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Transient Reduction in Pulse-Based Impedance Measurements

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Abstract- Impedance measurements based on voltage pulse injection and current detection are faster and easier to implement than measurements based on sine waves, but transients produced in the current-to-voltage conversion may lead to large deviations in the measurement result. Because those transients depend on the same impedance being measured, solutions based on specific circuit values fail for large measurement ranges. We propose to reduce those transients by controlling the rise time of the pulse being applied to the impedance under test. For a particular implementation of the method, transient amplitude has been reduced to less than 1 % of the full scale voltage, as compared to 40 % overshoot when faster voltage pulses were applied.

I. Introduction

Pulses are easier to generate than sine waves hence can be advantageous for electrical impedance measurement. If the pulse rise time is short enough, its high-frequency components short the capacitances in the impedance and only the series resistance is measured, which is particularly convenient in two-electrode impedance measurements as their impedance becomes negligible [1] [2] [3]. Usually, a voltage pulse is injected because it is easier to generate than a current pulse, and the resulting current is detected with a transimpedance amplifier [2] [3]. For impedances modelled by a simplified Randles cell, the series resistance equals the voltage amplitude divided by the initial value of the resulting current pulse. Ideally, this current pulse shows an exponentially decaying slope, but in practice the output voltage of the transimpedance amplifier displays transients unrelated to the target impedance. Those transients arise from parasitic capacitances or inductances (for example in cables), or from the transimpedance amplifier itself, which transfer function is second-order or higher [4]. Large parasitic capacitances from the input and output of the transimpedance impedance to signal ground can even result in amplifier instability. Therefore, even though short wiring helps in reducing parasitic capacitances and inductances, it cannot avoid undesired transients that affect the initial value of the resulting current pulse. Transients can be reduced by adding feedback capacitors or by increasing the amplifier output resistance [4], but the component values required to ensure stability may depend on the impedance being measured. In this paper we propose to reduce that transient by lengthening the rise time of the voltage pulse being injected.

II. Circuit Model

Figure 1(a) shows the equivalent circuit when the impedance under test Z_m is measured by applying a voltage v_{in} across it and the resulting current is converted into a voltage v_{out} by an op-amp-based transimpedance amplifier. When measuring in liquids, Z_m is usually modelled by a Randles cell [1] as in Figure 1(b); R_p and C_p model the two electrodes and R_s is the bulk resistance of the electrolyte, here an aqueous ionic dissolution.

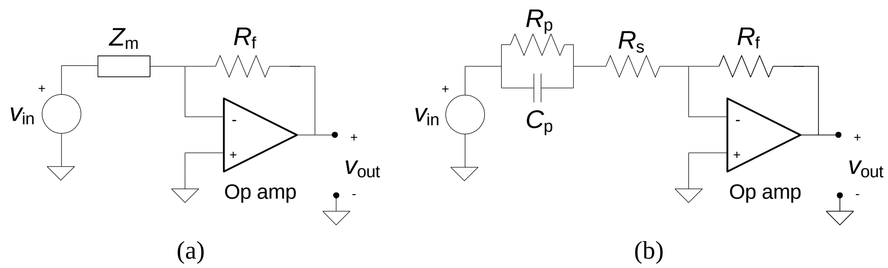


Figure 1. (a) Circuit to measure Z_m by injecting a voltage pulse and detecting the current through it. (b) Randles model for Z_m.

If the voltage step V_p in Figure 2(a) is applied to the input terminal in Figure 1, the output voltage is,

$$v_{\text{out}}(t) = \frac{-R_f V_p}{R_s + R_p}\left(1 + \frac{R_p}{R_s} e^{-\frac{t}{\tau}}\right) \tag{1}$$

where $\tau = (R_s \| R_p)C_p$. At $t = 0$, $v_{out}(0) = -R_f V_p/R_s$; therefore, from V_p and R_f we can obtain R_s. The high-frequency components associated to the initial transition short C_p and only R_s is measured. The shape of v_{out} is an exponentially decaying curve whose initial (and final) value are independent from C_p, as shown in Figure 2(b).

