19th Symposium IMEKO TC 4 Symposium and 17th IWADC Workshop
Advances in Instrumentation and Sensors Interoperability
July 18-19, 2013, Barcelona, Spain

Model of Errors Caused by Discrepancies of Input Channels in Multiresolution ADC

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***Abstract* -** Multiresolution analog-to-digital converter (MRADC) is usually used in the Time Domain ElectroMagnetic Interference (TDEMI) measuring system for very fast signal sampling with a sufficient dynamic range. The properties of the spectrum measured by TDEMI influenced by imperfections in MRADC are analyzed in the paper. Errors are cause by imperfect matching of the offset and gain of circuits used in parallel input channels typical for MRADC. For deep analyses of MRADC behavior, two mathematical models have been created using the concept of additive error pulses. The basic model is simpler. It uses rectangular pulses to describe the error component of the digitized signal. Furthermore, employing pulses of cosine shape too, the second extended mathematical model has been proposed covering both the slope and offset discrepancies of the input channels. For both models the resulting estimated error spectra are shown and compared with really measured ones.
Keywords- multiresolution quantization, time domain EMI measurement, offset and slope errors, spectrum measurement.

I. Introduction

Measuring of electromagnetic interference (EMI) spectra is obviously a time consuming procedure. Super heterodyne principle used in conventional analog EMI receivers or spectrum analyzers provides a high dynamic range. However, only one narrow frequency band is transferred to the detector via the intermediate frequency amplifier at a time. Therefore time-consuming sweeping through the whole bandwidth is needed and several tens of minutes are often required to complete the whole EMI spectrum measurement.

For commercial production of electronic devices, EMI measurements according to standards are required. Thus introduction of new faster EMI measuring principles leads to reduction in the production cost and time. The time domain EMI (TDEMI) system [1] was introduced quite recently. It is based on a multiresolultion analog-to-digital converter (MRADC) technology, which engages several parallel input channels to achieve the required qualities of the system. Principal block structure of TDEMI device is depicted in Figure 1. The power splitter distributes the analog signal to all paths, while 3 channels are usual today. The limiter protects ADC input from overvoltage. Separate amplifiers/attenuators provide different ranges and voltage resolutions of individual channels. All channels are simultaneously sampled and converted by identical 8 or 10 bit very fast flash ADCs. The final discrete value is created by extracting the output from that ADC offering the best resolution but with the range still covering the actual input value. Short time Fast Fourier Transform (FFT) is finally applied to the sampled data [2] to obtain the whole frequency spectrum.

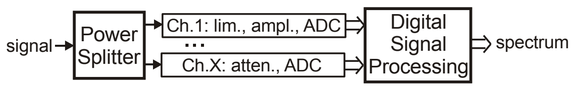


Figure 1. Block diagram of generalized TDEMI system.

TDEMI devices process the complete spectral content at a time and parallel structure of MRADC should still allow high dynamic range of measurements [3]. However, new error sources arise in the system, where parallel channels with different characteristics are used. Errors caused by those differences seem to be a severe source of spurious spectral components in the result of EMI spectrum measurement. Two models of error spectrum are discussed in the paper, which should help to understand the emergence of spurious higher harmonics in the measured signal spectrum. The situation is analyzed on an example of a harmonic input signal. The basic model allows simpler calculation of error components while the extended model precisely describes the impact of both the slope and offset discrepancies of the channels.

