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Assessment of the uncertainty associated with synchronization error in analog to digital conversion with dither and CAV

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Abstract- The paper discusses the problem of cumulative averaging (CAV) of digital signals after the analog to digital conversion with dither signal. The CAV algorithm performs three operations: *division* of the signal into a set of repetitions, *synchronization* of sectors of each repetition and *cophasal averaging* of samples. The influence of non-ideal synchronization of digital sectors on the value of samples of signal being a result of CAV was assessed. A dependence determining the variance of the signal value after averaging was given because of synchronization error. In the case of sinusoidal signal, the variation depends on: synchronization error variance, signal amplitude, relation between signal frequency and sampling frequency, and the number of signals averaged. With non-zero synchronization error, CAV – reducing additive noise – causes distortions in actual values of signal being a result of averaging, and increases the uncertainty value of such a-d conversion result.

I. Introduction

Cumulative averaging (CAV) is one of the methods of reducing noise in a signal. It can be applied to periodic signals or repeatable signals when they are acquired many times and are disturbed with additive random signal (noise) not correlated with the examined signal. CAV is used for example in the processing of biomedical signals, the testing of driving machines and systems, examination of cyclic phenomena. The digital algorithm CAV is used in a-d conversion systems with dither signal, *inter alia* in order to reduce the randomized quantization error and the differential nonlinearity DNL of ADC converter, generating an error with noise structure. Then, the quantization error would have desired physical and statistical properties.

The influence of CAV on the a-d conversion result with dither signal is significant in systems with a digital measurement algorithm. Then, the samples of recorded signal are the data utilized to estimate the value of a measured quantity. Hence, it is postulated – with regard to digitization – that the values of samples correspond as faithfully as possible to the actual values of analog signal from which they come. Due to such "destination" of the data, the assessment of CAV influence on preserving this correspondence becomes crucial.

An important advantage of CAV that distinguishes it from a simple digital filtration is that, as a result, we have noise reduction also in the signal band, which is conductive to fulfill the above-formulated postulate. In simple filtration the noise eliminated/reduced is the noise out of the filter pass band; and the noise inside the pass band, i.e. also in the signal band, remains.

Ideally, CAV affects the noise without influencing the value of the recovered signal. A disadvantage of CAV is the extension of processing duration and the requirement to synchronize the averaged sectors. Practically, the effectiveness of CAV depends on to which extent this requirement is met.

Methods of reduction of synchronization error can be divided into two groups:

- reduction method in the time domain – alignment methods – consisting in estimation/detection of shifts of signals being averaged, in order to their re-synchronization before CAV [1,3,4,8],
- reduction methods in the frequency domain – consisting in the estimation of filter characteristic being a Fourier transform of the probability density function of the randomness of position of the beginnings of averaged signals. The deconvolution of filter characteristic and signal averaged as a result of CAV processing makes it possible to reconstruct the frequency characteristic of a signal averaged without synchronization errors [9].

Methods of practical performance of CAV (both in software and hardware) are presented in [5], where also various aspects of the design of measurement instruments for synchronous digital averaging are discussed.

The aim of this paper is to determine the influence of non-ideal synchronization of averaged sectors of signal on the CAV result. The quantity assessment of this influence was expressed through the variance of the value of a sample being a result of CAV.

With respect to the uncertainty value of the result of a-d conversion with dither signal, CAV (if it can be applied) is more useful than filtration (e.g. moving average, MAV). In the other case, data correlated to a certain extent undergo averaging. The variance of average value is then greater.

II. CAV algorithm in a real system of a-d conversion

CAV processing was applied to a **recorded digital signal,** being a sum of a usable deterministic signal ***x*** and a random signal ***n*** not correlated with ***x*** signal. The recorded signal is divided into disjoint repetitions, or those repetitions can be a result of separate signal registrations.

sector 1	space 1	sector 2	space 2	**…**	sector *M*

repetition 1	repetition 2	**…**	repetition *M*-ta

Fig. 1. Structure of registered signal undergoing CAV

It was assumed that the set of repetition, and at the same time the set of sectors, has ***M*** elements. In each sector there are ***I*** samples (i=1, …, I). All sectors have identical length, in the sense of the number of samples. Similarly, spacing between them has also identical lengths. The lengths of sector and spacing, however, do not need to be the same. Sectors are disjoint (they come from disjoint repetitions). The duration of each repetition is an integer multiple of the period of signal x.