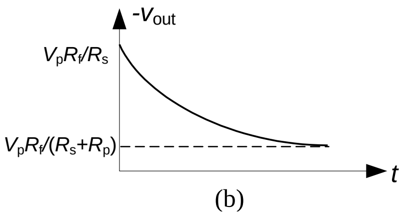


Figure 2. (a) Step voltage input applied to measure Z_m in Figure 1. (b) Voltage output response.

If the voltage step has finite rise time t_r, as in Figure 3(a), then the output voltage is

$$v_{\text{out}}(t) = \frac{-R_f V_p}{R_s + R_p}\left(\frac{t}{t_r} + \frac{R_p}{R_s}\frac{\tau}{t_r}\left[1 - e^{-\frac{t}{\tau}}\right]\right) \tag{2}$$

for $0 \le t \le t_r$ and

$$v_{\text{out}}(t) = \frac{-R_f V_p}{R_s + R_p}\left(1 + \frac{R_p}{R_s}\frac{\tau}{t_r}\left[e^{-\frac{t-t_r}{\tau}} - e^{-\frac{t}{\tau}}\right]\right) \tag{3}$$

for $t \ge t_r$. If $t_r << \tau$, we can approximate $\exp(-t_r/\tau) \approx 1 - t_r/\tau$. Then, when $t = t_r$ (2) and (3) yield

$$v_{\text{out}}(t_r) \approx \frac{-R_f V_p}{R_s + R_p}\left(\frac{t_r}{t_r} + \frac{R_p}{R_s}\frac{\tau}{t_r}\left[1 - 1 + \frac{t_r}{\tau}\right]\right) = \frac{-R_f V_p}{R_s} \tag{4}$$

which is the same voltage obtained in (1) when $t = 0$. That is, if the rise time is short enough relative to τ, R_s can be obtained by measuring the output voltage when the leading edge of the pulse ends. However, parasitic capacitances in Figure 1 can result in transients at that time.

Figure 3(b) shows the simplified equivalent circuit when impedance R_s is measured by applying a voltage v_{in} across it with two electrodes which impedance is negligible at the measurement frequency. C_L is the load capacitance and C_{in} is the op amp input capacitance.

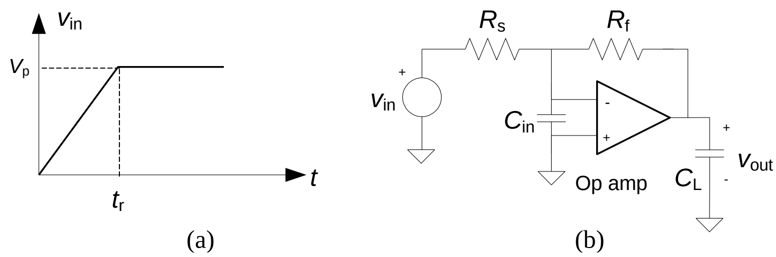


Figure 3. (a) Injected voltage pulse with controllable rise time (t_r). (b) Stray capacitances in the circuit to measure R_s by injecting a voltage pulse and detecting the current through it.

The complete transfer function when both C_{in} and C_L are considered is

$$\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = -\frac{R_{\text{f}}}{R_{\text{s}}} \frac{1}{1+\left(1+\frac{R_{\text{f}}}{R_{\text{s}}}\right)\frac{s}{\omega_{\text{t}}}+\left(R_{\text{f}}C_{\text{in}}+\left[1+\frac{R_{\text{f}}}{R_{\text{s}}}\right]R_{\text{o}}C_{\text{L}}\right)\frac{s^2}{\omega_{\text{t}}}+R_{\text{f}}C_{\text{in}}R_{\text{o}}C_{\text{L}}\frac{s^3}{\omega_{\text{t}}}} \tag{5}$$

This low-pass third-order transfer function can be decomposed in a first-order system (real pole) connected to a second-order system (two real or complex poles) [5]. Therefore, if v_{in} is a voltage pulse, transient analysis only needs to consider the step response of the second-order system because the first-order system will contribute only some attenuation to that response [5].