II. Real MRADC system and new model of errors

The concept of MRADC lies in the use of several parallel ADCs each with uniform but different quantization step. The minimum quantization step for the multiresolution quantization corresponds to the step of the channel with the lowest range. Actually, the system range is divided into subranges with different quantization steps. Even if perfectly realized such system generates disturbance in the quantized spectrum, as was shown in [4] for a harmonic signal. However practical experience shows that in real systems there are more serious sources of errors. Amplitude and phase frequency characteristics in each channel are not perfectly flat in the whole frequency range. Moreover it is hardly possible to avoid the offset and slope difference between channels as the gain must be different in each channel. So serious signal discontinuities could arise in points where the system switches from one ADC output to another. Spurious spectrum components generated by those discontinuities significantly restrict spurious free dynamic range (SFDR) of a real TDEMI device.
Harmonic input signal may be considered as suitable for modeling the measured interference of devices operating on switched mode power supply principle, where disturbance is like a mixture of sinusoids [5]. Differences between channels result in disturbances similar to time-domain error pulses like sketched in Figure 2 (in the graph the error is dotted gray line) for the harmonic input signal. For theoretical analysis of discontinuities present in the waveform reconstructed from sequence of samples the erroneous waveform could be simply modeled as an additive impulsive error signal. Within the frame of our study we consider two error models. Simplified error model is based on rectangular pulses (marked as REC). However rectangular error pulses precisely describe only discontinuities caused by the offset between channels. To embrace also the slope error the model of errors is extended with pulses of cosine shape (COS model) – Figure 2.

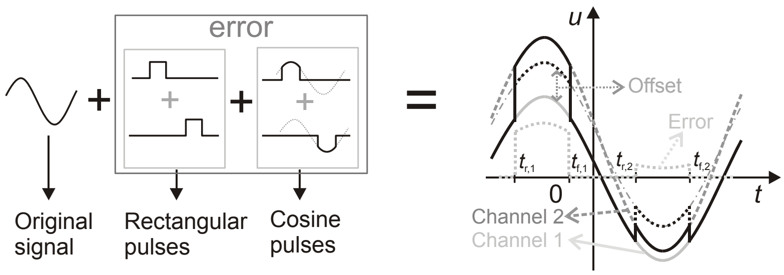


Figure 2. Modelling of errors of MRADC system.

III. Analytical expression of error models

Digital systems calculate spectrum by Discrete Fourier transform (DFT) usually using FFT algorithm. DFT output approximates coefficients of Fourier series, which decomposes the given periodic function $u(t)$ with frequency f_0 into the sum of harmonic functions. Therefore, speaking about spectrum, we are trying to find coefficients of Fourier series which could be written in the following exponential form

$$u(t) = U_0 + \mathrm{Re}\left\{ \sum_{1}^{\infty} 2\mathbf{U}_n e^{jn\omega_0 t} \right\} = \sum_{-\infty}^{\infty} \mathbf{U}_\mathrm{n} e^{jn\omega_0 t} \qquad (1)$$

where $\mathbf{U}_\mathrm{n}$ is complex spectral component often expressed in form $\mathbf{U}_n = a_n - \mathrm{j}b_n$ and some useful rules could be applied: $a_{-n} = -a_n$; $b_{-n} = -b_n$; $b_0 = 0$. In the complex coefficient $\mathbf{U}_n$ both the signal amplitude $2|\mathbf{U}_n|$ and the phase $\varphi_n = \arctan(-b_n/a_n)$ are included. Theoretical model proposed in this paper considers pure cosine harmonic input signal $u_\mathrm{in}(t) = A_\mathrm{in}\cos(\omega_0 t)$ with the amplitude A_in and frequency $\omega_0 = 2\pi f_0$. The spectrum of the ideal harmonic signal has only one component at f_0. As spectrum of sum of two signals is the sum of both signal spectra the higher harmonics present in the MRADC output waveform are just higher harmonics of error pulses. In other words, if we identify higher harmonics of error pulses, those could be regarded as theoretical spurious components in the measured spectrum. The first harmonic of the error waveform still means distortion of the value of the first harmonic shown by the measuring unit. To write analytical expression for spectrum of the MRADC error we need to know the spectrum of particular pulses: of the rectangular or cosine shape.