The CAV algorithms perform three operations: the *division* of signal into a set of repetitions, *synchronization of* sectors from every repetition and *cophasal averaging* of samples. In ideal situation and full synchronization, all averaged i-th samples from every sector have the same phase in relation to the beginnings of the sectors from which they come. Then, the i-th sample of the k-th sector can be written down as follows:

$$y_k(t_i) = x_k(t_i) + n_k(t_i) \tag{1}$$

$n_k(t_i)$ - noise burdening the i-th sample of the k-th repetition.

The CAV result is the following:

$$\overline{y(t_i)} = \frac{1}{M}\sum_{k=0}^{M-1} y_k(t_i) = \frac{1}{M}\sum_{k=0}^{M-1}\left[x_k(t_i) + n_k(t_i)\right] = x(t_i) + \frac{1}{M}\sum_{k=0}^{M-1} n_k(t_i) \qquad i = 1,..., I \tag{2}$$

$$x_1(t_i) = x_2(t_i) = ... = x_M(t_i) = x(t_i)$$

In a real system of a-d conversion there is a phenomenon of instable phase relations among signal samples, named timing jitter. Its source is: S/H jitter (aperture, key management system, clock jitter) and jitter of analog signal source. In CAV processing there is also a **synchronization error**, called also frame jitter [6, 7]. Synchronization error can be considered as a special case of the timing jitter [6, 11].

In a real system of a-d conversion the i-th sample of the k-th sector can be described in the following way:

$$y_k(t_i) = x(t_i + \tau_{ik} + \Delta_k) + n_k(t_i + \tau_{ik} + \Delta_k) \tag{3}$$

τ_{ik} - error being result of jitter, burdening the i-th sample of the k-th sector

Δ_k - synchronization error in the k-th sector (the same for all samples of that sector)

The sources of synchronization errors and jitter are independent. We can, therefore, consider the impact of these errors on CAV result separately. Further part of the discussion is focused on an analysis of the influence of synchronization error on the accuracy of CAV result.

Jitter does not influence the power of the input noise [2, 6, 7]. Errors being results of jitter move onto the CAV result through the deterministic component; therefore, our consideration can be limited to this component. In the case of assessing the results of the very synchronization error, the set of samples undergoing CAV takes the following form:

$$\left\{y_k(t_i) = x(t_i + \Delta_k), \qquad i = 1,..., I, \qquad k = 0,..., M-1\right\} \tag{4}$$

It was assumed that synchronization error is a stochastic process, independent and ergodic. Spacing among sectors is appropriately long. The repetition frequency of the frames is generally much less than the ADC sampling frequency within each frame. The signals undergoing a-d conversion and the system are stationary.

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III. Variance of the values of signal samples, being a result of synchronization error

As a result of non-ideal synchronization of sectors, the nominal beginning of a successive sector can be actually accelerated or delayed. Acceleration/delay of the beginning of a given sector results in the same acceleration/delay for all following samples of that sector. The phases of *i*-th samples, however, of different sectors have dispersion such as the dispersion of the beginnings of sectors from which these samples come. Because the synchronization errors of *i*-th samples taken for averaging come from different sectors, i.e. from independent experiments, the random variable Δ_k is independent. It can be assumed that its average value is equal to 0, and the probability is uniformly distributed [6]. Synchronization concerns **digitized** signals, therefore synchronization error is a **discrete** random variable whose values are multiplicities of the sampling period T_S. The probability distribution of that variable is given in Fig. 2.