The transfer function of a low-pass second-order system is

$$\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = -\frac{R_{\text{f}}}{R_{\text{s}}} \frac{1}{1+\frac{2\zeta}{\omega_{\text{n}}}s+\frac{s^2}{\omega_{\text{n}}^2}} \tag{6}$$

where ζ is the damping factor and ω_{n} is the natural angular undamped frequency. If $C_{\text{in}} = 0$, (5) becomes a second-order transfer function with $\zeta = 0.5[(1 + R_{\text{f}}/R_{\text{s}})/R_{\text{o}}C_{\text{L}}\omega_{\text{t}}]^{1/2}$ and $\omega_{\text{n}} = [\omega_{\text{t}}/(1 + R_{\text{f}}/R_{\text{s}})R_{\text{o}}C_{\text{L}}]^{1/2}$. If $C_{\text{L}} = 0$, (5) also becomes a second-order transfer function but now $\zeta = 0.5[(1 + R_{\text{f}}/R_{\text{s}})/(R_{\text{f}}C_{\text{in}}\omega_{\text{t}})]^{1/2}$ and $\omega_{\text{n}} = [\omega_{\text{t}}/R_{\text{f}}C_{\text{in}}]^{1/2}$. ω_{t} is the full power bandwidth and R_{o} is the op amp's open loop output resistance. Whenever $\zeta < 1$, the two poles of the transfer function (6) are complex, the step response displays an overshoot [4] and the time response is a damped oscillation around the final value [5]. Therefore, because both ζ and ω_{n} depend not only on op amp parameters but also on R_{s}, whenever $C_{\text{in}} \neq 0$ or $C_{\text{L}} \neq 0$, it is difficult to guarantee $\zeta \geq 1$ hence no overshoot. As a result, R_{s} cannot be obtained from v_{in} and the current (hence v_{out}) at $t = 0^+$.

When $\zeta < 1$, the normalized step response of a low-pass second-order system is [6],

$$\frac{v_{\text{out}}(t)/V_{\text{p}}}{-R_{\text{f}}/R_{\text{s}}} = 1 + \frac{e^{-\zeta\omega_{\text{n}}t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_{\text{d}}t+\varphi\right) \tag{7}$$

where $\omega_{\text{d}} = \omega_{\text{n}}(1 - \zeta)^{1/2}$ and $\varphi = \cos^{-1}\zeta$. The overshoot is the maximum of the right-side term added to 1. If this overshoot results from $C_{\text{in}} \neq 0$, the problem can be solved by shunting the feedback resistor with a capacitor C_{f} [7]. However, this is not practical as the appropriate C_{f} value depends on the resistance R_{s} being measured. If the oscillation is caused by $C_{\text{L}} \neq 0$, the usual solution, termed "capacitive load isolation" [7], is still more difficult to implement as it requires an additional feedback capacitor and an output resistor which values depend on the load capacitance C_{L} and R_{s}. Therefore, that solution, which is effective to stabilize voltage drivers for capacitive loads, is not useful here.

The overshoot in (7) results from the bigger gain of the circuit at frequencies around its resonance frequency. A sudden change, such as a step with zero rise time like that leads to (7), includes high-frequency components that are amplified by a gain larger than that for lower frequencies. Therefore, slowing the transition lessens those high-frequency components and could eliminate the overshoot.

If the applied voltage pulse has finite rise time t_{r}, as shown in Figure 3(a), the output voltage of a low-pass second-order system is [6]

$$\frac{v_{\text{out}}(t)/V_{\text{p}}}{-R_{\text{f}}/R_{\text{s}}} = \frac{t}{t_{\text{r}}} - \frac{2\zeta}{\omega_{\text{n}}t_{\text{r}}} + \frac{e^{-\zeta\omega_{\text{n}}t}}{\omega_{\text{d}}t_{\text{r}}}\sin\left(\omega_{\text{d}}t+2\varphi\right) \tag{8}$$

