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Average Value Evaluation of the Distorted Pulse Signal

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Abstract-Average value of the distorted pulse signal has been evaluated for two very often encountered distortions, namely finite rise and fall edges durations and oscillations (ringing) during the pulses. Relative errors of these average values are also given. The results can be applied e.g. in class D power amplifiers.

Keywords-distorted pulse signal, average value, rectangular pulse, trapezoidal pulse, exponential pulse, ringing.

I. Introduction

Average value of a pulse signal is used in many applications, e.g. in some methods of phase measurement, [1], in vector impedance meters, [2], in PLL systems, [2], in class D power amplifiers, [3], [4], [5], [6], and others. The majority of papers concerning pulse signal evaluation or measurement are aimed at the evaluation of pulse amplitude (peak value), duty cycle, and frequency spectra. Moreover, average value evaluation concerns mainly rectangular, [7], [8], [9], [10], [11], or trapezoidal pulses, [12], [13], [14], [15]. The effect of ringing on the frequency spectra is investigated in [12], [13], a relation between a percentage overshoot and the damping ratio of the ringing is in [16]. The definition of a function (or signal) average value can be found e.g. in [17], the definition of a pulse train average value in [18], [19]. Generally, pulse signal can be distorted as explained in [20] and shown in [7]. In the following, rounding of the signal corners is replaced by intersecting straight lines.
The errors of the average value of distorted pulse signals caused by finite rise and fall time and by the exponential edges of the pulses have been investigated. The pulse train has been first considered to be rectangular, which implies zero rise and fall time. In this case reference value without errors has been achieved. Finite rise and fall time leads to trapezoidal pulse train. Now, errors are caused by finite rise and fall time of the pulse edges. Similar effect is caused also by the exponential edges of the pulses. Second kind of errors are errors caused by oscillations (ringing) superimposed on the pulse and the zero level. This has also been investigated.

II. Finite rise and fall time

In ideal case, the pulses are rectangular, Fig. 1. The period of the pulses is T, the pulse duration is t_p and its amplitude is V. The ratio t_p/T is the duty cycle D. Then the voltage average value is given by the equation, [21],

$$V_{ar} = \frac{Vt_p}{T} \qquad (1)$$

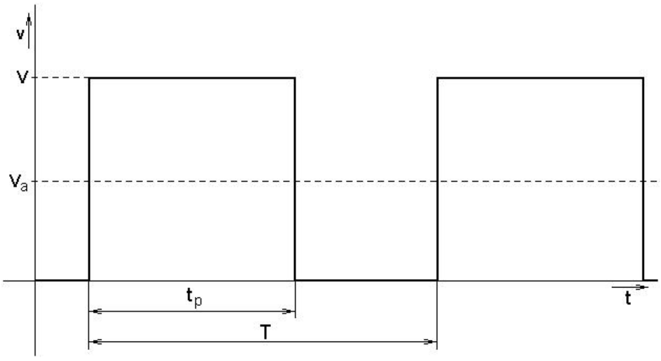

Figure 1. Ideal rectangular pulses

In real case, the pulses are trapezoidal, Fig. 2. Here, the pulse duration which is mostly encountered in practical use, is shown. Such situation is caused by finite rise and fall time of circuits triggered in the instants of starting of the edges. The durations of the rising, t_r, and the falling, t_f, edges are supposed to be constant and for the sake of simplicity, pulse edges have been considered to be linear, without rounding, and these time durations start and end at the zero level and at the pulse amplitude level. The voltage average value, V_{at}, is now given by the equation

$$V_{at} = \frac{V}{2T}\left(2t_p - t_r + t_f\right) \tag{2}$$

The error voltage, V_{aterr}, can be expressed as the difference of the equations (2) and (1)

$$V_{aterr} = V_{at} - V_{ar} = \frac{V}{2T}\left(t_f - t_r\right) \tag{3}$$

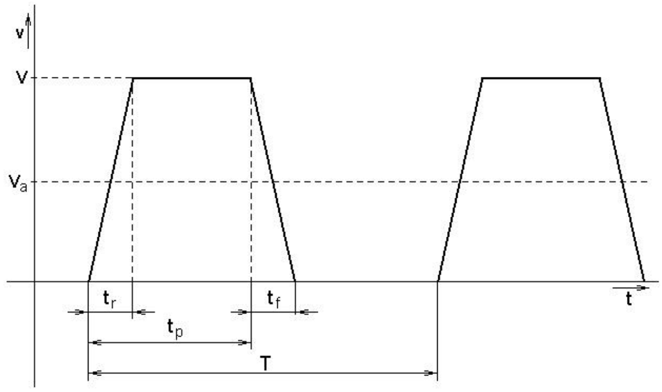

Figure 2. Trapezoidal pulses

and the relative error is

$$\delta_{Vat} = \frac{V_{aterr}}{V_{ar}} = \frac{1}{2}\frac{t_f - t_r}{t_p} \tag{4}$$

