16th IMEKO TC4 Symposium
Exploring New Frontiers of Instrumentation and Methods for Electrical and Electronic Measurements
Sept. 22-24, 2008, Florence, Italy

Laboratory current transformer based on Rogowski coil

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Abstract – This paper cover the analysis and construction of current to voltage transducer based on Rogowski coil which satisfy the requirements of high-accuracy measurement of AC current (up to 20 A at power supply frequency, with the aiming uncertainty of 100 parts per million). Primary source of ac current uncertainty measured by this type of transducer is nonuniform density of turns which, in case of eccentricity or shift of primary conductor results in deviations of mutual inductance. Self capacitance and self resistance, temperature dependence of coil geometry and electromagnetic interferences affect the accuracy as secondary source of uncertainty. With respect of influencing parametres, the current transducer of aim accuracy can be realized.

I. Requests on transformer properties

Electromotive force, induced in Rogowski coil with arbitrary, most often flexible toroidal shape which enclose the primary conductor, is proportional to derivation of total magnetic flux in coil, i.e. time derivation of measured current in primary conductor and mutual inductance of that geometrical system:

$$e(t) = -\frac{\mathrm{d}\Psi(t)}{\mathrm{d}t} = -M \cdot \frac{\mathrm{d}i(t)}{\mathrm{d}t} \qquad (1)$$

The current with strictly sinusoidal shape can be measured without integrator but then their frequency should be measured. However, for the purpose of reconstruction of complex current waveform, the appliance of integrator is obligatory. Thus, the integration and amplification of voltage induced in secondary coil is equally important part of complete current transducer (Fig.1).

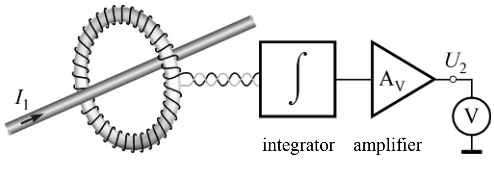

Fig. 1. The principle of ac current measurement by transducer based on Rogowski coil

In view of requests, the laboratory current transformer should be capable for measurement of ac current up to 20 A in frequency range from 20 Hz to 1 kHz, with relative uncertainty in order of 10^{-4}. Moreover, it must be persistent on influences of electromagnetic interferences and disturbances..

II. Selection of transformer geometry

The construction of transformer depends greatly of geometrical relationship between its primary and secondary part. Stability of mutual inductance of this system is achieved by firm mechanical connection between primary conductor and secondary coil, and especially by delicate geometry of secondary turns. Therefore, secondary turns shall be wound on rigid body with accurately known dimensions, which precise construction is the most simplified by adopting standard toroidal geometry of non-magnetic insulator with rectangular cross-section. The version with circular cross-section is not discussed, as the requested accuracy of body dimensions are much harder to achieve. In selection of dimensions and construction of precise coils, the results of the previously analysis [1,2] are taken into consideration. This verdicts can be condense into three elementary claims:

1. for obtaining the largest possible mutual inductance, the coil must be positioned as close as possible to primary conductor, and must have greatest cross-section of each turn
2. the coil must be wounded in one layer, and in this case the number of turns depends on diameter of used wire
3. The influence of eventual discontinuity, caused by first and last turn, and varying density of turns along the perimeter of toroid are minimised by centered mutual position and firm mechanical connection between primary conductor and coil

III. Conception of the construction relating to influences of strange electromagnetic fields

Small achieved mutual inductance of the system consists of coil and conductor indicating its high sensitivity on electromagnetic interferences, as well as possible unwished influence on accuracy in precise current measurements. In principle, the disturbances derived from strange magnetic fields induce additional electromotive force E_m in leakage inductance L_m, while the disturbances derived from strange electrical fileds effects as injection of small current I_E in measuring circuit through effective parasitic capacitance C_E between coil and whole surroundings. (Fig.2).

