

## Comparative Testing of Dynamic Parameters DACs with Multi-Tone Signals

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**Abstract** - In this paper is presented methods for testing dynamic parameters DA converters with Multi-Tone signals. Results of this test with discrete multi-tone signal and equivalent amplitudes, AM and FM signals compared with standard Single-Tone Fourier Transform Test.

### I. Introduction

Standardized methods for *Single Tone Test* of ADCs and DACs are *Sine Wave Fit Test* and *Discrete Fourier Transform Test* [1], [2]. Typical dynamic parameters are *ENOB* (Effective Number of Bits), *THD* (Total Harmonic Distortion), *SNHR* (Signal Non-harmonic Ratio), *SINAD* (Signal Noise and Distortion), *SFDR* (Spurious Free Dynamic Range) and for *Double Tone Test* is defined the *IMD* (Inter Modulation Distortion). *Effective Number of Bits* is determined by standard deviation of difference between reconstructed and fitted signals  $\epsilon$

$$ENOB = n - \log_2 \frac{\epsilon}{q} \text{ (bit)} \quad (1)$$

where  $q = 2^{-n} / \sqrt{12}$  is ideal quantization error of  $n$  bit ADC. Following ADC parameters can be identified from spectral analysis of digitized single sine wave signal.

*Total Harmonic Distortion* is to the ratio of the first harmonic frequency  $U_1$  to root-mean-square (RMS) of the harmonics  $U_i$

$$THD = \frac{U_1}{\sqrt{\sum_{i=2}^n U_i^2}} \quad (2)$$

*Signal Non-Harmonic Ratio* is the ratio of the RMS value of the first harmonic frequency  $U_1$  to the RMS amplitude of the output noise  $U_{NF}$  (*Noise Floor*)

$$SNHR = \frac{U_1}{U_{NF}} \quad (3)$$

*Signal Noise and Distortion* is the ratio of the RMS value of the basic frequency to the mean value of the root-sum-square of all other spectral components, but excluding dc component.

$$SINAD = \sqrt{SNHR^2 + THD^2} \quad (4)$$

Interdependency between *Signal Noise and Distortion* and *Effective Number of Bits* is for single sine wave signal with FS DAC range given by following equation

$$SINAD = 6,02ENOB + 1,76[\text{dB}] \quad (5)$$

*Dynamic range* of tested DACs can be characterized by parameter *Spurious Free Dynamic Range*, which is ratio of the RMS value of the basic frequency  $U_1$  to RMS value of next higher harmonic or non-harmonic component  $U_m$ .

$$SFDR = \frac{U_1}{U_m} \quad (6)$$

*Inter-Modulation Distortion* can be several determined from spectrum of *Multi-tone* signals. *IMD* is the ratio of mean value of the RMS of the inter-modulation products. Frequencies of multi-tone signals are chosen as *comprise numbers* due to interference reduction between frequencies

When it's necessary to analyze frequency characteristic of mentioned parameters, these tests are relatively time-consuming. One possible way how to decrease duration of the test is to drive input of DAC by poly-harmonic signal [3]. Application of frequency swept signal which is suitable for economic tests of DAC testing are described in [4], [5].

## II. Test with Multi-Tone Signal

The multi-tone signal with discrete frequency components is defined by formula

$$u_{MT} = \sum_{i=1}^m U_i \sin(\omega_i t) \quad (7)$$

where  $U_i$  and  $\omega_i$ ,  $i = 1, 2, \dots, m$  are amplitudes and frequencies of  $m$  spectral component. *Signal Noise and Distortion* of tested DAC is dedicated

$$SINAD_{MT} = \sqrt{\frac{\sum_{i=1}^m U_i^2}{\sum_{i=kM/2, k=1, 2, \dots}^m U_i^2 - U_i^2}} \quad (8)$$

The *effective number of bits* is given by equation

$$ENOB_{MT} = \frac{SINAD_{MT} - 4,77 + 20 \log CF_{MT}}{6,02} \text{ (bit)} \quad (9)$$

The *Crest Factor*  $CF_{MT}$  of this poly-harmonic signal is

$$CF_{MT} = \frac{\sum_{i=1}^m U_i}{\sqrt{\sum_{i=1}^m \frac{U_i^2}{2}}} \quad (10)$$

For equivalent frequency components  $U_i = 1/2m$  is *Crest Factor*  $\sqrt{2m}$ .

In Table I. is summarized *Crest Factors*  $CF$ , depending of *Signal Noise and Distortions*  $\Delta SINAD$  and *Effective Number of Bits*  $\Delta ENOB$  for multi-tone test signals.