for $0 \leq t \leq t_{\text{r}}$ and

$$\frac{v_{\text{out}}(t)/V_{\text{p}}}{-R_{\text{f}}/R_{\text{s}}} = 1 + \frac{e^{-\zeta\omega_{\text{n}}t}}{\omega_{\text{d}}t_{\text{r}}}\sin\left(\omega_{\text{d}}t+2\varphi\right) - \frac{e^{-\zeta\omega_{\text{n}}(t-t_{\text{r}})}}{\omega_{\text{d}}t_{\text{r}}}\sin\left[\omega_{\text{d}}\left(t-t_{\text{r}}\right)+2\varphi\right] \tag{9}$$

for $t \geq t_{\text{r}}$. The exponential terms in (9) are divided (attenuated) by $\omega_{\text{n}}t_{\text{r}}$ as compared to those in (7) hence the overshoot will be smaller [6]. Now, the v_{out} value from which the "initial" current value can be calculated is not that at $t = 0^+$ but that at $t = t_r$, that is, when the ramp ends.

III. Experimental Results and Discussion

We have built the circuit in Figure 3(b) with $R_s = R_f = 1$ kΩ and a fast op amp (OPA2355), and injected a -200 mV pulse (5 µs duration) from a function generator (Agilent 33220A), with adjustable rise time from $t_r \approx 35$ ns to $t_r = 0.5$ µs. The tolerance of passive component was ±1 % for resistors and ±10 % for capacitors. The results have been recorded by an Agilent DSO-X 2012A oscilloscope.

Figure 4 shows the step response of the circuit in Figure 3(b) when 100 pF have been added to C_{in} (1.5 pF according to its specifications [8]) and C_L is from about 10 pF to 15 pF. Such a large C_{in} could be the capacitance of 1 m of coaxial cable [9] connected to a distant measurement cell, whereas C_L is within the range of the equivalent input capacitance of a 10:1 oscilloscope probe [10]. $C_f \approx 1$ pF is the parallel equivalent of stray capacitance of R_f and the land pattern of the printed circuit board [11].

When $t_r \approx 35$ ns, the overshoot is about 40 %, Figure 4(a); adding 10 pF to C_f reduces overshoot to 3 % (solid line in Figure 4(b)). If $R_s = 510$ Ω overshoot decreases to 1.6 %, if $R_s = 2$ kΩ overshoot increases to 4.8 %, and if $R_s = 7.5$ kΩ overshoot increases to 9.6 %. If $C_f = 22$ pF, there is no overshoot, as the broken line in Figure 4(b) shows, but the output value at t_{max} (Figure 4(b)) is 16.5 % below the final (correct) value, hence the deviation is larger than that because of the overshoot when $C_f = 10$ pF. These results corroborate that overshoots result from excessive gain at some high frequencies, but if C_f is increased to reduce overshoot, on the one hand the amplifier bandwidth decreases and, on the other hand, if R_s increases overshoot can reappear.

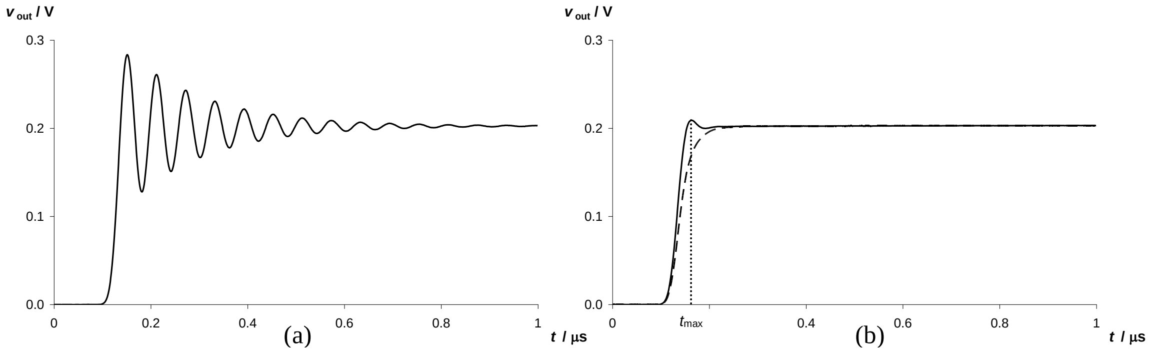


Figure 4. Output voltage from the circuit in Figure 3(b) with $C_{in} = 100$ pF when the input voltage v_{in} is a -200 mV step with $t_r \approx 35$ ns. (a) When $C_f = 1$ pF, there is overshoot. (b) When $C_f = 10$ pF the overshoot is smaller (solid line) and when $C_f = 22$ pF, there is no overshoot (broken line).