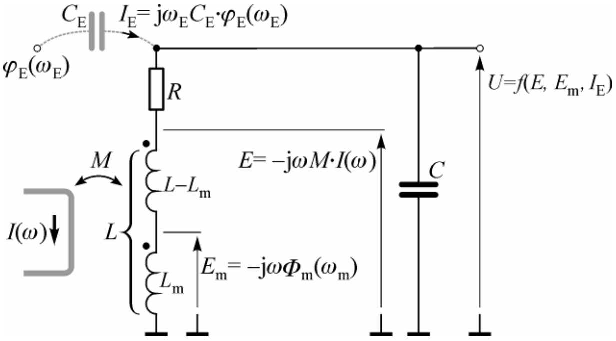

Fig. 2. Equivalent circuit in analysis of influence of strange electromagnetic fields

The basic problem is unwished axial contour area of coil, which in electrical manner act as mentioned leakage inductance L_m. Neutralisation of this effect can be performed by using the additional loop with equal cross-section fitted close to (Fig.3a) or into coil.

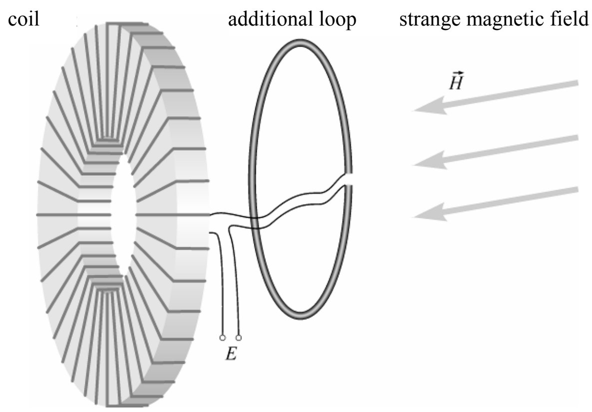

Fig. 3a. Cancelation of influence of strange ac magnetic field by using an tertiary loop

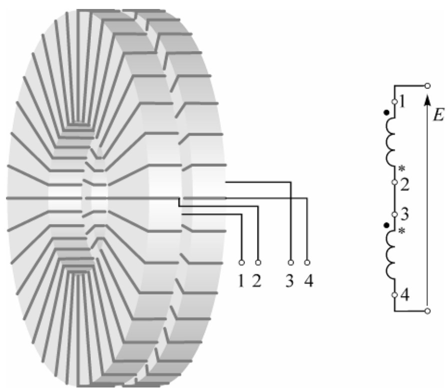

Fig. 3b. Solution with astatic configuration which has two coils wounded in opposite directions

Electromotive forces induced by strange magnetic field in this two loops are canceled by its properly serial connection. The same performance, but with double transducer sensitivity (i.e. double mutual inductance) can be obtained by astatic configuration (Fig.3b). In this case, the addition of electromotive forces induced by measured current in primary conductor is given by two identical coils which are wounded in opposite directions. Here are the coil terminals with same polarity of electromotive force induced by measured current pointed by dots, while the terminals with the same polarity of electromotive force induced by interferences pointed by asterisk. On the other side, owing to large area of secondary coils (approximately 500 cm^2) it can be expect relatively high sensitivity on disturbances of capacitive type induced by strange electric field. Thus, the conception with two parallel and identical measuring channels is adopted. Their opposite output voltages are lead to voltage amplifier of instrumentation type (Fig.4), whose common-mode rejection ratio (CMRR) is very high .