TABLE I. DEPENDING OF *SINAD* AND *ENOB* BY MULTI-TONE SIGNALS

Number of Tone	1	2	4	8	16	32	64	128	256
$CF_{MT}$	$\sqrt{2}$	2	$2\sqrt{2}$	4	$4\sqrt{2}$	8	$8\sqrt{2}$	16	$16\sqrt{2}$
$\Delta SINAD$ (dB)	0	- 3.01	- 6.02	- 12.04	- 15.05	- 18.06	- 21.07	- 24.08	- 27.0
$\Delta ENOB$ (bit)	0	- 0.5	- 1.0	- 1.5	- 2.0	- 2.5	- 3.0	- 3.5	- 4.0

For input voltage equal to full-scale of the DAC it is necessary to satisfy following condition.

### III. Test with Amplitude Modulated Signal

The amplitude modulation is created by forming product of carrier signal and modulation signal. If we considered zero phase shifts in both signals, waveform is given by next equation

$$u_{AM} = (U_n + U_m \cos \omega_m t) \cdot \sin \omega_n t = U_n \sin \omega_n t + \frac{U_m}{2} [\sin(\omega_n - \omega_m) \cdot t + \sin(\omega_n + \omega_m) \cdot t] \quad (11)$$

Modulation depth  $m_{AM} = U_m / U_n$  affects character of frequency spectrum of the signal. For  $m_{AM} < 1$  we consider *Dual Side Band* DSB amplitude modulation, its spectrum contains carrier with frequency  $\omega_n$  and amplitude  $U_n$  and two sideband components with frequencies  $\pm \omega_m$  and amplitude  $U_m/2$ .

If the amplitude of the carrier signal  $U_n = 0$  the carrier frequency is eliminated but the sidebands are preserved. This is called Double-sideband suppressed-carrier transmission. In fact we can use it as Dual Tone test with symmetrically distributed spectral components. Signal processing is the same as in classic Dual Tone methods. Signal generated by this way is possible to fit using LMS error method Multi tone Fit Test. Easier way to obtain results is to do spectral analysis of AM signal.

The *Signal-to-Noise and Distortion*  $SINAD_{AM}$  is from amplitude modulated signal defined by formula

$$SINAD_{AM} = \sqrt{\frac{\frac{U_n^2}{2} + \frac{U_m^2}{4}}{\sum_{i=KM/2, k=1, 2, \dots}^p U_{f_i}^2 - \frac{U_n^2}{2} - \frac{U_m^2}{4}}} \quad (12)$$

*Effective number of bits* is equivalent defined by formula (9). The *Crest Factor*  $CF_{AM}$  of amplitude modulated signal is given

$$CF_{AM} = \frac{1 + 2m_{AM}}{\sqrt{1 + m_{AM}^2}} \quad (13)$$

For input voltage equal to full-scale of the ADC it is necessary to satisfy following condition.

In Table II. is summarized *Crest Factors*  $CF$ , depending of *Signal Noise and Distortions*  $\Delta SINAD$  and *Effective Number of Bits*  $\Delta ENOB$  for AM signals.

Table II. Depending of  $SINAD$  and  $ENOB$  by AM signals

$m_{AM}$	0.25	0.5	1.0
$CF_{AM}$	$\sqrt{2}$	$2/\sqrt{1.25}$	$3/\sqrt{2}$
$\Delta SINAD$ (dB)	0	-2.09	-3.52
$\Delta ENOB$ (bit)	0	-0.35	-0.58

### IV. Test with Frequency Modulated Signal

Principle of frequency modulation consists in modulation of carrier frequency according the expression  $\omega_n(t) = \omega_{n0} + \Delta\omega \cos \omega_m t$ , where  $\Delta\omega$  is frequency deviation of modulated signal and  $m_{FM} = \Delta\omega / \omega_m$  is modulation index. Waveform of the frequency modulated signal is

$$u_{FM} = U_n \sin \left( \omega_n t + \frac{\Delta\omega}{\omega_m} \sin \omega_m t \right) \quad (14)$$

Spectrum of the *FM* consists of carrier frequency  $\omega_n$  and symmetrically displaced spectral component around carrier  $\omega_n$  with multiples of frequency  $\omega_m$ . Amplitudes of spectral components are given by Bessel's function first order with argument  $\Delta\omega / \omega_m$ .

In Table III. is amplitudes of the spectrum of frequency modulated signal are shown for *modulation index*  $m_{FM} = \Delta\omega/\omega_m$  in range from 0 to 2.

Table III. Spectral magnitudes of FM signal

$m_{FM}$	$n$	$n \pm m$	$n \pm 2m$	$n \pm 3m$
0	1.00			
0.25	0.98	0.12		
0.50	0.94	0.24	0.03	
1.00	0.77	0.44	0.11	0.02
1.50	0.51	0.56	0.23	0.06
2.00	0.22	0.58	0.35	0.13

Modulation indexes 0.25, 0.5 and 1 are suitable for dynamic DAC testing. Hence, the amplitudes of all other spectral components are smaller than the amplitude of carrier frequency.

