible accuracy for f_x has a numerator of the form $\sum_m P_n = 1 \times 10^r$. That is, the numerator is in the form of "one with *r* zeros".

It is easy to achieve a desired accuracy in hardware when the form of "*r* zeros" of least significant bits is registered using the unknown frequency's period counter. This is a stop signal for the end of the measurement process. It allows constructing time keeping systems or a frequency meter with a set accuracy and duration of measurement. A simplified functional diagram of a frequency meter is presented in Figure 2.

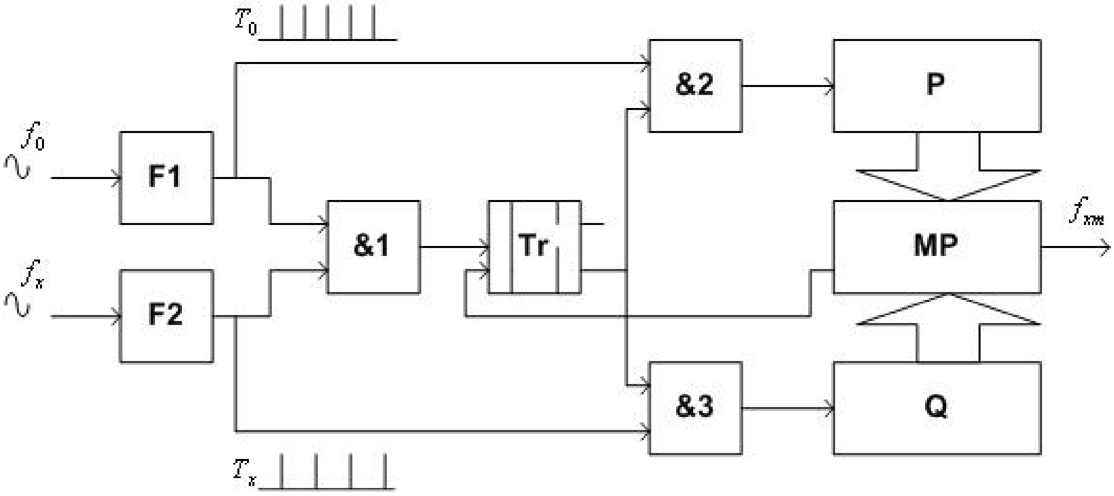

Figure 2. The frequency meter block diagram.

The desired and standard trains of narrow pulses are compared for coincidence using the AND-gate &1. A coincidence pulse train is generated. A coincident pulse is used as triggers to start the pair of digital counters P and Q (start event). The standard and desired pulse trains are applied to the counters. Counters P and Q keep a count of the pulses of both frequencies until counter P receives a result in the form of $\sum_m P_n = 1 \times 10^r$. This signal results in feedback that resets the RS-trigger to its initial state. The measurement is completed. A measure of the desired frequency is obtained by multiplying the known standard frequency by the ratio between the desired count and the standard count obtained in the two digital counters, see equation 4.

IV. Simulation

In the simulation two pulse trains of unitary amplitude are generated. The value of reference frequency was accepted as $f_0 = 1 \times 10^7$ Hz. The hypothetical value of unknown frequency $f_X = 5878815.277629991$ Hz is a result of the accepted value of the period $T_X = 1.701023 \times 10^{-7}$ s, and value of pulse width in both pulse trains is accepted as $\tau = 1.5 \times 10^{-9}$ s. T_X and T_0 are mutually prime numbers and have the common denominator $\Delta T = 1 \times 10^{-13}$. The simulation algorithm provided continuous formation of segments $T_x \sum_m P_n$ and $T_0 \sum_m Q_n$ and compares the magnitude of their difference with parameter 2τ. When the value of the specified difference was less than 2τ on corresponding steps, it was identified like a coincidence of pulses and the integer number, $\sum_m P_n$ and $\sum_m Q_n$ are stored. The unknown frequency is calculated using equation (4) and is stored also. The best approximation is selected (in this case) using the condition, $,\sum_m P_n = 1 \times 10^6$. The simulation results are partially presented in the Table 1. This table represents an interesting fact. For thousands of data we have the same uncertainty range 10^{-13}, as for first and third rows. And only when the stop condition takes a form of 1 with six zeros (in this case, second row of a Table 1) we are getting up to 10^{-17}.