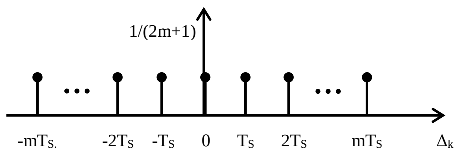

Fig. 2. Probability distribution of random variable Δ_k

The variance of random variable Δ_k amounts to:

$$Var[\Delta_k] = \frac{1}{2m+1}\sum_{l=-m}^{m}(lT_S)^2 = \frac{m(m+1)}{3f_S^2} \tag{5}$$

Possible correlation of signal samples in a single sector is not important. They do not constitute a set of samples undergoing CAV averaging.
In general, the variance of signal value *y* of instant t_i, because of error Δ can be determined as below [10]:

$$y(t_i) = x(t_i + \Delta) \;\rightarrow\; Var[y(t_i)] = Var[x(t_i + \Delta)] = \left[x'\left(\overline{t_i + \Delta}\right)\right]^2 \cdot Var[t_i + \Delta] = [x'(t_i)]^2 \cdot Var[\Delta] \tag{6}$$

Δ - random variable and $\overline{\Delta} = 0$
x - signal, continuous, differentiable function.

In the case of sinusoidal signal:

$$y(t) = A\sin[\omega(t+\Delta)] = A\sin(\omega t + \omega\Delta) \;\rightarrow\; y(i) = A\sin\left(2\pi\frac{f_y}{f_S}i + 2\pi f_y\Delta\right) \tag{7}$$

f_y - signal frequency *y*
f_S - sampling frequency.

Utilizing formula $\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$ and dependence (6), we can determine the variance of the *i*-th sample of the *k*-th sector as follows:

$$Var[y_k(i)] = A^2(2\pi f_y)^2\cos^2\left(2\pi\frac{f_y}{f_S}i\right)\cdot Var[\Delta_k] \tag{8}$$

Because synchronization error is an independent random variable, random variable $\{y_k(i),\ k=0, \ldots, M-1\}$ being a function of that variable (conf. (4)) is also independent [10]. Hence, the variance of CAV result because of synchronization error takes the form:

$$\begin{aligned} Var\left[\overline{y(i)}\right] &= Var\left[\frac{1}{M}\sum_{k=0}^{M-1}y_k(i)\right] = \frac{1}{M^2}\sum_{k=0}^{M-1}Var[y_k(i)] = \\ &= A^2(2\pi f_y)^2\cos^2\left(2\pi\frac{f_y}{f_S}i\right)\frac{1}{M^2}\sum_{k=0}^{M-1}Var[\Delta_k] = A^2(2\pi f_y)^2\cos^2\left(2\pi\frac{f_y}{f_S}i\right)\frac{1}{M}Var[\Delta_k] \end{aligned} \tag{9}$$

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With (5) taken into account, this dependence is reduced to the form:

$$Var\left[\overline{y(i)}\right] = \frac{m(m+1)}{3M}(A\pi)^2\left(2\frac{f_y}{f_S}\right)^2\cos^2\left(2\pi\frac{f_y}{f_S}i\right) \tag{10}$$

The maximum value of CAV result variance (10), assuming the fulfillment of sampling theorem, is limited:

$$Var\left[\overline{y(i)}\right]\Big|_{MAX} < \frac{m(m+1)}{3M}(A\pi)^2\frac{1}{(OSR)^2} < C = \frac{m(m+1)}{3M}(A\pi)^2 \tag{11}$$

$OSR = f_S/(2f_y))$

while $f_S > 2f_y \rightarrow (2f_y/f_S)^2 < 1$.

In particular, when m=1, i.e. when the maximum synchronization error is equal to $\pm 1/f_S$, the maximum variance value of CAV result will not be greater than $C=2(A\pi)^2/(3M)$.
The dependence of CAV result variance on the size of the synchronization error *m* (conf. Fig. 2) and on the sampling frequency f_S, for a different number of sectors *M* of sinusoidal signal (*A*=1, f_y =20 Hz) is given in Fig. 3. The influence of maximum synchronization error $\Delta_{max} = m/f_S$ is relevant at low over-sampling (low oversampling ratio, $OSR=f_S/(2f_y)$); then it strongly depends also on the number of averaged samples (i.e. on the number of sectors *M*). And if *OSR*>5 (f_S >200 Hz), the CAV result variance is significantly reduced. At *OSR*>10 (f_S >200 Hz) it becomes negligible and practically independent from *m* and *M*. As it results from Fig. 3a, below a certain number of averaged sectors (*M*=10) the variance value becomes unacceptable because of synchronization error. Based on dependence (11), we can determine such *M* at which this value does not exceed a given limit.

a) b)