A. Rectangular pulse model

For evaluation of model of errors emerging in spectrum measured by TDEMI unit we need to find analytical expression for harmonic components of error pulse signals present in time representation of samples collected at the MRADC output. Let us consider a periodic rectangular pulse signal with a general amplitude or high level A, frequency f_0 and time shift (against start of period) of rising and falling edge t_r and t_f. Following Fourier coefficients could be derived for the pulse signal

$$\mathbf{U}_{\mathrm{RP},n}(\omega_0, A, t_r, t_f) = \begin{cases} \frac{\omega_0 A}{2\pi}(t_f - t_r); & n = 0 \\ \frac{A}{n2\pi}\left[\sin(n\omega_0 t_f) - \sin(n\omega_0 t_r)\right] + j\frac{A}{n2\pi}\left[\cos(n\omega_0 t_f) - \cos(n\omega_0 t_r)\right]; & n > 0 \end{cases} \quad (2)$$

We can see, that those coefficients are functions of parameters $\mathbf{U}_{\mathrm{RP},n}=f(\omega_0,A,t_r,t_f)$ mentioned above. Two square waveforms form the rectangular model of error (see Figure 2). Therefore error spectral components occurring by measurement could be theoretically estimated as

$$\mathbf{U}_{\mathrm{REC},n} = \mathbf{U}_{\mathrm{RP},n}(\omega_0, A_{\mathrm{REC1}}, t_{r1}, t_{f1}) + \mathbf{U}_{\mathrm{RP},n}(\omega_0, A_{\mathrm{REC2}}, t_{r2}, t_{f2}) \quad (3)$$

Amplitudes A_{REC1}, A_{REC2} represent error pulses of the model in the positive (A_{REC1}) and negative (A_{REC2}) half-waves of the input harmonic signal. The angular frequency of both pulses is ω_0 while the time shifts of rising and falling edge are different: t_{r1}, t_{r2} for the first pulse and t_{r1}, t_{r2} for the second one. Real error pulses are of square shape if offset between channels is the only discrepancy. Other imperfections usually cause that the real error value changes between t_r and t_f. Therefore some method should be chosen for determination of both amplitudes A_{REC1}, A_{REC2} (like the mean value estimation between rising and falling edge of the pulse).

B. Cosine error model

Rectangular pulses allow precise estimation of the error for the case when offset is the prevailing imperfection of input channels. The gain error of input circuits reflects in the stretched or shrunken slope of signals obtained from ADCs and the resulting error could be modeled by cosine pulses. To extend the error model for the case of significant slope error we need to analytically formulate the spectrum of pulses of the cosine shape. Like in the previous case complex coefficients of a cosine pulse will be function of amplitude, period and edge times

$$\begin{aligned} &\mathbf{U}_{\mathrm{RP},n}(\omega_0, A, t_r, t_f) = \\ &= \frac{A}{2\pi}\left[\sin(\omega_0 t_f) - \sin(\omega_0 t_r)\right]; && n = 0 \\ &= \frac{A\omega_0(t_f - t_r)}{4\pi} + \frac{A}{8\pi}\left[\sin(2\omega_0 t_f) - \sin(2\omega_0 t_r)\right] - j\frac{A}{8\pi}\left[\cos(2\omega_0 t_r) - \cos(2\omega_0 t_f)\right]; && n = 1 \\ &= \frac{A\left[n\cos(\omega_0 t_f)\sin(n\omega_0 t_f) - \sin(\omega_0 t_f)\cos(n\omega_0 t_f) - n\cos(\omega_0 t_r)\sin(n\omega_0 t_r) + \sin(\omega_0 t_r)\cos(n\omega_0 t_r)\right]}{2\pi(n^2-1)} \\ &+ j\frac{A\left[n\cos(\omega_0 t_f)\cos(n\omega_0 t_f) + \sin(\omega_0 t_f)\sin(n\omega_0 t_f) - n\cos(\omega_0 t_r)\cos(n\omega_0 t_r) - \sin(\omega_0 t_r)\sin(n\omega_0 t_r)\right]}{2\pi(n^2-1)}; && n > 1 \end{aligned} \quad (4)$$

