

Impedance Estimation of a Vibrating Wire Viscosity Sensor Using Multi-Harmonic Signals

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Abstract-This paper presents a method to estimate the impedance response of a vibrating wire viscosity sensor. The method is based on the application of a multi-harmonic signal at the terminals of the sensor. The signal is composed by the harmonics that correspond to the frequencies at which one wants to obtain the impedance of the sensor. The impedance is determined using least-squares (LS) multi-harmonic fitting algorithms to estimate the amplitudes and phases of the harmonics of the impedance current and voltage. The measurement and estimation procedure is performed by a dedicated developed measurement system. It includes a digital signal processor (DSP) to perform all the calculations, programmable gain instrumentation amplifiers (PGIA) and analog to digital converters (ADC) to amplify and acquire the signals across the sensor and a reference impedance used to limit and sample the current flowing through the wire of the sensor. The system is connected via USB to a personal computer (PC) where the measurement results can be stored, interpreted and further processed.

Keywords: Multi-Harmonic Least-Squares Fitting, Impedance Estimation, Viscosity Sensor, DSP.

I. Introduction

Viscosity measurements play a key role in several scientific and industrial fields. There is a significant variety of fluids with an extensive range of viscosity with great economic importance, and for many industrial processes the speed at which the viscosity measurement can be performed is critical. For example, in medicine, studies have shown that increases in the viscosity of blood and plasma can constitute a risk factor for certain vascular diseases [1]. The treatment of such diseases requires the control of blood fluidity, which is achieved by monitoring its viscosity. In the petroleum industry [2], reservoir fluid properties, including viscosity, are important for the exploration process. Prior knowledge of the behaviour of fluids under a wide range of pressure and temperature conditions is required.

There are several methods to measure the viscosity of fluids. For example, capillary viscometers [3] operate based on the Hagen-Poiseuille equation of fluid dynamics. The viscosity is obtained from the measurement of the flow rate produced by a pressure difference along a tube of known diameter. The rolling ball viscometer is based on the falling body method [3], where Stokes' law is used to relate the free fall time of an object (e.g., a sphere or cylinder) under the influence of gravity through the fluid of interest to the fluid's viscosity. The vibrating wire viscometer [3, 4] consists on injecting an alternating electrical current through a wire that is subject to an externally applied transverse magnetic field and immersed in the fluid of interest. The interaction between the current and the magnetic field causes periodic oscillations in the wire and these oscillations depend on the viscosity of the fluid. Using a set of equations, based on the physical behaviour of the sensor, the viscosity of the fluid can be derived.

The typical measurement procedure, when using a vibrating wire viscometer, is to perform a single-tone frequency sweep around the resonance frequency of the sensor to obtain its impedance frequency response [4]. This can be time consuming, depending on the throughput capability of the measurement system, the number of points in the frequency sweep and the number of samples acquired. The aim of this work is to obtain the impedance response of the sensor in a single acquisition/measurement by using a multi-harmonic signal whose harmonics correspond to the frequencies at which the impedance response of the sensor is to be analysed. LS multi-harmonic fitting algorithms [5] are used to estimate the amplitudes and phases of the harmonics and obtain the sensor's impedance. In this paper, the traditional measurement approach, referred to as the sweep method, will be compared to the multi-harmonic approach, which will be referred to as the MH method.

II. Vibrating Wire Viscosity Sensor

The vibrating wire viscosity sensor used in this work is represented in Fig. 1. The vibrating wire is made of tungsten, the body of the sensor is composed by stainless steel and the magnetic field is generated by permanent magnets placed transversely to the wire inside a cylindrical encapsulation.

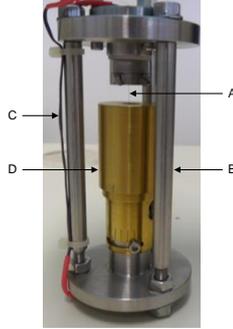


Figure 1. Vibrating wire sensor. A: Tungsten vibrating wire; B: Stainless steel spacers; C: Electrical wiring; D: Magnetic circuit.

