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On the measurement of the stray capacitance of single layer air solenoids

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Abstract- Low stray capacitance solenoids are air solenoids, possibly single layer, with significant turn-to-turn spacing. The problem of the correct estimation of the stray capacitance is relevant both during design and to validate measurement results. The expected value is so low that any stray capacitance of the external instruments is relevant; a simplified method is proposed that doesn't perturb the stray capacitance of the solenoid under test. The method is based on the resonance with an external capacitor and on the use of a linear regression technique.

I. Introduction

The present work originated from the need of determining the stray capacitance terms of a high voltage Rogowski coil with a wide frequency range [1][2]. This solenoid was designed for high voltage immunity by selecting a coaxial conductor instead of a single conductor and the stray capacitance is represented by an articulated scheme of capacitance terms between adjacent and non-adjacent turns. This design is different from the more usual solenoid with an external cylindrical shield, that features a preponderant winding-to-shield (or self-) capacitance term, that reduces the relevance of the said turn-to-turn (or mutual) capacitance terms.
The capacitance terms and their distribution along the solenoid are relevant for two reasons: they determine the overall frequency response of the solenoid and how an external electric field couples to the solenoid elements, thus determining its susceptibility.

II. Stray capacitance measurement

There are a few references where closed form expressions and some measurement results are presented [3]-[5]. Yet, there is no agreement among the said results and the case of wide separation between adjacent turns is rarely treated accurately enough. For this reason the problem of the design of an experimental setup and a measurement procedure is considered here. In the following only the literature results that are judged trustable are reported for reference. Medhurst [4][5] presented a semi-empirical formula, that fits the results of several measurements on different solenoids:

$$C_{Med1} = D\left[11.26 l / D + 8 + 27\sqrt{D/l}\right] \quad (1)$$

where D [m] is the solenoid diameter and l [m] is the solenoid length. The first term is derived from the Nagaoka formula for the solenoid inductance, the other two are determined on an empirical basis.
Another formula credited to Medhurst, but for which a precise bibliographic reference couldn't be found, is

$$C_{Med2} = \frac{4\varepsilon_0 l}{\pi}\left[0.71 D/l + 1 + 2.4 (D/l)^{1.5}\right] \quad (2)$$

The case considered in [3] regards inductors with an external shield and is not applicable to the present problem, as it was evident by inspection of the curves behavior, opposite to Medhurst findings and to what has been observed experimentally.

III. Experimental results and setup

Two air solenoids are considered with two very different wire diameters: solenoid A with r_1=0.4 mm of enameled copper wire, and solenoid B with r_2=4 mm, using the external shield of a RG58 coaxial cable. The number of turns is made variable between 10 and 40 turns. The measurement of the stray capacitance has been done using different techniques.

The influence of the inductive part of the solenoid and the value of the natural self resonance frequency were evaluated first [6]. The application of the Webster-Havelock formula (with Rosa correction for turns separation), with wire diameter *r* [cm], turn pitch *p* [cm], solenoid radius *R* [cm] and length *l* [cm], *N* the number of turns)

$$L_{W,H,R} = 4\pi^2 \frac{R^2 N^2}{l}\left[1-\frac{8}{3\pi}\frac{R}{l}+\frac{1}{2}\frac{R^2}{l^2}-\frac{1}{4}\frac{R^4}{l^4}+\frac{5}{16}\frac{R^6}{l^6}-\frac{35}{64}\frac{R^8}{l^8}+\cdots\right]-4\pi RN(A+B) \tag{1}$$
$$A = 3.4904\ln(r/p) \qquad B = 0.25 \text{ if } n<15,\ 0.3 \text{ if } n \geq 15$$

gives a variable inductance of 1 to 5 μH for the number of turns ranging between 10 and 40. The self resonance frequency is then estimated between 20 and 110 MHz for the expected stray capacitance values, so that a direct measurement of the stray capacitance by a LCR bridge is not possible, since the inductive reactance in parallel shunts the capacitive reactance under measurement over too an extended frequency range.
Two setups have then been preferred and are briefly described here below.

A. Inductively coupled source and clamp-on current measurement

The excitation signal is applied to the s.u.t. (solenoid under test) by means of a coupling coil with a very small number of turns, so that its self resonance frequency is moved well above the frequency range of interest. The s.u.t. is connected to an external capacitor C_{ext} as further explained below in subsection IV.C. The output variable is the current circulating in the s.u.t. measured with a current probe featuring a negligible insertion impedance (< 1 Ω), that shunts any further stray capacitance term in parallel due to the internal probe circuitry (see Fig. 1a). The use of a current probe avoids the influence of any oscilloscope voltage probe connected to the s.u.t. itself, represented by the stray capacitance terms and ground coupling, that compromise the circuit symmetry and may produce a common mode to differential mode transformation.