In this case the model of error spectrum consists from four members

$$\begin{aligned} \mathbf{U}_{\mathrm{COS},n} = &\mathbf{U}_{\mathrm{RP},n}(\omega_0, A_{\mathrm{COS\text{-}O}}, t_{r1}, t_{f1}) + \mathbf{U}_{\mathrm{RP},n}(\omega_0, A_{\mathrm{COS\text{-}O}}, t_{r2}, t_{f2}) \\ &+ \mathbf{U}_{\mathrm{CP},n}(\omega_0, A_{\mathrm{COS\text{-}S}}, t_{r1}, t_{f1}) + \mathbf{U}_{\mathrm{CP},n}(\omega_0, A_{\mathrm{COS\text{-}S}}, t_{r2}, t_{f2}) \end{aligned} \quad (5)$$

First two members represent rectangular error pulses caused by offset. We could take the signal from the finer channel 1 for reference to analyze the spectrum disturbance caused by switching between non-identical channels. Then the amplitude of square pulses is determined just by the difference in offset O of values obtained from two channels (index "R" means rescaled for the common grid)

$$A_{\mathrm{COS\text{-}O}} = O_{\mathrm{R},2} - O_{\mathrm{R},1} \quad (6)$$

where index 1 means the first channel and 2 the second and higher voltage range channel. Two cosine pulses represent the slope error between channels and the amplitude is

$$A_{\mathrm{COS\text{-}S}} = A_{\mathrm{R},2} - A_{\mathrm{R},1} = A_{\mathrm{in}}(G_{\mathrm{R},2} - G_{\mathrm{R},1}) \quad (7)$$

where $A_{\mathrm{R},1}$, $A_{\mathrm{R},2}$ are amplitudes of signals in corresponding channel and G is the gain. Number $A_{\mathrm{COS\text{-}S}}$ should be used with the sign – positive or negative – if the right phase of components $\mathbf{U}_{\mathrm{COS},n}$ obtained from the model is of interest. If the channel 1 is ideal, like considered later in simulation, expressions (6)(7) get a simplified form.

C. Time shift of rising and falling edge

To be able to calculate error spectral components a time shift of the rising and falling edge of error pulses has to be expressed. It could be estimated from experimental data or also analytically under some assumptions about the system behavior. We will also assume that the MRADC system switches from the first channel to the second, or vice-versa, when the value from the first channel is just crossing its range R_1. For a periodic input signal we are trying to find all impulses occurring within one period T_0. To start with the rising edge for the case of the considered cosine function (time shift towards the cosine) we will delimitate the reference period starting from a negative time $-T_0/4$. Therefore we need to find t_r and t_f within the interval $(-T_0/4;3T_0/4)$. Actually there are up to two pulses per period for the harmonic signal, like depicted in Figure 2. Theoretically the time shift of rising/falling edge is (index 1/2 means the first/second half-wave)