Table 1. Results of frequency measurement process simulation.

$,\sum_m P_n$	$\sum_m Q_n$	$\left\| T_x \sum_m P_n - T_0 \sum_m Q_n \right\|$,s	f_{Xm} , Hz
957087	1628027	1.00×10^{-13}	5878815.277633602
1000000	1701023	2.78×10^{-17}	5878815.277629991
1042913	1774019	1.00×10^{-13}	5878815.277626677

In Figure 3 the calculated frequency relative error is presented for a simulation time of 0.2 s. In this graphic, we can see a global convergence to zero of this parameter. An alternated convergence and a non monotone decreasing characteristic are evident. However, we can identify a point where absolute value of β is minimum for an approximated time of 0.17 s.

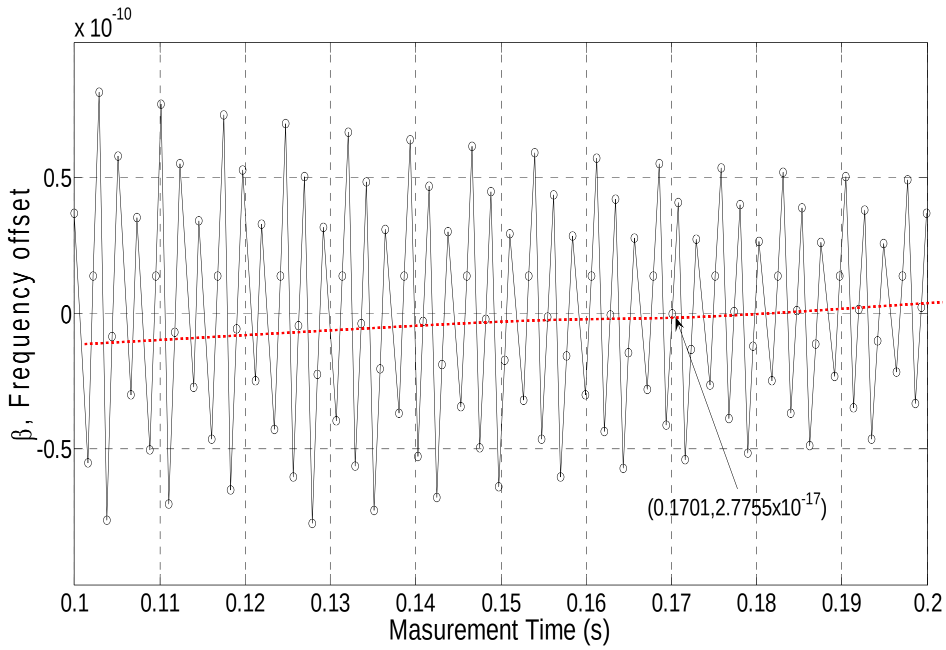

Figure 3. MathLab screenshot for discrete aproximation points set (marks °).

V. Conclusions

In the offered model for fast frequency measurement, the result is fixed on the equality of intervals $n_0 T_0$ and $T_x \sum_m P_n$. Therefore the model is independent to the parameters of coincidence circuits, duration and the shape of coincidence pulses, and the parameters of “zero-crossing” pulses in both sequences.

Offered method gives exact frequency measurement result in various coincidence cycles. For MHz frequencies range or more the time of this phenomena is relatively short.

For measuring systems which can be constructed on the basis of the specified model, systematic and instrumental errors have the same infinitesimal order. Instrumental errors are caused only by the reproducibility of the reference frequency. For measurements of high frequency values it is expedient to use higher values of reference frequency in order to have an equivalent reduction of measurement time.

In other words: the higher frequencies, the less time of measurement.

Also it is impotant to note, that this theorical method permits to measure unknown frequency value in a case, when the reference frequency value is less than unknown frequency. For classical methods it is impossible completely.

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