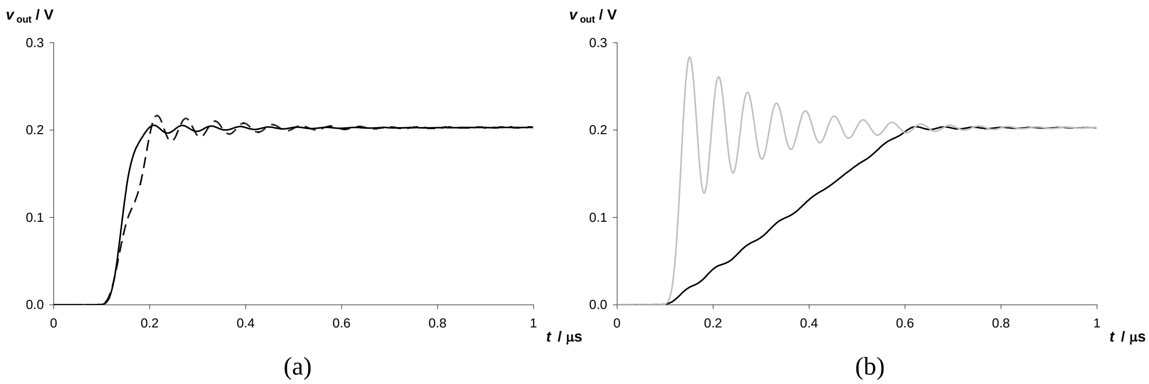
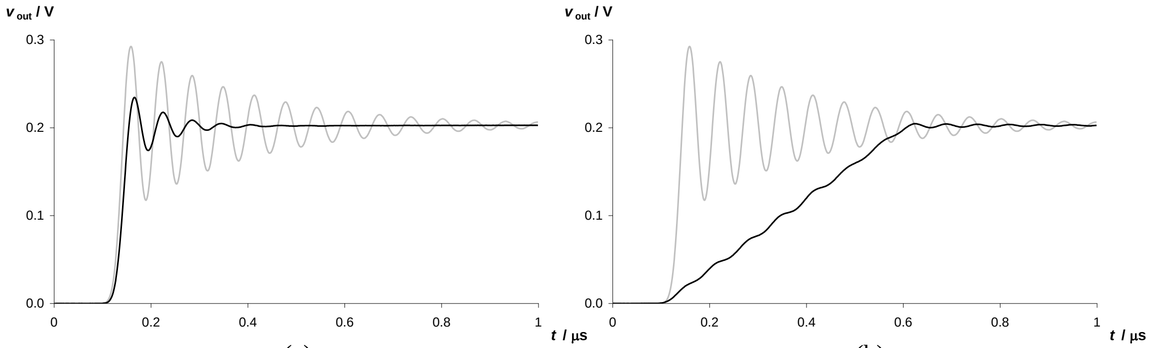


Figure 5. Output voltage from the circuit in Figure 3(b) with $C_{in} = 100$ pF and $C_f = 1$ pF when a -200 mV pulse has been applied. (a) With $t_r = 60$ ns (solid line) and $t_r = 100$ ns (broken line). (b) With $t_r = 0.5$ µs (black line); the grey line has been redrawn from Figure 4(a) for reference.