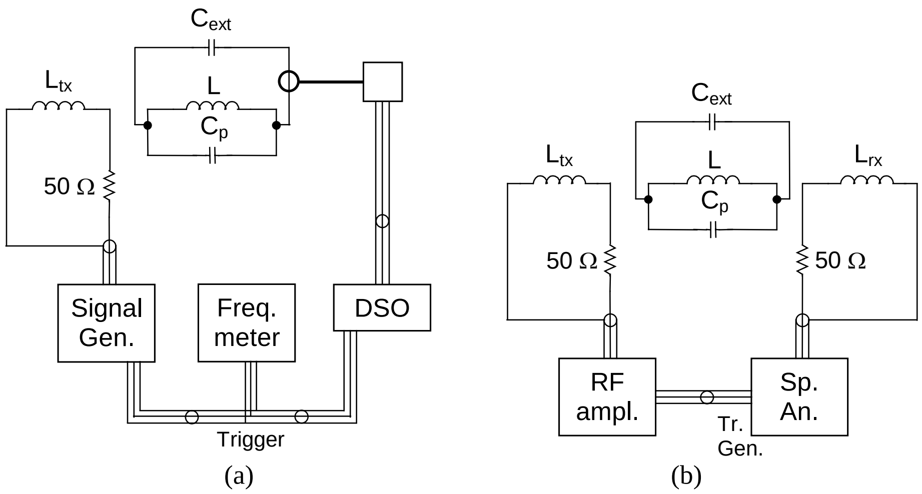

Fig. 1. Measurement setups (a) with inductively coupled source and current probe, (b) with coupled coils and interposed solenoid

B. Inductively coupled source and receiver

An alternative measurement method has been also considered, that measures the resonance frequency of the solenoid by the change in the transfer function that couples a transmitting and a receiving coil (see Fig. 1b).
The transmitting and receiving coils are like the one adopted in the previous method, with a self resonance frequency well above the frequency range of interest. Both coils are terminated on a 50 Ω resistor to normalize the reflection coefficient over the frequency interval of interest at the interfaces with the feeding RF power amplifier and the spectrum analyzer. Two measurements are made for each s.u.t. configuration, one without and one with the interposed s.u.t.; when the interposed s.u.t. resonates the shunting effect on the TX and RX coils reduces to a minimum and the picked up RX voltage reaches its maximum.

C. Measurement with external capacitor

For both the measurement setups described in the previous subsections, since the self resonance of the solenoid is scarcely visible and positioned at too a high frequency, a linear regression technique has been followed [5], using external resonating capacitors. The resonance frequency is determined by the joint effect of the total equivalent parasitic capacitance C_p and the external capacitor C_{ext}:

$$f_r = \frac{1}{2\pi\sqrt{L(C_p + C_{ext})}} \qquad (3)$$

It is convenient to express the external capacitor as a linear function of the square of the wavelength with the unknown stray capacitance C_p as the intercept:

$$C_{ext,i} = \frac{1}{4\pi^2 c^2 L}\lambda_i^2 - C_p = A\lambda_i^2 - C_p \qquad (4)$$

where c is the speed of light and the pairs (λ_i^2, $C_{ext,i}$) indicate the measurement points.
Several resonating capacitors C_{ext} have been connected to the solenoid output and the resonance frequency values recorded. The adopted Least Mean Square algorithm for the determination of the linear regression coefficients (slope A and intercept $-C_p$) shows a significant sensitivity to the location of the reference points (see again next section for details), so the C_{ext} values were selected clustered at the two extremes.
The instrumental uncertainty is minimized by the fact that the spectrum analyzer is used only to determine the resonance frequency f_r and its accuracy is then better than ±(5 ppm + 1% span/f_r + 10% RBW/f_r, where span and RBW stand for frequency span and resolution bandwidth), as declared by the manufacturer with negligible aging [7]; with successive readings the frequency span and resolution bandwidth may be reduced to conveniently low values, so that the total accuracy never exceeds 100 ppm. A better uncertainty is attainable if the measurement of the resonance frequency is done with a frequency meter (as shown in Fig. 1); for the used Hewlett-Packard model the manufacturer declares a ±1 LSD (Least Significant Digit) ±time base stability [8]; for the 11 digit instrument with the Option 001 (for high stability and temperature compensation of the reference clock) the accuracy amounts thus to a few units times 10^{-10}. The attention is thus on the more significant setup uncertainty, that is considered in Section IV.

D. Measurement results

The tested case correspond to one solenoid made of thick wire (the external shield of a RG58 coaxial cable, with a radius of 4 mm) with a progressively smaller number of turns (N=10, 15, 20, 25, 30, 40), the cable in excess being cut away. The measurements are made with eight C_{ext} values between 27 and 30 pF and eight C_{ext} values between 2.17 and 2.21 nF, for a total n=16 measurements.