Fitting this signal is similar as in previous case. It is also possible to use *LMS* error method *Multi-Tone Fit Test*, but for this case is very hard to optimize much more parameters. Another way is to evaluate a signal in spectral domain and determine *Signal Noise and Distortion SINAD<sub>FM</sub>*

$$SINAD_{FM} = 6,02ENOB_{FM} + 4,77(dB) \quad (15)$$

*Crest Factor C<sub>FM</sub>* for FM signal is independent modulated index and is equal 1.

### V. Experimental Test System and Results

For verification of this method has been used *Data Acquisition System* with PXI Controller PXIe8130 from National Instruments. For generation of multi-tone signals has been used *Multifunction Data Acquisition Module* NI PXIe-6251 with 16 bit ADCs and 16 bit DACs. For all experiments has been reconstructed basic sine signal with frequency 5 kHz with sample frequency 1,048576 MS/s. The output signal from 16 bit DAC under test has been used digitizer NI PXI 5922 with nominal resolution 16 bit and with sample frequency 15 MSa/s.

Examples of time and frequency spectral plots for multi-tone signals are shown in Fig.1 to Fig. 8.

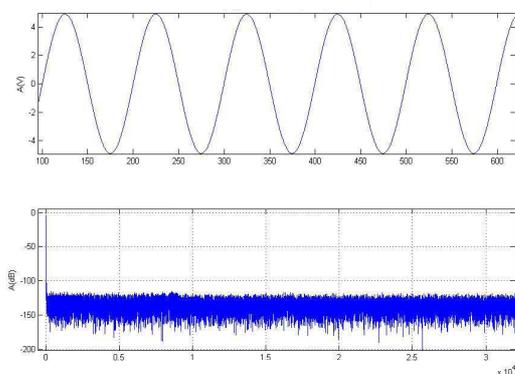


Figure 1. Time and FFT plot for 1 tone test

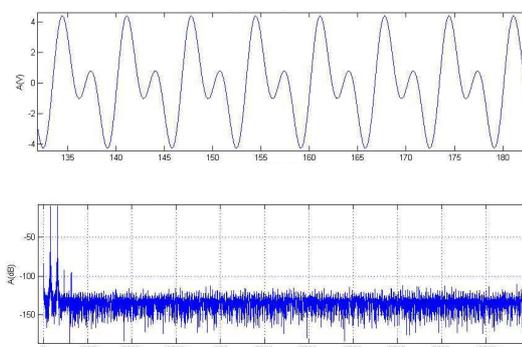


Figure 2. Time and FFT plot for 2 tone test

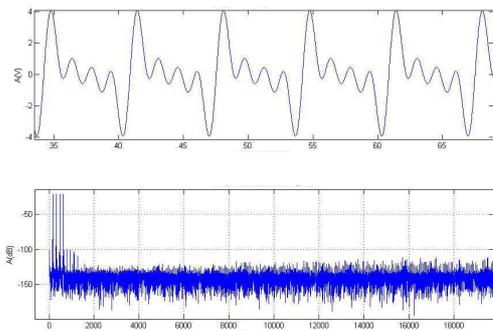


Figure 3. Time and FFT plot for 4 tone test

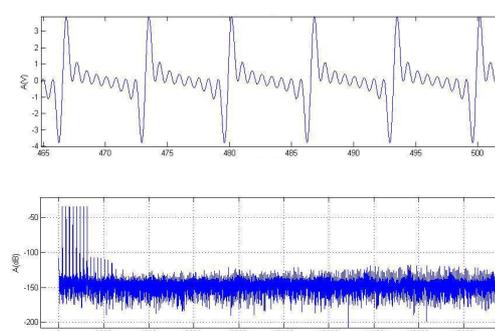


Figure 4. Time and FFT plot for 8 tone test

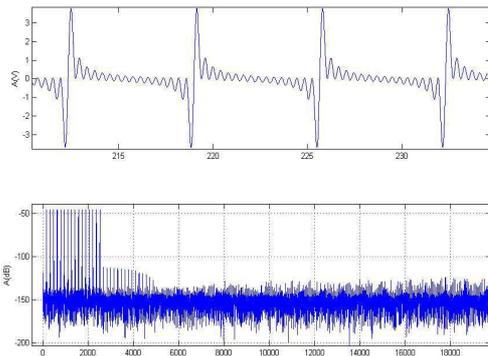


Figure 5. Time and FFT plot for 16 tone test

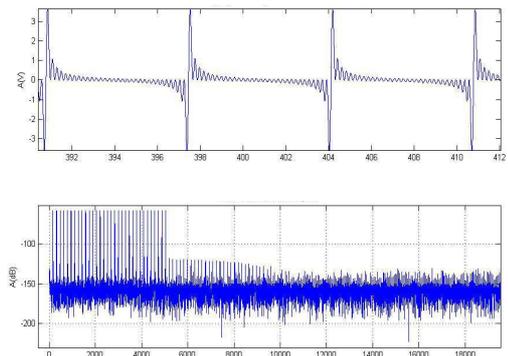


Figure 6. Time and FFT plot for 32 tone test

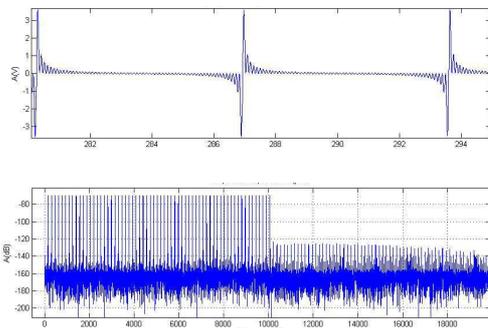


Figure 7. Time and FFT plot for 64 tone test

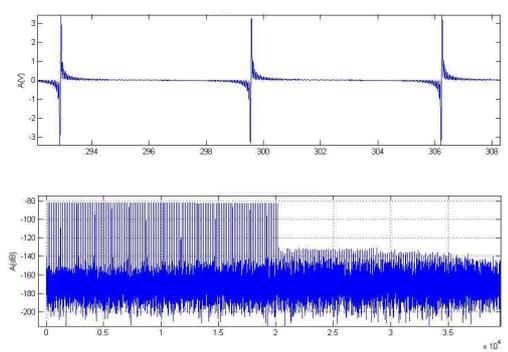


Figure 8. Time and FFT plot for 128 tone test

Experimental results of these methods are presented in Table III. It's obvious that *ENOB* is decreasing with number of used tones in multi-tone methods in comparison with single-tone method.

Table III. Results for discrete multi-tone DAC test

