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Measurement of Frequency Dependences of Low-Value Four-Terminal Resistance Standards

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Abstract – A 10 Ω quadrifilar resistor with calculable frequency dependence served as a reference in measurement of frequency dependences of resistance standards ranging from 1 Ω to 0.01 Ω in a frequency range up to 10 kHz. In this measurement, a step-down method based on successive 10:1 comparisons of the standards by means of a transformer bridge was used. Ratio arms of this bridge being formed by an eight-decade, two-stage inductive voltage divider, a small variable voltage injected into one connecting lead of the ratio winding of the divider serves for quadrature balancing.
Keywords – Current shunt, calculable resistor, transformer bridge.

I. Introduction

Four-terminal resistance standards of low values are often used as current-to-voltage converters in measurements of both DC and AC currents. All relevant metrological parameters and characteristics of these standards should be known when evaluating the results of such measurements. In the case of standards for AC measurements, their frequency dependences in ranges covering frequencies of all significant harmonic components of the measured currents have to be considered.

Frequency dependence of a resistance standard can most easily be determined by its 1:1 comparison with a standard of the same nominal value having negligible, known or calculable frequency dependence [1]. In this paper, measurement of frequency dependences of 1 Ω (RS 1), 0.1 Ω (RS 0.1) and 0.01 Ω (RS 0.01) resistance standards is described for which only one 10 Ω calculable resistor (CR 10) was available and in the course of which successive 10:1 comparisons of the standards (CR 10 with RS 1, RS 1 with RS 0.1 and, finally, RS 0.1 with RS 0.01) were performed.

II. Reference resistor with calculable frequency dependence

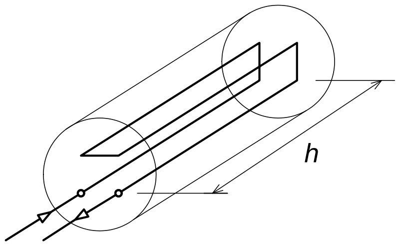

Figure 1. Resistive element of a quadrifilar resistor

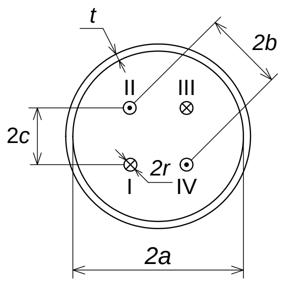

Figure 2. Cross-section of a quadrifilar resistor

A 10 Ω Gibbings´s type quadrifilar resistor [2] served as the reference standard. As can be seen from Figures 1 and 2, this resistor is provided with a thin cylindrical shield and its resistive element (resistance wire circular in cross-section) takes the form of a double loop arranged so that the current flows in opposite directions in its two halves. With such an arrangement, magnetic fluxes produced by the two halves of the loop tend to cancel. In calculating frequency dependence of this resistor, changes in resistance arising from parasitic inductances and capacitances, as well as from eddy currents induced both in the wire itself and in the shield, have to be evaluated.

A uniform transmission line model was used in the calculation of the effect of parasitic inductances and capacitances, and the parallel equivalent resistance of quadrifilar resistor has been found to be

$$R_{\mathrm{p}} \approx R\left\{1+\frac{\omega^2}{R^2}\left(L-8M_1+4M_2\right)^2+\frac{\omega^2R^2}{11520}\left[240C_0^2-15\left(C_0+8C_1+8C_2\right)^2-\left(C_0+16C_1\right)^2\right]\right\} \qquad (1)$$

where ω is the angular frequency,
R is the resistance of the wire,
L is the self inductance of the wire,

M_1 is the mutual inductance between a pair of adjacent parts of the wire (e.g. between I and IV – see Figure 2),
M_2 is the mutual inductance between a pair of diagonally opposite parts of the wire (e.g. between I and III),
C_0 is total capacitance between the wire and the shield,
C_1 is the capacitance between a pair of adjacent parts of the wire,
C_2 is the capacitance between a pair of diagonally opposite parts of the wire, and
C_t is the capacitance between the terminal leads.

Time constant τ of the resistor is

$$\tau \approx \frac{1}{R}\left(L-8M_1+4M_2\right)-\frac{R}{6}\left(5C_1+3C_2-C_0+6C_t\right) \tag{2}$$

The increase in resistance due to the skin effect was calculated from

$$R \approx R_0\left[1+\frac{1}{192}\omega^2\left(\frac{\mu_1\,\mu_0\,r^2}{\rho_1}\right)^2\right] \tag{3}$$

where R is the resistance of the wire at an angular frequency ω, R_0 is dc resistance of the wire, r is the wire radius, ρ_1 is the wire resistivity, μ_1 is relative permeability of the wire and $\mu_0 = 4\,\pi \times 10^{-7}$ H/m is the magnetic constant.