This error depends on the rise and fall time of the pulses.
The edges of the pulses are often shaped by charging and discharging of capacitors through resistors, so they are exponential, Fig. 3. It is supposed that time duration of the edges is short enough not to influence the pulse amplitude. The pulse voltage average value, V_{ae}, can be expressed by the equation

$$V_{ae} = \frac{V}{T}\left[t_p + \tau\left(e^{-\frac{t_p}{\tau}} - e^{-\frac{T-t_p}{\tau}}\right)\right] \tag{5}$$

where τ is the time constant of the exponential functions of both, rising and falling edges.

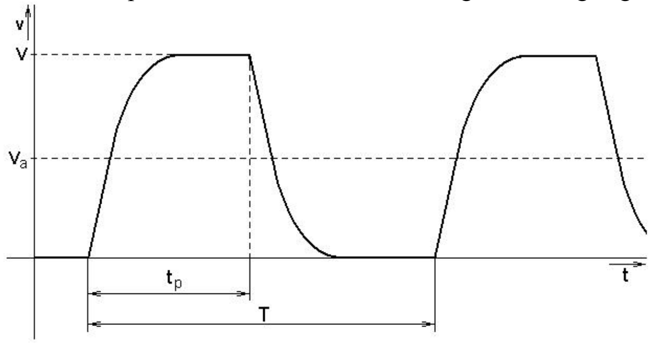

Figure 3. Exponential pulses

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The error voltage, V_{aeerr}, can be expressed as the difference of the average voltages in the real exponential and the ideal case

$$V_{aeerr} = V_{ae} - V_{ar} = V\frac{\tau}{T}\left(e^{-\frac{t_p}{\tau}} - e^{-\frac{T-t_p}{\tau}}\right) \tag{6}$$

and the relative error is

$$\delta_{Vae} = \frac{V_{aeerr}}{V_{ar}} = \frac{\tau}{t_p}\left(e^{-\frac{t_p}{\tau}} - e^{-\frac{T-t_p}{\tau}}\right) \tag{7}$$

This relative error depends on the time constant of the exponential functions of both edges.

III. Errors Caused by Oscillations

The oscillations, also called ringings, start by overshoots at the ends of steep transition edges of the pulses and are usually caused by resonant phenomena. Because of energy dissipation in the circuits, these oscillations decay, so their average value is not zero. This average value depends on the pulse duration, because of a different number of periods of the oscillations during the pulse, Fig. 4. The initial amplitudes, V_o, of the oscillations in both, the positive pulse and the zero level, are supposed to be equal and so are the periods, T_o. The average value, V_{arp}, of the oscillations during the positive pulse, t_p, is expressed as

$$V_{arp} = \frac{V_o}{T}\frac{1}{k^2 + \frac{4\pi^2}{T_o^2}}\left\{\frac{2\pi}{T_o} - e^{-kt_p}\left[k\sin\left(\frac{2\pi}{T_o}t_p\right) + \frac{2\pi}{T_o}\cos\left(\frac{2\pi}{T_o}t_p\right)\right]\right\} \tag{8}$$

where k is the decay coefficient. The average value, V_{arn}, of the oscillations during the zero level, $T - t_p$, is

$$V_{arn} = -\frac{V_o}{T}\frac{1}{k^2 + \frac{4\pi^2}{T_o^2}}\left\{\frac{2\pi}{T_o} - e^{-k(T-t_p)}\left[k\sin\left(\frac{2\pi}{T_o}(T-t_p)\right) + \frac{2\pi}{T_o}\cos\left(\frac{2\pi}{T_o}(T-t_p)\right)\right]\right\} \tag{9}$$