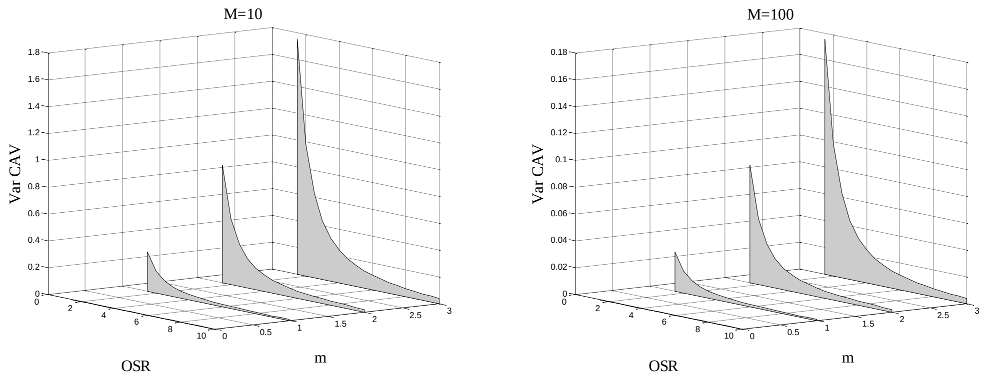
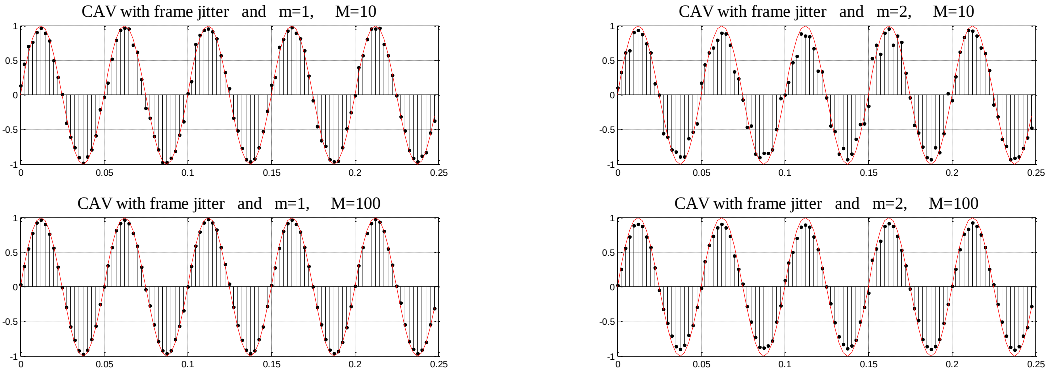


Fig. 3. Maximum variance of CAV result for different number of averaged sectors: a) M=10, b) M=100

a1) a2)



a3)

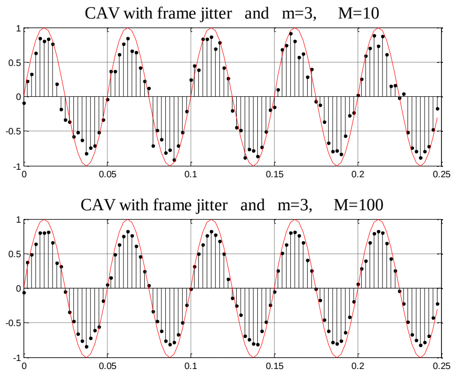


Fig. 4. CAV with frame jitter and set of repetition M=10, M=100: a1) $\Delta_{max} = T_S$, a2) $\Delta_{max} = 2T_S$, a3) $\Delta_{max} = 3T_S$

IV. Conclusions

In the CAV digital algorithm synchronization error moves onto sample value error, and, consequently, onto CAV result error.

If there is a non-zero synchronization error, CAV, together with a reduction in additive noise, causes distortions in actual values of recovered signal and increases the uncertainly value of the result of a-d conversion with dither signal. It is possible to determine a limit of the CAV result variance because of that error.

In the case of sinusoidal signal, the CAV result variance depends on: synchronization error variance, signal amplitude, relations between signal frequency and sampling frequency as well as on the number of averaged signals. Its limit depends on: maximum value of synchronization error, signal amplitude and the number of signals averaged. It does not depend, however, on signal frequency and sampling frequency.

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