For calculating the increase in resistance due to eddy-current losses in the shield the following formula was used:

$$R \approx R_0\left[1+\frac{8\mu_2}{\pi}\frac{m\omega}{R/h}\frac{1}{4+m^2}\left(\frac{b}{a}\right)^4\right] \tag{4}$$

where $m=\omega\mu_2\,\mu_0\,a\,t/\left(2\rho_2\right)$, ρ_2 is the shield resistivity and μ_2 is relative permeability of the shield. For dimensions a, t and h see Figures 1 and 2.

Resistive double loop of the 10 Ω quadrifilar resistor being made from bare Zeranin wire, 0.165 mm in diameter, the distance between adjacent parts of the wire is 10 mm and the length of the folded loop is 116 mm. The loop is surrounded by a copper cylindrical shield having an inner diameter of 116 mm and a wall thickness of 4 mm.

With the exception of the capacitance C_t which was measured, all other parameters necessary for determining R_p and τ from (1) and (2) were calculated.

By the method of capacitive coefficients, the following formulas were found for C_0, C_1 and C_2 [3]:

$$C_0 = 4h\,\frac{1}{p_{11}+p_{12}+2\,p_{13}} \tag{5}$$

$$C_1 = h\,\frac{p_{13}}{\left(p_{11}+p_{12}\right)^2-4\,p_{13}^2} \tag{6}$$

and

$$C_2 = h\,\frac{p_{12}\left(p_{11}+p_{12}\right)-2\,p_{13}^2}{\left(p_{11}-p_{12}\right)\left[\left(p_{11}+p_{12}\right)^2-4\,p_{13}^2\right]} \tag{7}$$

where

$$p_{11} = \frac{1}{2\pi\varepsilon\,\varepsilon_0}\ln\frac{a^2-b^2}{a\,r} \tag{8}$$

$$p_{12} = \frac{1}{2\pi\varepsilon\,\varepsilon_0}\ln\frac{a^2+b^2}{2\,a\,b} \tag{9}$$

and

$$p_{13} = \frac{1}{2\pi\varepsilon\,\varepsilon_0}\ln\frac{\sqrt{a^4+b^4}}{2\,a\,b} \tag{10}$$

In (8) – (10), ε is the relative permittivity of the medium surrounding the wire (ε = 1 for air) and ε_0 is the electric constant.

Inductances L, M_1 and M_2 were calculated from the well-known formulas for strait filaments [4]:

$$L \approx \frac{2\mu_0}{\pi}\,h\left(\ln\frac{2h}{r}-1+\frac{\mu_1}{4}\right) \tag{11}$$

$$M_1 \approx \frac{\mu_0}{2\pi} h \left[\ln\frac{h}{c} - 1 + \frac{2c}{h} - \frac{1}{4}\left(\frac{2c}{h}\right)^2 \right] \tag{12}$$

and

$$M_2 \approx \frac{\mu_0}{2\pi} h \left[\ln\frac{h}{b} - 1 + \frac{2b}{h} - \frac{1}{4}\left(\frac{2b}{h}\right)^2 \right] \tag{13}$$

where μ is relative permeability of the resistance wire (μ = 1 for Zeranin) and μ_0 is the magnetic constant.
In Table 1 the results of calculation of parasitic capacitances and inductances of the 10 Ω quadrifilar resistor are summarized.

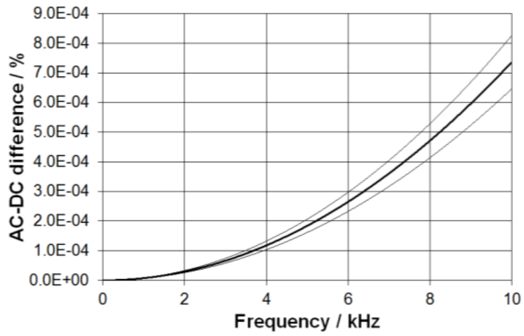

Figure 3. Frequency dependence of the 10 Ω quadrifilar resistor

Table 1. Calculated parasitic inductances and capacitances of the 10 Ω quadrifilar resistor

Capacitance / pF		
C_0	C_1	C_2
2.25	0.22	0.13
Inductance / nH		
L	M_1	M_2
667	51.7	44.5

Calculated AC-DC differences of this resistor (relative changes of its parallel equivalent resistance from the DC value) and their uncertainty band for a coverage factor of 2 are shown in Figure 3. Time constant is $(4.32 \pm 0.26) \times 10^{-8}$ s .