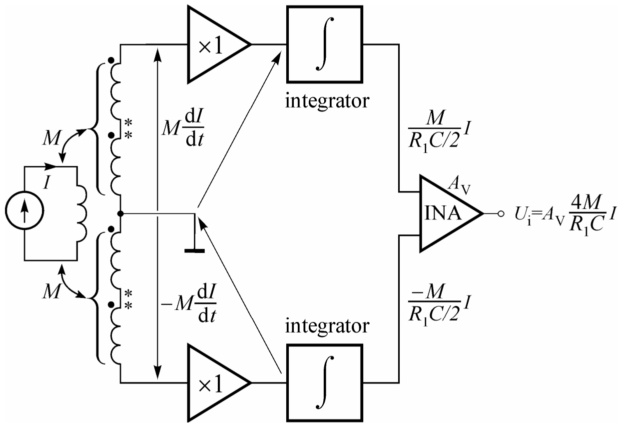

Fig. 4. Conception of current measurement by two parallel measuring chanells

If those chanells are well balanced (which can be achived by careful designing), the transformer would be immunity on influences and disturbances caused by strange electromagnetic fields. In this manner, the usage of electromagnetic shields and screens are avoided.

A. Primary circuit of the transformer

The main parameter in designing of geometry and estimating of primary conductor is density of measured current and waste heat, so the conductor should be treated as dissipative element of the system. The thermal stabilisation of entire transducer is more simplest task if the heat dissipation are minimised, which is reduce the temperature gradient in the transformer volume. The highest expected current are I_{PM} = 20 A and the cross-section of the primary conductor must be relatively large, so the diameter of 18 mm was chosen. As the primary conductor also serves like a frame of the coil centrators, it must be precise processed, so the best choise of material was duralumine alloy (AlCu5Mg1). According to room required for secondary coils, the lenght of the conductor is 350 mm. The influence of temperature variations on transformer properties is controled by dissipation limit, which is 50 mW at highest expected current.

B. Secondary circuit of the transformer

Design of the secondary part of transformer was based on desired electrical properties and geometrical parameters. Like starting point, the transimpedance (i.e. sensitivity) Z_T of 1,25 mV/A at frequency f = 50 Hz was chosen. Thus, the mutual inductance is:

$$M_T = \frac{Z_T}{2\pi \cdot f} \approx 4\ \mu\text{H} \tag{2}$$

As the rejection of interferences is solved by double astatic construction (i.e. secondary is composed of four coils), the system of conductor and single coil should have mutual inductance of 1 μH. In real case, the effective cross-section of each turn is somewhat larger than rectangular cross-section of toroidal body, so the mutual inductance could be expressed as:

$$M = \mu_0 \cdot \frac{N \cdot h'}{2\pi} \cdot \ln\left(\frac{r_V'}{r_U'}\right) = \mu_0 \cdot \frac{N \cdot (h+d)}{2\pi} \cdot \ln\left(\frac{2r_V + d}{2r_U - d}\right), \tag{3}$$

where N is number of turns, d is wire diameter, h is body thickness, while r_U and r_V are inner and outer body radius, respectively. Furhermore, it is easily to verify that the inductance of toroidal coil is $L = N \cdot M$. With known desired mutual inductance, the estimation should be start with consideration of influences which might became obstacle in the process of production. In the first place, this is wire diameter, which should be chosed in respect to precision of winding, as well as inner diameter of coil body because of winding technology. As the coil must be single-layered, the wire should be of satisfactory strenght, but also sufficiently thin so the aimed number of turns (i.e. mutual inductance) can be realized. The optimal diameter d of copper wire was settled at roughly 0,3 mm so, owing to thickness of insulation lacquer, the external diameter of wire is d = 0,322 mm). The coil is performed on the body of toroidal shape, made of polymethyl methacrylate (plexiglas), which was proven as good choice during the construction of test probes, which was used in verification of analitycal methods for estimation of mutual inductance of real Rogowski transducers. Due to standardized dimension of plexiglas sheets, the height of toroidal body is h = 19,24 mm, while the inner and outer radius is r_U = 15 mm and r_V = 37 mm, respectively (Fig.5).