Signal	<i>BW</i> (kHz)	<i>SINAD</i> (dB)	<i>ENOB</i> (bit)	<i>SNHR</i> (dB)	THD (dB)	<i>SFDR</i> (dB)
$m = 1$	5	88.4	14,4	88.5	- 96.9	94.5
$m = 2$	10	86.2	14,0	87.3	- 94.5	91.3
$m = 4$	20	84.1	13,7	84.8	-	86.5
$m = 8$	40	82.9	13,5	84.3	-	84.4
$m = 16$	80	79,2	13,0	81,5	-	83,5
$m = 32$	160	76,4	12,6	79,2	-	81,4
$m = 64$	320	73,8	12,1	76,4	-	79,6
$m = 128$	640	70,6	11,6	75,3	-	77,5
AM signal $m_{AM} = 0,5$	$50 \pm 10$	86,1	14,3	74,5	-	75,7
AM signal $m_{AM} = 1$	$50 \pm 10$	85,6	13,9	74,5	-	75,7
FM signal $M_{FM} = 0,5$	$50 \pm 20$	85,2	14,2	73,6	-	67,3
FM signal $M_{FM} = 1$	$50 \pm 30$	84,6	14,1	73,4	-	62,4

- not defined

The parameters *ENOB* and *SINAD* in Table III is *average value* in frequency bandwidth *BW* of tested signals.

## VI. Conclusions

The aim of this work is to verify possibilities of DAC testing using poly-harmonics signals. These signals are generated by internal 16 bit DACs in *Multifunction Data Acquisition Module NI PXIe-6251*. Signal processing of these signals in time domain using fitting methods is quite complicated.

For Multi-tone signals with equivalent amplitudes the theoretical dependence of Signal Noise and Distortion  $\Delta SINAD = -3$  dB/ton and dependence of Effective Number of Bits  $\Delta ENOB = 1/\sqrt{m}$  bit, where  $m$  is number of tones. For amplitude modulated signals it depends  $\Delta SINAD = -2,1$  dB and  $\Delta ENOB = 0,35$  bit for modulation depth 0,5 and  $\Delta SINAD = -3,5$  dB and  $\Delta ENOB = 0,6$  for modulation depth 1. For frequency modulated signals it is not dependent of  $SINAD$  and  $ENOB$ . The practical results are approximately equivalent with theoretical results with differences  $\Delta SINAD \approx 0,5$  dB.

The advantage of this method is time reduction and ascertaining parameters in wider frequency range without need of measuring their frequency characteristic. Disadvantage of this method is indeterminacy  $THD$  and  $IMD$ . Therefore it is possible to suppose, that stated methods find application in industry in less demanding economical tests.

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