III. Transformer bridge and measurement results

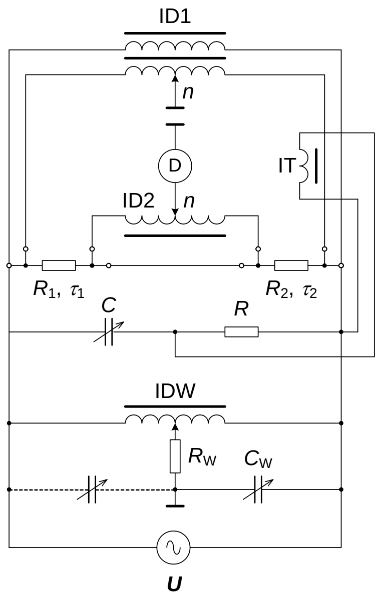

Figure 4. Transformer bridge

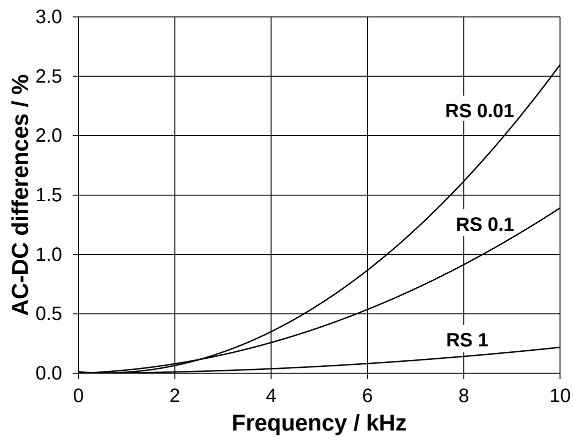

Figure 5. AC-DC differences of RS 1, RS 0.1 and RS 0.01

Transformer bridge [5] of Figure 4 has been used in the above 10:1 comparisons. Ratio arms of this bridge being formed by an eight-decade, two-stage inductive voltage divider (ID1), another eight-decade divider serves as a simple combining network. In addition, a Wagner earthing device consisting of a seven-decade divider IDW, a fixed-value resistor R_W and a capacitance box C_W is incorporated into the bridge circuit. The output voltage of an RC phase shifter (resistor R and capacitance box C), which is injected into one connecting lead of the ratio winding of ID1 via a 100:1 injection transformer (IT), serves for quadrature balancing. The value of the injected voltage is

$$\boldsymbol{U}_{\text{inj}} = p\,\boldsymbol{U}\left(\frac{\omega^2 C^2 R^2}{1+\omega^2 C^2 R^2} + \mathrm{j}\frac{\omega C R}{1+\omega^2 C^2 R^2}\right) \tag{14}$$

where p is the ratio of the injection transformer (p can be +0.01 or –0.01 depending on the relative polarity of the primary and the secondary) . For $\omega^2 C^2 R^2 \ll 1$ (13) reduces to

$$\boldsymbol{U}_{\text{inj}} \approx \mathrm{j}\, p\, \omega C R \boldsymbol{U} \tag{15}$$

where imaginary unit j indicates a phase shift of $\pi/2$ radians between $\boldsymbol{U}_{\text{inj}}$ and $\boldsymbol{U}$. The bridge is balanced when

$$\frac{R_2}{R_1} = \frac{1-n}{n}\left(1 + \frac{\alpha}{1-n}\right) \tag{16}$$

and

$$\tau_2 - \tau_1 = \frac{1}{\omega}\frac{\beta}{1-n} \tag{17}$$

where $\alpha = p\,\omega^2 C^2 R \big/ \left(1+\omega^2 C^2 R^2\right)$ and $\beta = p\,\omega C R \big/ \left(1+\omega^2 C^2 R^2\right)$.

Figure 5. Photograph of the bridge.

Results of measurement of AC-DC differences of RS 1 (Tinsley type 1659 standard), RS 0.1 (Tinsley type 1682 standard) and RS 0.011 (Tinsley type 3111 standard) are shown in Figure 4. At 10 kHz, the respective absolute expanded uncertainties (k=2) of these differences are 1.4×10^{-3} %, 3.7×10^{-2} % and 7.1×10^{-2} %. Time constants are $(9.01\pm 0.50)\times 10^{-7}$ s , $(1.35 \pm 0.05)\times10^{-6}$ s and $(1.47 \pm 0.05)\times 10^{-6}$ s , respectively.

IV. Conclusions

In a situation where suitable reference standards for 1:1 comparison were not available, a 10 Ω calculable resistor primarily realized for other purposes was used as reference in the measurement of frequency dependences of 1 Ω, 0.1 Ω and 0.01 Ω resistance standards. Even though resistances of these standards were several orders lower than that of the calculable resistor, acceptable accuracy of the measurement was achieved by preferring successive 10:1 comparisons of the standards to their direct comparisons with the calculable resistor.

Acknowledgement

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References

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