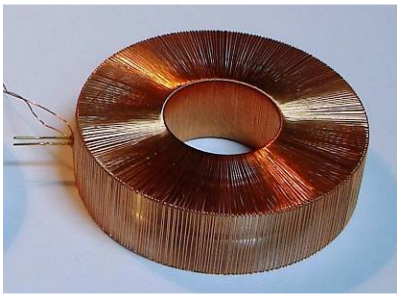

Fig. 5. The view of toroidal coil (r_U = 15 mm, r_V = 37 mm, h = 19,24 mm, N = 287)

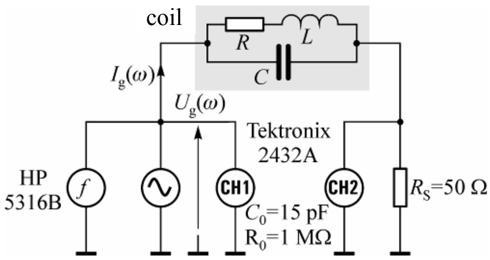

Fig. 6. The resonance method for determining of the coil self capacitance C

Each coil have 287 turns and, according to (3), the calculable mutual inductance is M = 1,0308 µH while the calculable inductance is L = 295,84 µH. Besides this, the self resistance was measured to be R = 6,03 Ω. In order to analyse the frequency dependence of induced electromotive force, it is also necessary to determine the coil self capacitance C. Since the self capacitance cannot be measured directly, the resonance method (Fig. 6.) was utilized. The resonance appear on frequency on which the zero phase shift between generator voltage $U_g(\omega)$ and generator current $I_g(\omega)$ were set, or when the generator current is minimal (which is sometimes appropriately). In this case, the coil impedance is very high:

$$Z_0 = \frac{L - CR^2}{CR} \tag{4}$$

Parallel coupling of input impedance of second channel (CH2) of Tektronix 2432A oscilloscope (R_o = 1 MΩ, C_o = 15 pF) and resistance R_S = 50 Ω have no influence on current in measuring circuit, nor on its phase shift for frequencies near the resonance. Expression of the current $I_g(\omega)$ as vector sum of inductive and capacitive component, leads to the following equation:

$$I_g(\omega) = \frac{U_g(\omega)}{jX_L + R} - \frac{U_g(\omega)}{jX_C} = U_g(\omega) \cdot \left[\frac{(1-\omega^2 LC)R + \omega^2 LCR}{R^2 + (\omega L)^2} + j\frac{\omega R^2 C - \omega L + \omega^3 L^2 C}{R^2 + (\omega L)^2} \right] \tag{5}$$

Thus, by equalizing the imaginar component of current with zero, follows to equation for resonance frequency, and self capacitance of the coil is then:

$$C = \frac{L}{(2\pi f_0)^2 L^2 + R^2} \tag{6}$$

The coil self capacitance measured directly on the each coil terminal is C = 12 pF. In the process of the transformer assembly, each secondary tap was connected by thin interwinted copper wires to common terminal, while the terminal should be linked to integrator by coaxial cable. Obviously, all of this connections increase effective secondary capacitance of transformer. Therefore, the measurement was made on terminals which are appropriated to connection with integrator. The resonance was appeared on frequency of f_{0S} = 869 kHz, so the effective self capacitance of the each pair of coils with connection cables was 57 pF. So, considering that the capacitance of the each pair of the coils itself amounts 6 pF, to the capacitance of connection cables belongs 51 pF. Finally, the parameters of the each secondary part (i.e. each pair of the coils) are:

M_S = 2,062 µH, L_S = 591,67 µH, R_S = 12,06 Ω, $C_S \approx$ 57 pF,

while the calculable mutual inductance of entire transformer is M_T = 4,123 µH.

IV. Measurement results obtained on realised transformer

Based on carried analysis and testings, the current transformer based on Rogowski coils was realised (Fig. 7.). The base and all frame components, except the primary conductor and coils itself, was made of plexiglas, while the entire transformer was placed in hermetically sealed case. In order to achieve stability and accuracy of the mutual inductance, as well as immunity on electromagnetic disturbances, the secondary terminals are electrically balanced and made with special care.