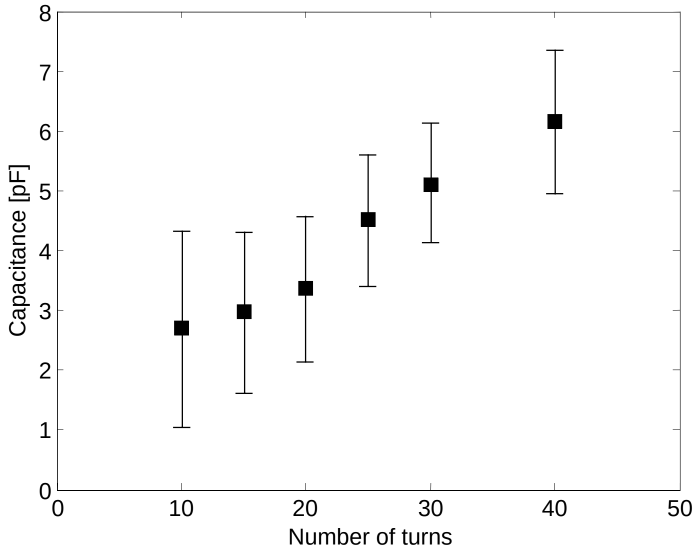

Fig. 2. Measured stray capacitance for solenoid A (r=0.5 mm) and B (r=4 mm) vs. number of turns N

The measurement results confirm an almost linear relationship of the capacitance value with respect to the number of turns. The exposed confidence interval is estimated on the basis of (12) in the next section; a future activity is to test this estimation of the confidence interval by means of a Type A approach, that is by repeating the measurements on the same solenoid in different days, by accounting for all the sources of uncertainty.

IV. Details of the measurement method

When the stray capacitance C_p is determined as the intercept of the linear regression operated on the measured pairs of values (C_{ext}, λ^2), the uncertainty is considered by defining the confidence intervals at $100\times(1-\alpha)$ % [9]. The expression (4) in reality identifies a linear relationship between the observed resonance frequency (in terms of λ^2, expressed as l in the following, by means of a change of variables) and the value of the external capacitor C_{ext}. The same expression written for the pairs ($C_{ext,i}$, l_i) of observed values takes the form

$$C_{ext,i} = \hat{A}\lambda_i^2 - \hat{C}_p + e_i = \hat{A}l_i - \hat{C}_p + e_i \quad (5)$$

where e_i represents a sample of the error variable e, and $\hat{A}$ and $\hat{C}_p$ correctly indicate the estimations of the slope and intercept.

It is known that the estimations of the slope A and of the intercept $-C_p$ taken as linear combinations of the observed values, are unbiased estimators and thus bring to the "exact" values:

$$E\left[\hat{A}\right] = A \quad E\left[\hat{C}_p\right] = C_p \quad (6)$$

An unbiased estimator of the variance of e is

$$\hat{\sigma}_e^2 = \frac{\sum_{i=0}^{n-1} e_i^2}{n-2} \quad (7)$$

For the variance of the slope and of the intercept we may write

$$\mathrm{var}\left[\hat{A}\right] = \frac{\hat{\sigma}_e^2}{S_{ll}} \quad (8)$$

$$\mathrm{var}\left[\hat{C}_p\right] = \hat{\sigma}_e^2\left[\frac{1}{n} + \frac{\bar{l}^2}{S_{ll}}\right] \quad (9)$$

where

$$S_{ll} = \sum_{i=0}^{n-1}(l_i - \bar{l}) = \sum_{i=0}^{n-1} l_i^2 - \bar{l}^2 \quad \bar{l} = \frac{1}{n}\sum_{i=0}^{n-1} l_i \quad (10)$$

The variance of the intercept identifies directly the uncertainty of the estimation of the stray capacitance C_p. While the first term between square brackets in (9) makes the variance decrease for an increasing number of observations n, the second term indicates that the distribution of the resonance frequency values is relevant to the reduction of the variance. In other terms, for the degenerate case of all the l_i terms equal to a constant value l^*, the variance tends correctly to infinity. On the contrary, the wise distribution of the designed resonance frequency values at the two extremes of the frequency range, gets the minimum variance. Let's suppose that the set of l_i values is subdivided into $n/2$ points at l_{dn} (the lower value) and $n/2$ points at l_{up} (the upper value), with the two values symmetrical with respect to the central point, that represents the average value $\bar{l}$. While keeping constant the average value, the term S_{ll} is made larger by increasing the number of points n and by maximizing the distance of the two clusters $l_{up}-l_{dn}=2b$. Since l_{up} and l_{dn} are chosen symmetric around the average value $\bar{l}$, then $S_{ll} = nb^2$; if $\bar{l} \cong b$ (such as in the adopted case where l_{dn}=27 pF and l_{up}=2.2 nF), then (9) simplifies to

$$\mathrm{var}\left[\hat{C}_p\right] \cong \sigma_e^2 \frac{2}{n} \quad (11)$$

The constraints for l_{up} and l_{dn} are represented by the maximum measurable frequency of the frequency meter and by the extreme values of the external capacitors, that must be larger than the expected stray capacitance and smaller than a convenient maximum value, that ensures capacitance stability (with respect to temperature variations and ageing) by the availability of high quality dielectric materials (dielectric constant stability and low losses).