When the sensor is subject to an AC current in the presence of the magnetic field, the wire oscillates. In turn, due to the motion of the wire in the presence of the magnetic field, an electromotive force will be induced at the terminals of the wire, which is proportional to the velocity of the wire [6]. The velocity of the wire reaches its peak at its resonance frequency, and it decreases its value as the signal's frequency differs from the resonance frequency. By measuring the voltage at the terminals of the sensor, and the current flowing through the wire, it is possible to determine the impedance of the wire, and ultimately estimate the viscosity of the fluid.

III. Measurement System

The system used (Fig. 2) is an upgrade from the system previously developed and described in [7]. PGIA's and ADC's are used to amplify and acquire the signals across the sensor and a reference impedance connected in series with the sensor. The reference impedance used is a 3 kΩ resistor. The system has a direct digital synthesizer (DDS), used to generate sinusoidal signals with an amplitude of 0.6 V_{pp}, but it allows an external function generator (FG) to be used to inject current into the sensor. A DSP is used to control the hardware and run the algorithms to estimate the impedance of the sensor. The system is controlled via USB with a PC that runs a control application implemented in LabVIEW.

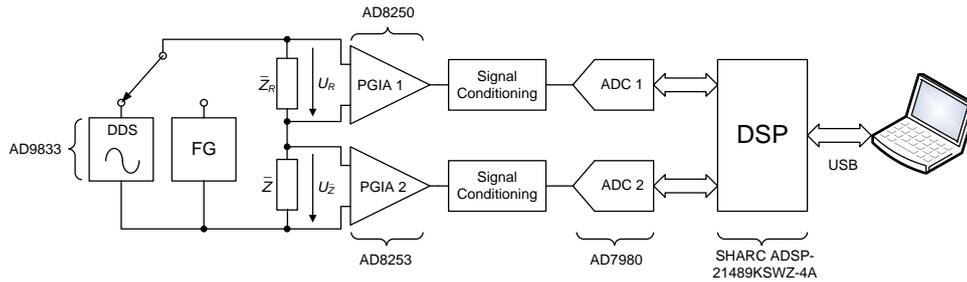


Figure 2. Measurement system. \bar{Z}_R is the reference impedance and \bar{Z} is the sensor's impedance.

The impedance magnitude ($|\bar{Z}|$) and phase ($\phi_{\bar{Z}}$) of the sensor are estimated by

$$|\bar{Z}| = |\bar{Z}_R| \times \frac{|U_{\bar{Z}}|}{|U_{\bar{R}}|}, \quad \phi_{\bar{Z}} = \phi_{\bar{Z}_R} + (\phi_{U_{\bar{Z}}} - \phi_{U_{\bar{R}}}) \quad (1)$$

where $|\bar{Z}_R|$ and $\phi_{\bar{Z}_R}$ are the magnitude and phase of the reference impedance at the measurement frequency, $|U_{\bar{Z}}|$ and $\phi_{U_{\bar{Z}}}$ represent the amplitude and phase of the signal across the wire, while $|U_{\bar{R}}|$ and $\phi_{U_{\bar{R}}}$ are the amplitude and phase of the signal across the reference impedance.

IV. Multi-harmonic Signal Generation

In this work an HP33120A arbitrary waveform generator is used to apply the multi-harmonic signal to the sensor. The multi-harmonic signals used were compressed using the clipping algorithm [8], which tries to minimize the crest factor of the signal by iteratively distributing the phases of the sine signals that constitute the multi-harmonic signal. Afterwards, the signals are centered and normalized to an amplitude of 1 V.

Three multi-harmonic signals composed by 21, 31 and 61 harmonics of equal amplitude and evenly spaced between 800 and 1100 Hz were used in this work. Fig. 3 presents the frequency spectrums of the signals. 8000 samples of the signals were acquired with a 16-bit NI 9215 data acquisition board at a sampling rate of 8 kS/s.