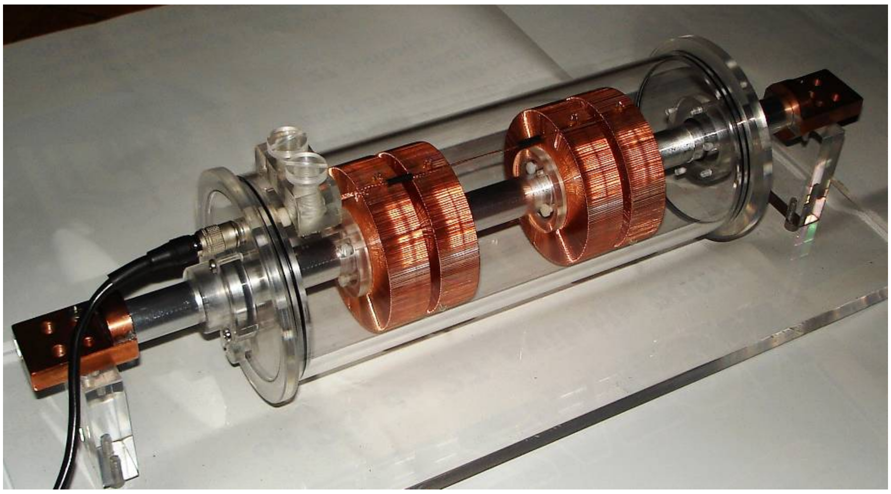

Fig. 7. View of realised current transformer; the massive primary terminals are placed on the left and right side, the two pairs of astatic secondary coils are located in the center, while the secondary terminals are placed on left side of the cylindrical case

The principle of mutual inductance measurement is presented in schematic diagram on Fig.8. The frequency of primary current is measured by HP 5316B Universal Counter, the induced voltage is measured by first HP 3458A Digital Multimeter marked as V1, driven by *Swerlein's* algorithm [4], while the primary current is determined as voltage on Fluke A40 shunt measured on second HP 3458A Digital Multimeter, marked as V2.

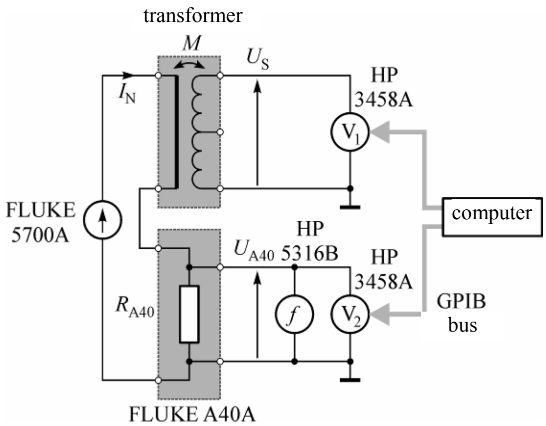

Fig. 8. Principle of measurement of the transformer mutual inductance

The uncertainty of this measurement depends mostly of voltage uncertainty. In quoted algorithm, for frequencies up to 100 Hz it is around 20 ppm, rising to 60 ppm for the frequency of 1 kHz. Mutual inductance of the entire transformer is estimated from following expression:

$$M_{\mathrm{T}}(f) = U_{\mathrm{S}}(f) \cdot (2\pi\, f\, I_{\mathrm{P}})^{-1} \tag{7}$$

The detail of measuring points up to frequency of 2 kHz is given in table 1.