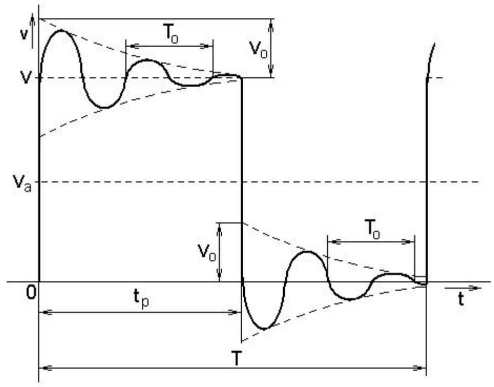

Figure 4. Rectangular pulses with oscillations

The total average value, V_{aro}, of the pulses with the oscillations is given by the sum of equations (1), (8) and (9)

$$V_{aro} = \frac{Vt_p}{T} + V_{arp} + V_{arn} \tag{10}$$

The error voltage, V_{aroerr}, is the difference of equations (10) and (1)

$$V_{aroerr} = V_{aro} - V_{ar} = V_{arp} + V_{arn} \tag{11}$$

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The relative error is the ratio of V_{aroerr} and V_{ar}

$$\delta_{Varo} = \frac{V_{aroerr}}{V_{ar}} = \frac{T}{Vt_p}(V_{arp} + V_{arn}) \tag{12}$$

Fig. 5 shows the dependence of the average values, V_{arp}, V_{arn}, and their sum $V_{arsin} = V_{arp} + V_{arn}$, on the pulse duration, t_p, for the values of the pulse signal amplitude V = 5 V, the pulse signal period T = 10 µs, the oscillations amplitude V_o = 1 V, the oscillations period T_o = 3 µs and the decay coefficient $k = 0.15 \times 10^6$ s^{-1}. It is evident, that the average value, V_{asin}, depends on the pulse duration, t_p, and there are values of t_p where the average value is zero. That means, that in such points the oscillations cause no error in the pulse average value.

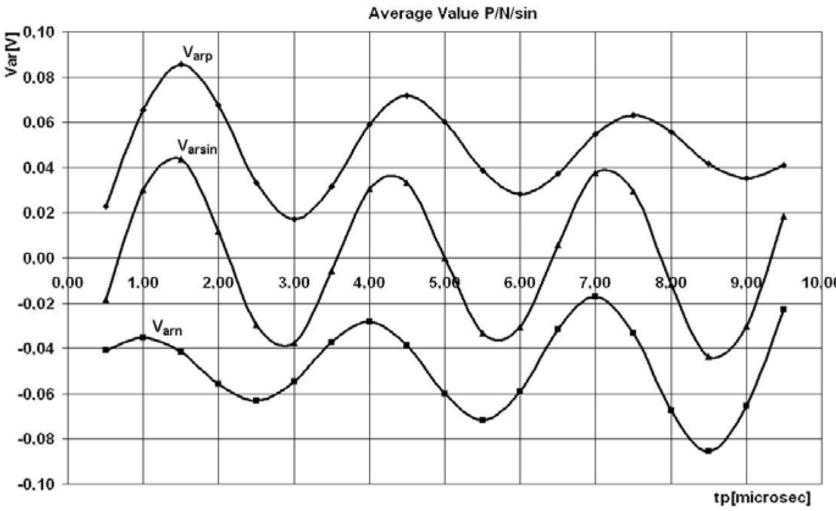

Figure 5. The average values of the oscillations: V_{arp}, during the positive pulse, V_{arn}, during the zero level and their sum V_{arsin}

Fig. 6 shows the dependence of the average value, V_{aro}, on the pulse duration, t_p. It is evident, that this dependence is nonlinear, so, there will be a distortion in situations, where the pulse signal with changing pulse duty cycle is used, e.g. in class D power amplifiers with PWM (pulse width modulation).