Finally, it is possible to derive a confidence interval for the estimator of the intercept $\hat{C}_p$ by observing that, if the observations are normally and independent distributed, the intercept is a t random variable with n–2 degrees of freedom; thus, the amplitude of the confidence interval around the estimated value of the intercept $\hat{C}_p$ that contains the true value C_p at 100×(1 – α)% probability is given by

$$\hat{C}_p - t_{\alpha/2,n-2}\sqrt{\sigma_e^2 \frac{2}{n}} \le C_p \le \hat{C}_p + t_{\alpha/2,n-2}\sqrt{\sigma_e^2 \frac{2}{n}} \tag{12}$$

A 95% prediction interval corresponds to α=0.025 and for $10 \le n < \infty$ $t_{\alpha/2}$=2.0±0.12. The half-width of the confidence interval, that identifies the uncertainty related to the calculation method $u(C_p)$, is then approximately twice the dispersion indicated by (9).

Until now the values of the pair ($C_{ext,i}$, l_i) have been considered exact; yet, they are characterized by the instrumental uncertainty related to the determination of the values of the external capacitors and of the resonance frequency (that determines the parameter $l_i=\lambda_i^2$).

The external capacitors are high stability high quality capacitors, whose values were measured with a custom made capacitance bridge, featuring a very low uncertainty and high repeatability [10]; the maximum uncertainty for the lowest C_{ext} values at k=2 coverage factor is lower than 100 ppm.

The instrumental accuracy of the resonance frequency reading was considered in Section III.C and found negligible.

A last issue on the determination of the resonance frequency is that it is in relationship with the detection of the minimum or maximum of a voltage or current, depending on the chosen method (see subsections III.A and III.B). The detection of the resonance condition is the more accurate: the larger the factor of merit of the resonant circuit. If the preferred method of subsection III.A is considered, the minimum detectable change of amplitude δV on the Digital Storage Oscilloscope (DSO) may be assumed about 1% of the vertical scale, provided that the driving signal is adjusted to span about 3/4 of the available vertical scale and that the peak is measured by the DSO with a noise rejecting time average. The error in the location of the resonance peak $e(V_p)$, that is a positively valued uniform random variable with $w(V_p)$=δV interval width, translates into a corresponding error in the resonance frequency $e(f_r)$, assumed to be a uniformly distributed random variable with an interval width $w(f_r)= w(V_p)/Q$; the associated expanded uncertainty at k=2 is thus $u(f_r)=2\delta V/(Q\, 2\sqrt{3})= \delta V/(Q\sqrt{3})$. The factor of merit $Q=\omega_r L/R$ ranges from 20 to 50 for most cases, since the series resistance R is between 0.05 and 0.5 ohm for the observed f_r values in the MHz to tens of MHz range and the inductance L was estimated to be a few μH at the beginning of Section III.

So, the term $u(f_r)$ represents the most significant instrumental factor affecting the uncertainty on the ($C_{ext,i}$, l_i) pairs and thus the estimated variance of the error variable e. With reference to (12), the total uncertainty on the stray capacitance C_p, $u(C_p)$, at k=2 is

$$u(C_p) = t_{\alpha/2,n-2}\sqrt{\sigma_e^2 \frac{2}{n}} \cong 2\frac{\delta V}{Q\sqrt{3}}\sqrt{\frac{2}{n}} \tag{13}$$

V. Conclusions

The paper has considered the problem of determining the very low stray capacitance of air solenoids with purposely separated turns. Measured values and expressions to be used as reference are very few in the literature, and they have been briefly reviewed to support the proposed measurement method. This method is based on linear regression applied to the pairs of values of the resonance frequency and the external resonating capacitance; the application of the driving signal and the measurement of the electrical variables is done without any connection to the solenoid under test, not to influence the stray capacitance terms, but the coupling is magnetic by means of an external launching coil and a current probe. The sources of uncertainty (both instrumental and related to confidence interval of the regression algorithm) have been considered and evaluated and the method has been applied to one test case with a solenoid made with 4 mm radius wire. As a future activity, the estimation of the confidence intervals developed in section IV will be validated by a Type A approach, repeating the measurement in different days on the same solenoid used as the reference solenoid.

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