Table 1. Measured values of mutual inductance M_{T} in frequency band from 20 Hz to 2 kHz

f / Hz	20	40	100	200	500	1000	2000
M_{T} / µH	4,12701	4,12702	4,12700	4,12700	4,12699	4,12700	4,12797

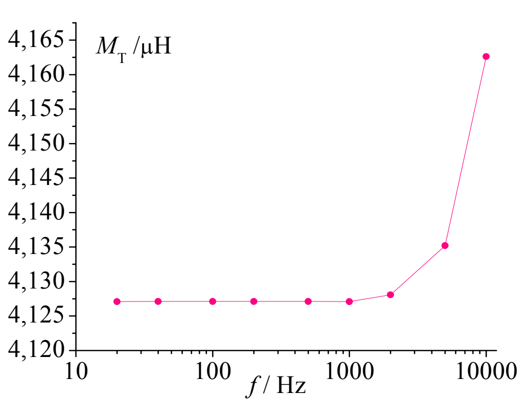

Fig. 9a. Mutual inductance M_T of the transformer in frequency domain

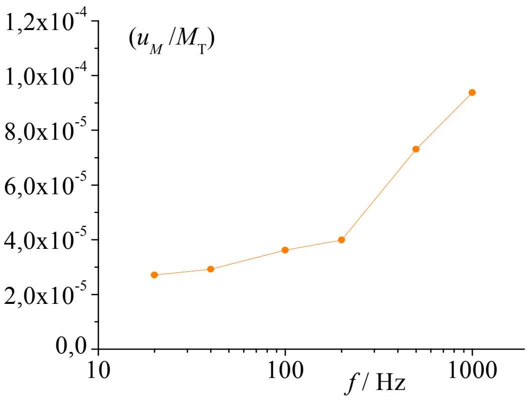

Fig. 9b. Uncertainty of M_T in frequency domain

The influence of HP 3458A Multimeter input capacitance of about 250 pF [3] is visible on Fig.9a. It raise effective secondary capacitance and reduce resonant frequency, which results in increase of voltage induced in frequency band above 1 kHz. During the operation of the transformer in ac current measurements, this effect will be solved by using specific voltage follower, which would act as impedance uncoupler between the secondary circuit of transformer and measuring device (i.e. integrator). Relative uncertainty of M_T depends of uncertainties of measured voltage, primary current and frequency. Due to the negligible uncertainty of frequency and *U-R* method of primary current measurement, the complex uncertainty of M_T is:

$$\left(\frac{u_{M_T}}{M_T}\right) = \sqrt{\left(\frac{u_{U_S}}{U_S}\right)^2 + \left(\frac{u_I}{I}\right)^2 + \left(\frac{u_f}{f}\right)^2} \approx \sqrt{\left(\frac{u_{U_S}}{U_S}\right)^2 + \left(\frac{u_{U_{A40}}}{U_{A40}}\right)^2 + \left(\frac{u_{R_{A40}}}{R_{A40}}\right)^2} \qquad (8)$$

As mentioned before, the uncertainties of measured voltages are frequency dependent, while the resistance of the Fluke A40 shunt in frequency band up to 1 kHz is $R_{A40} = (22{,}405 \cdot 10^{-3} \pm 4{,}13 \cdot 10^{-7})\ \Omega$. Hence the uncertainty of M_T raise with frequency, which is presented on Fig. 9b. Based on this analysis, the mutual inductance of entire transformer is $M_T = 4{,}1270 \cdot 10^{-6} \cdot (1 \pm 4 \cdot 10^{-5})$ H, which means that with frequency of $f = 50$ Hz, its transimpendance (i.e. sensitivity) would be $Z_T = 1{,}297$ mV/A.

V. Conclusions

The performed analysis shows that the true value of mutual inductance of each coil (i.e. the quarter of entire secondary) slightly differs from calculable value; the measured value is 1,032 μH, while the value obtained by equation (3) is 1,031 μH. So, if special care is taken, this results confirm that the precision current transformer with calculable mutual inductance can be realized. The further step in realization of precise current transducer is designing of precise integrator, which would ensures the frequency indepedence transfer function of induced voltage. The influence of transformer self resistance as well as input capacitance of such integrator might be neutralize by voltage follower. Thereby, the influence of small self capacitance of transformer, relating to high resonant frequency in this case, would be negligible in frequency band of interest.

References

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