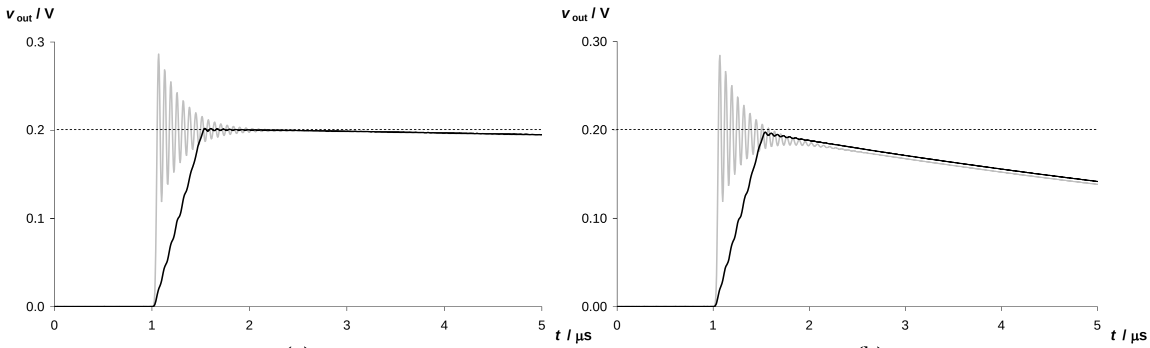
Figure 5 shows the step response of the circuit in Figure 3(b) in the same conditions as in Figure 4(a) with regard to capacitance values but for different rise times. When t_r = 60 ns the overshoot is 1.3 % (solid line in Figure 5(a)), down from 40 % when $t_r \approx 35$ ns. Surprisingly, when t_r is increased to 100 ns the overshoot does not decrease but increases to 7 % (broken line in Figure 5(a)). This is because the period of the damped oscillation ($T_d = 1/\omega_d$) is 60.6 ns, hence close to 60 ns, and the two sine waves in (9) near cancel out each other when t_r = 60 ns. The same would happen for other t_r values multiple of 60 ns. Otherwise, a longer the rise time yields a smaller overshot; in Figure 5(b) (black line) it is only 0.6 % for t_r = 0.5 µs. (black line), as compared to 40 % (grey line) for $t_r \approx 35$ ns. Since the rise time is known in advance, the output voltage can be sampled 0.5 µs after the pulse is applied in order to obtain the "initial" current value after the ramp ends. Ripple on the leading edge of the output will not affect.



(a) (b)

Figure 6. Output voltage from the circuit in Figure 3(b) when the input voltage v_{in} is a -200 mV step with $t_r \approx 35$ ns and C_L = 1 nF (grey line, C_f = 1 pF). (a) When C_f = 10 pF, there is overshoot. (b) With t_r = 0.5 µs (black line) the overshoot is largely reduced.

Figure 6 shows the effect on the step response of the circuit in Figure 3(b) to pulses with $t_r \approx 35$ ns caused by adding 1 nF to C_L when $C_{in} \approx 2$ pF (1.5 pF from the op amp plus 0.5 pF from the PCB lands). The overshoot is 44 % (grey line) and the ringing period is 63 ns (similar to that when $C_{in} \approx 100$ pF). If C_f is increased to 10 pF in order to compensate for C_{in}, the oscillation is reduced but a 16 % overshoot remains (black line in Figure 6(a)). However, if t_r = 0.5 µs, the overshoot reduces to 1 % (black line in Figure 6(b)), the same as in Figure 5(b). Therefore, long rise times can avoid transient effects from large C_{in} or C_L if the output pulse is measured just after the input reaches its maximum.



(a) (b)

Figure 7. Output voltage from the circuit in Figure 1(b) when the input voltage v_{in} is a -200 mV step with t_r = 35 ns and C_L = 1 nF (grey line) and with t_r = 0.5 µs (black line). (a) When C_p = 100 nF. (b) When C_p = 10 nF.

Figure 7 (grey lines) shows the output response of the circuit in Figure 1(b) to a -200 mV step voltage with $t_r \approx 35$ ns caused by adding 1 nF to C_L, when $R_s = R_f = 1$ kΩ, R_p = 10 kΩ and C_p = 100 nF, Figure 7(a), or C_p = 10 nF, Figure 7(b). If t_r is increased to 0.5 µs, the overshoot reduces from 43 % to 0.8 % (black line), in Figure 7(a) and to -0.6 % in Figure 7(b). In the four cases, after the transient dies out, the output voltage displays an exponentially decaying slope because of R_p and C_p. When C_p = 100 nF, the effect is negligible, but when

C_p = 10 nF (Figure 7(b)), $\tau \approx$ 9.1 µs and t_r = 0.5 µs, hence the condition $\tau >> t_r$ that leads to (4) is not longer acceptable and $v_{out}(t_r) \neq -R_f V_p / R_s$. Nevertheless, the deviation is only -0.6 %, much smaller than that when there is overshoot.