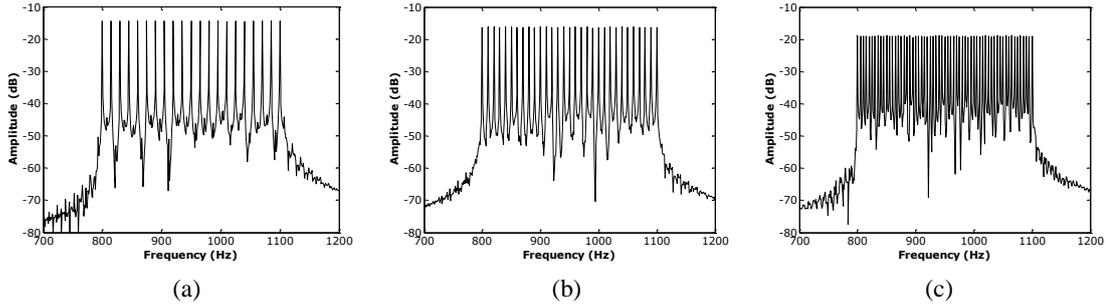


Figure 3. Frequency spectrums of the multi-harmonic signals composed by 21 (a), 31 (b) and 61 (c) harmonics.

V. Algorithms

A. Sine-Fitting Algorithms

Sine-fitting algorithms are quite useful to determine the set of parameters that better describe a sine signal given a finite set of acquired samples. The three-parameter and four-parameter sine-fitting algorithms were standardized in [9]. Both estimate the amplitude, phase and DC component of the signal. The three-parameter algorithm requires the knowledge of the frequency, while the four-parameter algorithm uses an iterative procedure to obtain the frequency, and the remaining parameters, based on an initial estimate, usually obtained with the three-parameter algorithm. In [10], a seven-parameter algorithm was proposed. In two-channel systems this algorithm uses the records of both channels to estimate the amplitude, phase and DC component of both signals, as well as the common frequency. In this work it was decided not to use the seven-parameter algorithm to provide a more accurate base of comparison between the sweep method and the MH method.

In the sweep method the three-parameter algorithm is initially applied to the signal with the largest amplitude. Next the four-parameter algorithm is applied to the same signal to estimate the signal's frequency, and finally the three-parameter algorithm is applied to the signal with the smallest amplitude using the previously estimated frequency. In this case the channel with the largest amplitude is always the channel of the reference impedance.

B. LS Multi-Harmonic Fitting Algorithms

A multi-harmonic periodic signal can be represented by

$$y(t) = C + \sum_{h=1}^H D_h \cos(2\pi h f t + \phi_h) = C + \sum_{h=1}^H [A_h \cos(2\pi h f t) + B_h \sin(2\pi h f t)], \quad (2)$$

with

$$D_h = \sqrt{A_h^2 + B_h^2}, \quad \phi_h = -\text{atan2}(B_h, A_h), \quad A_h = D_h \cos(\phi_h), \quad B_h = -D_h \sin(\phi_h), \quad (3)$$

where C is the DC component, f is the signal frequency, D_h and ϕ_h are the amplitude and phase of harmonic h , and A_h and B_h are the in-phase and in-quadrature amplitudes of harmonic h . H is the number of harmonics.

There are two kinds of LS multi-harmonic fitting algorithms: the non-iterative LS multi-harmonic fitting algorithm, referred to as the $2H + 1$ algorithm, and the iterative LS multi-harmonic fitting algorithm, also called the $2H + 2$ algorithm [5]. The $2H + 1$ algorithm is used when the signal's frequency is known. In this case the algorithm's $[(2H + 1) \times 1]$ estimated vector is

$$\hat{\mathbf{x}} = [\hat{A}_1 \quad \hat{B}_1 \quad \hat{A}_2 \quad \hat{B}_2 \quad \cdots \quad \hat{A}_H \quad \hat{B}_H \quad \hat{C}]^T = [(\mathbf{D}^T \mathbf{D})^{-1} \mathbf{D}^T] \mathbf{y}, \quad (4)$$

where \mathbf{y} is the $(M \times 1)$ vector with the acquired samples y_m ($m = 1, \dots, M$), to which the relative time stamps t_m are associated with. Matrix \mathbf{D} has M rows and $(2H + 1)$ columns and is defined as