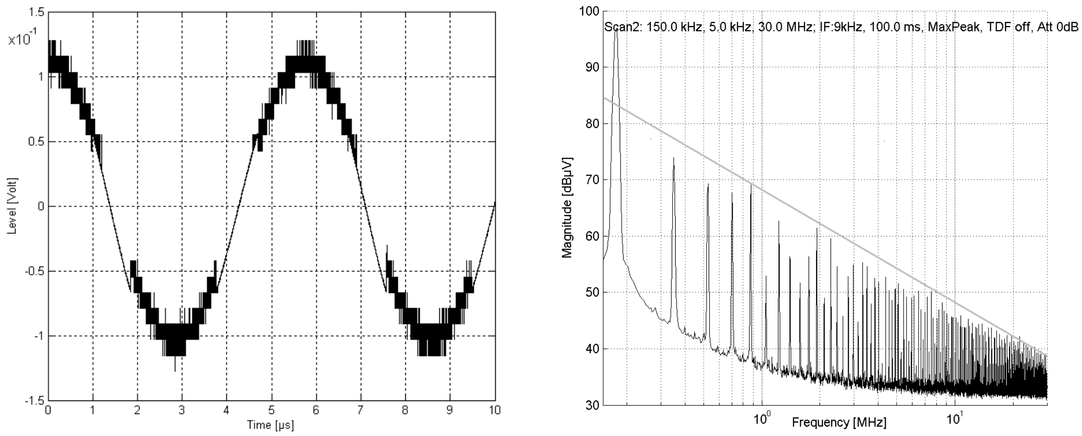
$$t_{r1} = -t_{f1} \; ; \; t_{f1} = \frac{\arccos\left(\dfrac{R_{R,1} - O_{R,1}}{G_{R,1}A_{in}}\right)}{\omega_0} \; ; \; t_{r2} = \frac{\arccos\left(\dfrac{-R_{R,1} - O_{R,1}}{G_{R,1}A_{in}}\right)}{\omega_0} \; ; \; t_{f2} = T_0 - t_{r2} \qquad (8)$$

The widths of error pulses are different if there is offset in the first channel.
If the first channel has no offset ($O_{R,1}$=0) a special relation between given time parameters could be found which allows some formal modification of the COS error model. For this case the edge times of the second pulse are just shifted towards the edge times of the first one by half of the signal period $t_{r2} = t_{r1}+T_0/2$, $t_{f,2}=t_{f,1}+T_0/2$ (like in Figure 5a). Because in the COS model both rectangular pulses have the same amplitude A_{COS-O}, only one square pulse signal but of double frequency $2f_0$ is now sufficient for modeling of errors caused by offset between channels. Contribution of this error part is then present only in members corresponding to even harmonics of analyzed signal while the odd error spectral components will be affected only by slope error modeled by cosine pulses. Finally for the COS model we could write

$$\mathbf{U}_{COS,n} = \begin{cases} \mathbf{U}_{RP,n/2}(2\omega_0, A_{COS-O}, t_{r1}, t_{f1}) + \mathbf{U}_{CP,n}(\omega_0, A_{COS-S}, t_{r1}, t_{f1}) + \mathbf{U}_{CP,n}(\omega_0, A_{COS-S}, t_{r2}, t_{f2}); & n \text{ even} \\ \mathbf{U}_{CP,n}(\omega_0, A_{COS-S}, t_{r1}, t_{f1}) + \mathbf{U}_{CP,n}(\omega_0, A_{COS-S}, t_{r2}, t_{f2}); & n \text{ odd} \end{cases} \qquad (9)$$

