

Extending the scale of inductance using 1:1 comparator bridge

Krzysztof Musiol, Tadeusz Skubis

*Institute of Measurement Science, Electronics and Control, Silesian University of Technology, ul. Akademicka
10, 44-100 Gliwice, Poland, phone +48 32 2371064, fax +48 32 237 2064, krzysztof.musiol@polsl.pl,
tadeusz.skubis@polsl.pl*

Abstract- An idea of using 1:1 comparison for 10:1 and 1:10 impedance transfer is described in the paper. Results of measurements performed for 10:1 transfer of inductance are discussed. The results achieved prove that using 1:1 comparator bridge and applying proposed procedure is possible to transfer unit to the objects of 10 times smaller values with relative uncertainty only a little higher than uncertainty estimated for 1:1 comparison.

I. Introduction

Since extremely high-permeability materials are used for toroidal cores' production, high-precision transformer-ratio-arm bridges began to be constructed for impedance measurement at National Metrology Institutes [1-5]. Though most of bridges are designed only for 1:1 comparisons, an idea of using these instruments for calibration of impedance standards of 1:10 ratio was developed. It gives the possibility to extend the scale of unit to the multiples and submultiples of the base value. The procedure of subdivision and multiplication value of reference standard using 1:1 comparison is known in the field of mass measurement [6, 7, 8]. In the paper a slightly different approach to the problem is proposed which takes a minimum number of comparison. It will be proved that proposed approach is good enough to transfer inductance unit to the multiples and submultiples with relative uncertainty on the level comparator's accuracy.

II. Comparison procedure

Let's assume, that there is seven inductance standards denoted by $S_1 \dots S_7$ having following nominal values: $S_1 = 10$ mH, $S_2 = S_3 = 5$ mH, $S_4 = S_5 = 2$ mH, $S_6 = S_7 = 1$ mH, and S_1 is reference standard of well-known value of inductance. We are looking for true value of S_7 standard which has ten times lower nominal value than S_1 . Using 1:1 comparator bridge following six comparisons are enough to enable inductance calculation of unknown S_7 :

- 1) $S_2 + S_3 \leftrightarrow S_1$
- 2) $S_4 + S_5 + S_6 \leftrightarrow S_2$
- 3) $S_4 + S_5 + S_6 \leftrightarrow S_3$
- 4) $S_6 + S_7 \leftrightarrow S_4$
- 5) $S_6 + S_7 \leftrightarrow S_5$
- 6) $S_6 \leftrightarrow S_7$

It's easy to notice, that the same comparisons performed in the other direction enable to calculation of unknown S_1 on the basis of the reference standard S_7 which is ten times lower than S_1 .

If we denote the inductances of $S_1 \dots S_7$ standards by $L_1 \dots L_7$, following equations corresponding with the above mentioned six comparisons can be written:

$$\begin{aligned}d_A &= L_6 - L_7, & (1) \\d_B &= L_6 + L_7 - L_5, & (2) \\d_C &= L_6 + L_7 - L_4, & (3) \\d_D &= L_4 + L_5 + L_6 - L_3, & (4) \\d_E &= L_4 + L_5 + L_6 - L_2, & (5) \\d_F &= L_2 + L_3 - L_1, & (6)\end{aligned}$$

where $d_A \dots d_F$ are difference results obtained using 1:1 comparator bridge.

Solving set of Eqs. (1)...(6) let us the calculation of inductances $L_2 \dots L_7$ on the base of well-known inductance L_1 and measured differences $d_A \dots d_F$:

$$L_2 = 0,5L_1 + 0,5d_D - 0,5d_E + 0,5d_F. \quad (7)$$

$$L_3 = 0,5L_1 - 0,5d_D + 0,5d_E + 0,5d_F, \quad (8)$$

$$L_4 = 0,2L_1 - 0,2d_A + 0,4d_B - 0,6d_C + 0,2d_D + 0,2d_E + 0,2d_F, \quad (9)$$

$$L_5 = 0,2L_1 - 0,2d_A - 0,6d_B + 0,4d_C + 0,2d_D + 0,2d_E + 0,2d_F, \quad (10)$$

$$L_6 = 0,1L_1 - 0,6d_A + 0,2d_B + 0,2d_C + 0,1d_D + 0,1d_E + 0,1d_F, \quad (11)$$

$$L_7 = 0,1L_1 + 0,4d_A + 0,2d_B + 0,2d_C + 0,1d_D + 0,1d_E + 0,1d_F, \quad (12)$$