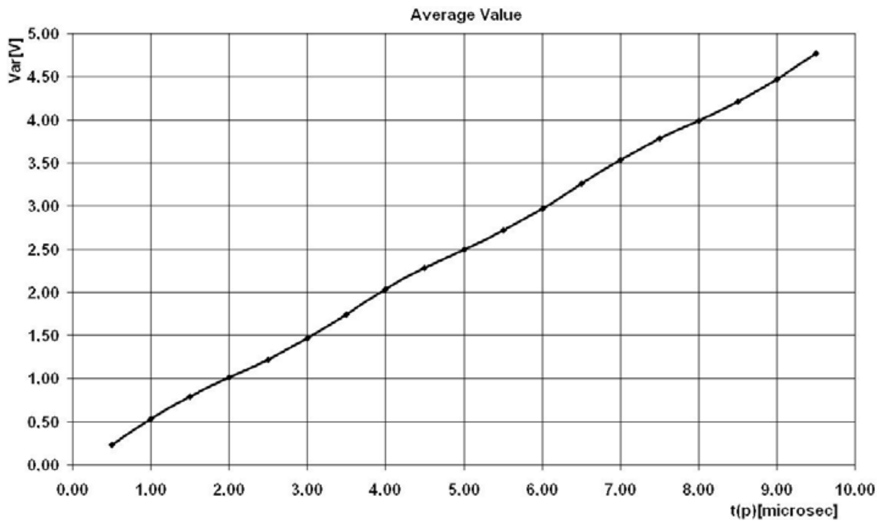

Figure 6. The total average value of the pulse signal with oscillations

In reality, the pulse signal is not rectangular, but approximately trapezoidal, with the oscillations superimposed on it, Fig. 7. The amplitudes, V_o, of the oscillations in both, the positive pulse and the zero level, are again considered to be equal and so are the periods, T_o.

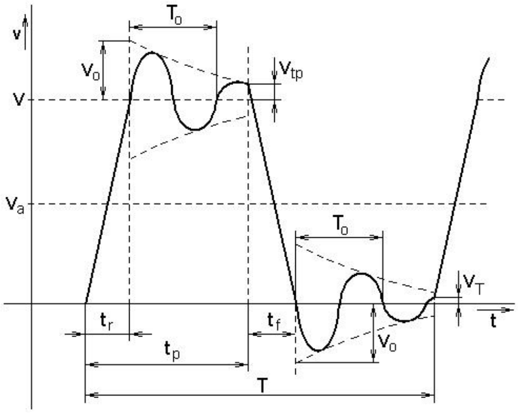

Figure 7. Trapezoidal pulses with oscillations

The average value, V_{atp}, of the oscillations during the positive pulse, t_p, can be expressed by the equation

$$V_{atp} = \frac{V_o}{T}\frac{1}{k^2+\frac{4\pi^2}{T_o^2}}\left\{\frac{2\pi}{T_o} - e^{-k(t_p-t_r)}\left[k\sin\left(\frac{2\pi}{T_o}(t_p-t_r)\right)+\frac{2\pi}{T_o}\cos\left(\frac{2\pi}{T_o}(t_p-t_r)\right)\right]\right\} \quad (13)$$

Similarly, the average value, V_{atn}, of the oscillations during the zero level, $T - t_p$, is

$$V_{atn} = -\frac{V_o}{T}\frac{1}{k^2+\frac{4\pi^2}{T_o^2}}\left\{\frac{2\pi}{T_o} - e^{-k(T-t_p-t_f)}\left[k\sin\left(\frac{2\pi}{T_o}(T-t_p-t_f)\right)+\frac{2\pi}{T_o}\cos\left(\frac{2\pi}{T_o}(T-t_p-t_f)\right)\right]\right\} \quad (14)$$

The total average value, V_{ato}, of the pulse with the oscillations is then given by the expression

$$V_{ato} = V_{at} + V_{atp} + V_{atn} = \frac{V}{2T}\left(2t_p - t_r + t_f\right) + V_{atp} + V_{atn} \quad (15)$$

The error voltage, V_{atoerr}, is the difference of equations (15) and (1)

$$V_{atoerr} = V_{ato} - V_{ar} = \frac{V}{2T}\left(t_f - t_r\right) + V_{atp} + V_{atn} \quad (16)$$

The relative error is the ratio of V_{atoerr} and V_{ar}

$$\delta_{Vato} = \frac{V_{atoerr}}{V_{ar}} = \frac{1}{2}\frac{t_f - t_r}{t_p} + \frac{T}{Vt_p}(V_{atp} + V_{atn}) \quad (17)$$

The dependence of the average values, V_{atp}, V_{atn}, and their sum $V_{atsin} = V_{atp} + V_{atn}$, on the pulse duration, t_p, is similar as in the rectangular pulses with oscillations. The difference is only in the total average value.

IV. Conclusions

Average value of the distorted pulse signal with finite rise and fall edges durations and with oscillations (ringing) during the pulses has been evaluated. The errors of the pulses with finite rise and fall edges durations depend on the rise, t_r, and the fall, t_f, times of the pulses. Similarly, the errors of the pulses with exponential edges depend on the time constant of the exponential functions. This is of importance e.g. in class D power amplifiers, where it is necessary to have the shortest possible edge duration of the pulse signal.

Errors caused by decayed oscillations have been investigated under a supposition that the oscillations have equal parameters in both, the positive pulse and the zero level. The average value of the pulses with oscillations depends on the pulse duration, t_p, because of a different number of periods of the oscillations during the pulse.

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