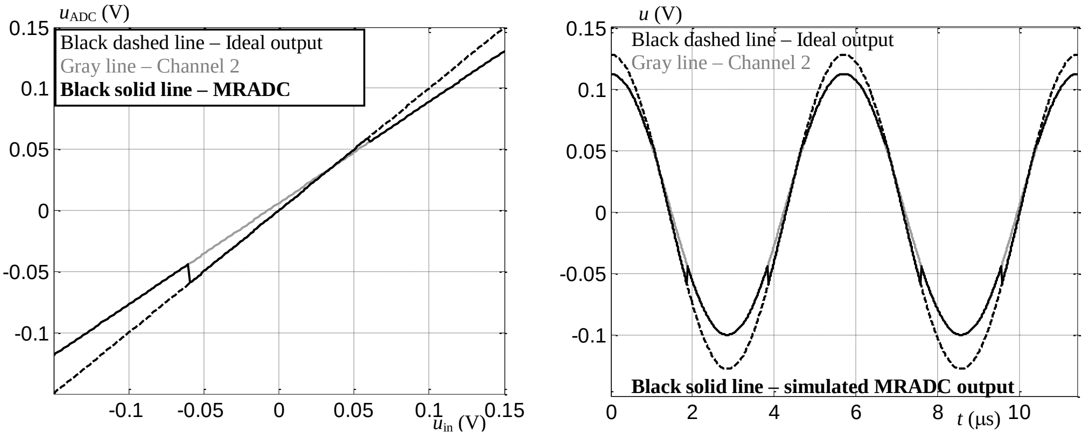
IV. Model and simulation

After mathematical derivation of theoretical models their properties were compared with simulation results. Parameters of the input signal and error of channels were set according to the previous experiment on a real TDEMI unit [5]. The measured waveform and spectrum are depicted in Figure 3. For the harmonic input signal of frequency f_0 = 175 kHz, amplitude above 100 mV and zero offset the measured waveform is shown in Figure 3a. The signal is visibly affected by error caused by differences of individual channels and by quantization error. From Figure 3b it is obvious that the measured spectrum is disturbed by spurious higher harmonics with decay close to -20dB/decade represented by gray line attached to 5th harmonics. We used simulation to show that the spurious components originate in discrepancies of channels.
Experimental waveform from Figure 3a is composed from samples of two channels. We set simulation parameters according to the properties of real channels using some simplifications. The amplitude of channel 1was considered ideal A_{in}=128 mV, $G_{R,1}$ = 1. Then, the gain of the second channel is $G_{R,2}$ = 0.83. The TDEMI switches between channels near the threshold $R_{R,1}$=60 mV. Actually, the threshold differs in the positive and negative half-waves because of offset of the channel 1 and system imperfections. However, for simulations we used the mean threshold $R_{R,1}$=60 mV and zero offset of channel 1 $O_{R,1}$ = 0 mV. Then, the offset between channels was assigned to the second channel $O_{R,2}$ = 6 mV. There is only little phase shift between channels which could be neglected in simulation. We used synchronous sampling (integer number of periods and rectangular window).
Described parameters of the simulated TDEMI system are roughly apparent from curves shown in Figure 4. In Figure 4a we can see corresponding transfer characteristics including combined characteristics of both channels of the MRADC. Mentioned parameters lead to signal waveforms depicted in Figure 4b where the reference cosine signal (dashed ideal waveform) is finally deformed within intervals delimited by edge times where the signal is crossing range of the channel 1. Two periods of the input signal were sampled in the simulation while the period was T_0=5.714 μs. The interval of reference period for calculation of pulse edge times is thus $(-T_0/4;3T_0/4)$ ~ (-1.428 μs; 4.286 μs). For given conditions values of times of rising and falling edge are t_{r1} = -0.9849 μs, t_{f1} = 0.9849 μs for the first pulse and t_{r2} = 1.8723 μs, t_{f2} = 3.8420 μs for the second one. Those times enter into equations of error models (3) and (5) or (9), while simulated edge times (Figure 5a) were the same or shifted by integer multiple of signal period.



a) b)

Figure 3. Experimental measured characteristics obtained from a real TDEMI unit (for f_0=175 kHz): a) Waveform composed from samples of two channels; b) Spectrum distorted by spurious components.



a) b)

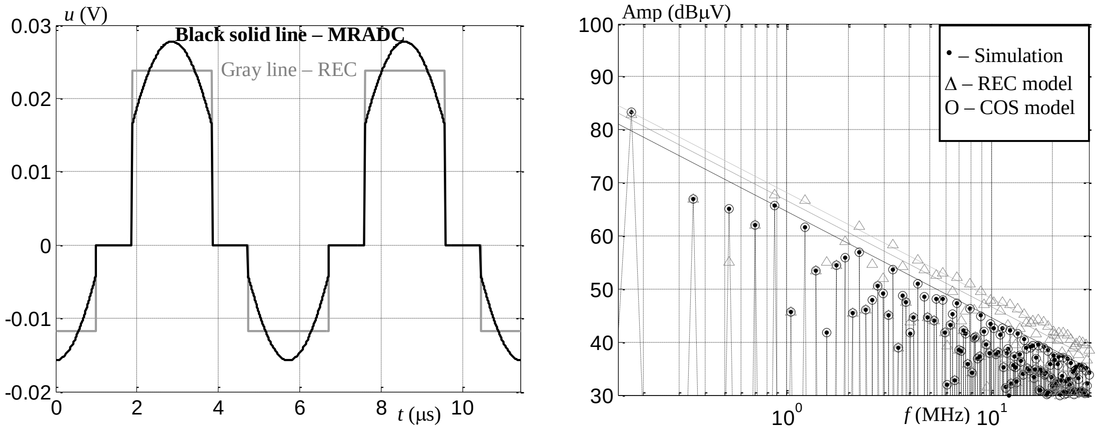
Figure 4. Transfer characteristics of simulated MRADC a) and simulated deformation of the harmonic test signal caused by discrepancies of input channels b).