III. Implementation

The KWL inductance comparator bridge [2] equipped with two inputs suitable for connection the compared inductors was used to evaluate the proposed procedure. A symmetric summing branch-joint, made of four similar flat bars placed on insulating plate (Fig. 1), was designed especially for adding in series up to three inductance standards. Special attention was given to minimize capacitive effects of the branch-joint. Therefore each of four sections was wrapped individually by foil copper screen, insulated from the inner conductor. Potentials of the screens of the branch-joint sections are controlled externally to be equal to the potential of the corresponding inner conductor. The self-inductance of the branch-joint, when all three standard inputs are shorted is nearly constant and very small. Measurements performed using KWL3 bridge showed that the value is 177 nH with standard deviation of 2 nH.

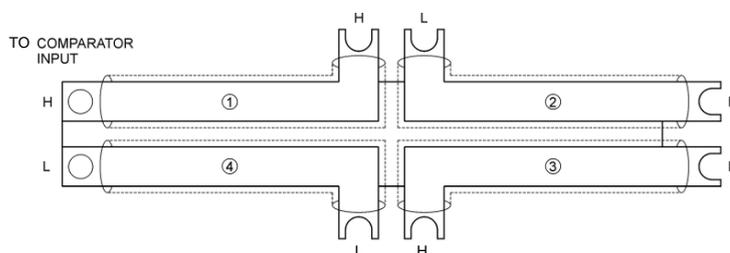


Figure 1. Summing branch-joint

IV. Results

According to comparison procedure presented in Section II six comparison (Figure 2) should be performed to enable subdivision and multiplication of the inductance unit. The theoretical considerations presented in the previous section omitted impedance of cables and residual parameters of measuring instrument, but these could be significant. It can be easily proved that series residual parameters of measuring circuit doesn't affect the measurement if procedure of interchange places of standards is applied. Hence, full procedure of the inductance transfer 10:1 consists of 12 comparison measurements. Individual steps of the measurement procedure are made

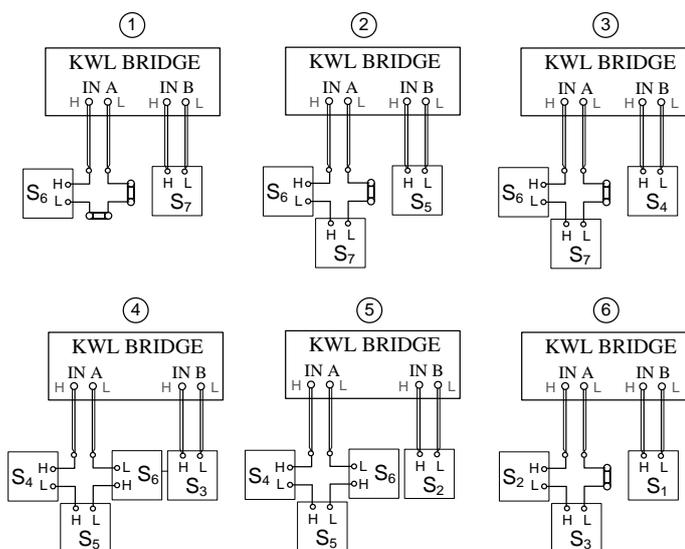


Figure 2. Measuring circuits needed for 10:1 inductance transfer

with one, two or three inductance standards connected to inputs of the branch-joint. If there is only one standard (or two standards) connected to the branch-joint, the remaining inputs have to be shorted.

Several measurement series of the transfer procedure were performed at frequencies 1 kHz and 1592 Hz. The procedure described in the paper was applied to transfer following values:

- 1) 10 mH → 1 mH,
- 2) 1 mH → 100 μH.

Comparisons were performed using the KWL bridge with standard uncertainty 4 nH. The measurements were performed in two-terminal circuit in which cases terminals of the standard inductors are shorted to LOW terminals. Regardless of the comparisons each standard inductor was measured by the Maxwell-Wien bridge. The values obtained from the Maxwell-Wien bridge can be treated as true values with relative uncertainties below 100 μH/H (Tables 1 and 2). The comparison results obtained at 1 kHz for 10 mH→1 mH and for 1 mH→100 μH transfer are presented in Tables 1 and 2. Inductance L_1 measured by the Maxwell-Wien bridge were taken as reference for the 10:1 transfer. Standard uncertainties of transfer were calculated according to the following equations:

$$u(L_2) = u(L_3) = \sqrt{0.25u^2(L_1) + 0.875u^2(d)}. \quad (13)$$

$$u(L_4) = u(L_5) = \sqrt{0.4u^2(L_1) + 1.98u^2(d)}, \quad (14)$$

$$u(L_6) = \sqrt{0.01u^2(L_1) + 0.98u^2(d)}, \quad (15)$$

$$u(L_7) = \sqrt{0.01u^2(L_1) + 1.22u^2(d)}, \quad (16)$$

The above uncertainty formulas were received by the use of general law of propagation of uncertainty for Eqs. 7 to 12, assuming the same absolute standard uncertainty for all difference measurements¹ $u(d_A) = u(d_B) = u(d_C) = u(d_D) = u(d_E) = u(d_F) = u(d)$.

An advantageous property of the 10:1 transfer are relatively low sensitivity coefficients occurring before $u^2(d)$ in the uncertainty formulas. The last achieved coefficient indicates that uncertainty component associated with the transfer is only 22% higher than absolute standard uncertainty of the 1:1 comparison.

For 10 mH→1 mH transfer final difference between inductance calculated from the transfer and measured by the Maxwell-Wien bridge is equal to -25.8 μH/H which is two times less than uncertainty estimated for Maxwell-Wien bridge. For 1 mH→100 μH transfer obtained final difference (-77.6 μH/H) is also less than accuracy of the Maxwell-Wien bridge.

Table 1. Results for transfer 10 mH→1 mH, $f=1$ kHz

Measured or calculated inductance	Nominal value of inductance standard	Inductance calculated from the transfer	Relative standard uncertainty (type B) of the transfer	Inductance measured by M-W bridge	Relative standard uncertainty (type B) of the M-W bridge	Difference between inductance calculated and measured	Relative difference between inductance calculated and measured
	(mH)						
L_1	10	-	-	10.00220	10	-	-
L_2	5	5.0000632	10.1	5.000130	60	-66.8	-13.4
L_3	5	4.9998927	10.1	4.999857	60	35.7	7.1
L_4	2	1.9994563	31.8	1.999457	50	-0.9	-0.5
L_5	2	1.9983678	31.8	1.998283	50	84.8	42.4
L_6	1	1.0003091	10.8	1.000343	50	-34.5	-34.5
L_7	1	0.9996426	11.0	0.999668	50	-25.8	-25.8

¹ this assumption is satisfied for KWL bridge

Table 2. Results for transfer 1 mH→100 μH, $f=1$ kHz

Measured or calculated inductance	Nominal value of inductance standard	Inductance calculated from the transfer	Relative standard uncertainty (type B) of the transfer	Inductance measured by M-W bridge	Relative standard uncertainty (type B) of the M-W bridge	Difference between inductance calculated and measured	Relative difference between inductance calculated and measured
	(mH)	(mH)	(μH/H)	(mH)	(μH/H)	(nH)	(μH/H)
L_1	1	-	-	0.9996685	50	-	-
L_2	0.5	0.4995625	50.6	0.4995673	60	-4.8	-9.6
L_3	0.5	0.498186	50.6	0.49811685	60	69.1	138.3
L_4	0.2	0.2000873	160.6	0.19986864	100	21.7	108.3
L_5	0.2	0.1998903	160.6	0.20006234	100	25.0	124.8
L_6	0.1	0.1001174	63.7	0.10012508	80	-7.7	-76.8
L_7	0.1	0.0999749	66.8	0.09998266	80	-7.8	-77.6

A few other measurement series with an interval of a few days were performed at 1000 Hz and 1592 Hz. The results of difference between inductance calculated and measured differed from those provided in Tables 1 and 2 no more than 10 nH. This proves very good repeatability of the results and confirmed usefulness of the presented method for calibration of inductance standards in National Metrology Institutes.

V. Conclusions

The analysis and the results presented in the paper prove that using 1:1 impedance comparator bridge with the developed summing branch-joint and applying proposed procedure it is possible to transfer the value of reference standard to the objects of 10 times smaller values with relative uncertainty only a little higher than uncertainty estimated for 1:1 comparison. This is a big advantage of the proposed method. High accuracy and resolution of the comparator bridge, high stability of the standard inductors and right screening of the summing branch-joint gave very good results of the transfer measurements. The procedure presented in the paper is relatively simple and can be used as an alternative method for calibration of the multi-range RLC bridges at points 1, 2, 5, and 10 at each range.

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