IV. Conclusions

The simplicity advantage of voltage step excitation as compared to sine wave excitation in impedance measurements can be counterweight by the difficulties in accurately measuring the response to that excitation. When a voltage pulse is applied to an impedance modelled by a simplified Randles cell, whose high-frequency equivalent is the resistance of interest, the output current pulse can display transient oscillations superimposed on a decaying exponential waveform. Because the relevant output value is the current just after the voltage pulse has been applied, those oscillations can lead to gross errors. Often, these transients arise in the transimpedance amplifier and depend on the op amp, the capacitance from its input and output to ground, feedback components, and the impedance being measured. Therefore, amplifier design alone cannot guarantee small transients amplitude.

We have shown, analytically and experimentally, that a controlled rise time is more effective. If the rise time t_r is relatively slow but much faster than the time constant τ of the impedance being measured, the deviation of the step response at t_r is smaller than that at $t \approx 0$ s when t_r is very fast. Analytically, if there were no transients, when $t_r \neq 0$ s the output response at $t = t_r$ is the same than the step response at $t \approx 0$ s when $t_r = 0$ s. If there are transients, their amplitude can be reduced by increasing the rise time of the pulse being injected.

Commonly, transients in transimpedance amplifiers because of C_{in} are reduced by designing a suitable feedback capacitor C_f. This C_f value, however, will work only for a narrow range of R_s values. Transients caused by capacitance C_L connected to the amplifier output are not effectively reduced. We have shown that increasing the rise time t_r of the voltage pulse can reduce transients that result from either C_{in} or C_L. If $t_r \approx T_d$, the period of the damped oscillation, the attenuation can be very small but shows large variations depending on how close is t_r to T_d, which depends on the square root of R_s. If $t_r > 10T_d$, transients decrease with increasing t_r. If $T_d << t_r << \tau$, transients are below 1 % for the R_s range tested (500 Ω to 7500 Ω), and the output response at t_r is very close to the ideal (transient-free) step response at $t \approx 0$ s. If the condition $t_r << \tau$ is not true, the output voltage at t_r is below the ideal response at $t \approx 0$ s.

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References

[1] A. Mejía-Aguilar and R. Pallas-Areny, "Electrical impedance measurement using voltage/current pulse excitation", XIX IMEKO World Congress, pp. 662-667, Lisbon, Portugal, 2009.
[2] D.E. Johnson and C.G. Enke, "Bipolar pulse technique for fast conductance measurements," Anal. Chem., vol. 42, No. 3, pp. 329-335, 1970.
[3] C.E.B. Neves and M.N. Souza, "A method for bio-electrical impedance analysis based on a step-voltage response", Physiol. Meas., vol. 21, pp. 395-408, 2000.
[4] J. Dostál, Operational Amplifiers, 2nd ed. Butterworth-Heinemann, Boston, 1993.
[5] K. Ogata, Modern Control Engineering, 2nd ed. Prentice Hall, New Jersey, 1990.
[6] E.O. Doebelin, Measurement Systems: Application and Design, 4th ed. McGraw-Hill, New York, 1990.
[7] S. Franco, Design with Operational Amplifiers and Analog Integrated Circuits, 2nd ed. McGraw-Hill, New York, 1998.
[8] OPA2355 datasheet, "200 MHz, CMOS Operational Amplifier with Shutdown", Texas Instruments, 2004.
[9] C. S. Walker, Capacitance, Inductance, and Crosstalk Analysis, Artech House, New York, 1990.
[10] ABC of Probes, Tektronix, 2005.
[11] H. W. Ott, *Noise Reduction Techniques in Electronic Systems*, 2nd ed. John Wiley & Sons, New York, 1988.