In Figure 5 only error parts of MRADC signals are shown. The time representation of errors has character of a pulse signals shown in Figure 5a. Plotted signal values were calculated according to idea of REC and COS model. The amplitudes of pulses in REC model are A_{REC1} = -11.75 mV, A_{REC2} = 23.75 mV determined as mean values from real error pulses. In the COS model square pulses correspond to offset between channels $A_{\mathrm{COS\text{-}O}}$ = 6 mV and cosine pulses of amplitude $A_{\mathrm{COS\text{-}S}}$ = - 21.76 mV correspond to the slope error, while sum of both forms the black line in Figure 5a.

Finally, spectrums of signals are depicted in Figure 5b. The simulated spectrum is compared with results from both models for the given input signal amplitude of $A_{\mathrm{in}}/\sqrt{2}$, i.e. cca 100 dBµV. Note that a real spectrum measuring system usually shows amplitude spectral components divided by $\sqrt{2}$. One can see that the spectral components obtained especially from the COS model are almost identical with simulation. The error of main the harmonic component is 83.3 dBµV and the DC error part – not visible in the figure – is 75.3 dBµV. The main spurious component is the second harmonic which could be precisely estimated from (5) or (9) $\mathbf{U}_{\mathrm{COS},3}$ = 1.58 mV (the complex part is zero) ~ 67.0 dBµV.

The trend and envelope of the spectrum obtained from the simulation or from the COS model well approximates the experimental spectrum drawn in Figure 3b. Little vertical shift should result from simplifications used in the simulated MRADC system. However, we can still draw a reference line of -20 dB/decade attached to the 5th harmonics – black dashed line for the simulation and COS model. Similarly to experiments (notice the interval from 10^0 MHz to 10^1 MHz) spurious components are placed under this reference line. On the other hand the

REC model is inaccurate for odd harmonics. It deeply underestimates the third harmonics while overestimates many higher odd harmonics which lie above the reference line drown for the REC model (the gray dashed line attached to its 5th harmonics) or even above the reference line copied from the experimental spectrum (the gray solid line).



a) b)

Figure 5. Errors of MRADC system (for f_0=175 kHz):
a) Error time pulses according to REC and COS representation; b) Spectrum of simulated error compared with spectra calculated by REC and COS error models.

V. Conclusions

In the paper two types of model were introduced for modelling the error spectral components generated by multiresolution ADC (MRADC) system discrepancies. The offset and slope differences between parallel input channels of the MRADC are considered. In a simple case a rectangular pulse model could be used for analysis of the error impact on the measured spectrum. The model extended with cosine impulses was proposed for more precise estimation of error spectral components if simple offset is not the only difference between two channels. Simulation of signal parameters, offset and slope error similar to a real TDEMI measurement system were accomplished and results have been compared with both models. Presented results have demonstrated correctness of extended model. From comparison to experimental results we can claim that channel discrepancy is the dominant error source in the experimental spectrum measured by a real TDEMI unit.

Acknowledgement

This work was supported by Slovak Ministry of Education under grant No. 2003SP200280802 and the Slovak Grant Agency VEGA under grant No. 1/0963/12.

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