$$\mathbf{D} = \begin{bmatrix} \cos(\omega t_1) & \sin(\omega t_1) & \cdots & \cos(H\omega t_1) & \sin(H\omega t_1) & 1 \\ \cos(\omega t_2) & \sin(\omega t_2) & \cdots & \cos(H\omega t_2) & \sin(H\omega t_2) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \cos(\omega t_M) & \sin(\omega t_M) & \cdots & \cos(H\omega t_M) & \sin(H\omega t_M) & 1 \end{bmatrix}, \quad (5)$$

where $\omega = 2\pi f$. When the frequency is unknown an iterative procedure takes place to estimate it and the rest of the signal's parameters. For this purpose, the $2H + 2$ algorithm is used. The initial estimates of the parameters are obtained with the $2H + 1$ algorithm, and the $[(2H+2) \times 1]$ estimated vector in iteration i is

$$\hat{\mathbf{x}}^{(i)} = \begin{bmatrix} \hat{A}_1^{(i)} & \hat{B}_1^{(i)} & \hat{A}_2^{(i)} & \hat{B}_2^{(i)} & \cdots & \hat{A}_h^{(i)} & \hat{B}_h^{(i)} & \hat{C}^{(i)} & \Delta\hat{\omega}^{(i)} \end{bmatrix}^T = \left[(\mathbf{D}^{(i)})^T \mathbf{D}^{(i)} \right]^{-1} (\mathbf{D}^{(i)})^T \mathbf{y}, \quad (6)$$

where $\Delta\hat{\omega}^{(i)}$ is the frequency increment in iteration i to the frequency estimation of iteration $(i - 1)$. Matrix $\mathbf{D}^{(i)}$ has M rows and $(2H + 2)$ columns and is defined as

$$\mathbf{D}^{(i)} = \mathbf{D} \begin{bmatrix} \sum_{h=1}^H -\hat{A}_h^{(i-1)} h t_1 \sin(\hat{\omega}^{(i-1)} h t_1) + \hat{B}_h^{(i-1)} h t_1 \cos(\hat{\omega}^{(i-1)} h t_1) \\ \sum_{h=1}^H -\hat{A}_h^{(i-1)} h t_2 \sin(\hat{\omega}^{(i-1)} h t_2) + \hat{B}_h^{(i-1)} h t_2 \cos(\hat{\omega}^{(i-1)} h t_2) \\ \vdots \\ \sum_{h=1}^H -\hat{A}_h^{(i-1)} h t_M \sin(\hat{\omega}^{(i-1)} h t_M) + \hat{B}_h^{(i-1)} h t_M \cos(\hat{\omega}^{(i-1)} h t_M) \end{bmatrix}. \quad (7)$$

The algorithm converges when the relative frequency adjustment, $|\Delta\hat{\omega}^{(i)}/\hat{\omega}^{(i)}|$, is below the preset threshold of 5×10^{-6} . This value results from a good compromise between obtaining a frequency estimate with a good accuracy and the number of iterations necessary for the algorithm to converge (typically two).

Note that the form of the algorithms presented here for the $2H + 1$ and $2H + 2$ algorithms is the same for the three-parameter and four-parameter sine-fitting algorithms, respectively, if one considers only one harmonic, i.e. $H = 1$. The order of the algorithms used in the MH method is analogous to the one in the sweep method. First the $2H + 1$ and the $2H + 2$ algorithms are applied to the signal with the largest amplitude, and then the $2H + 1$ algorithm is applied to the signal with the smallest amplitude.

VI. Results

The proposed method was tested with three sample fluids composed by distilled water and glycerol in different concentrations: S1 – 100% water, 0% glycerol; S2 – 90% water, 10% glycerol; S3 – 80% water, 20% glycerol. Samples with a lower content of glycerol in the water/glycerol mixture have a lower viscosity value, while samples with a higher content of glycerol have a higher viscosity value. Fig. 4 presents the impedance magnitude and phase of the sensor for the considered sample fluids obtained with the 3522-50 LCR HiTESTER from HIOKI. All samples were measured at ambient temperature and atmospheric pressure.

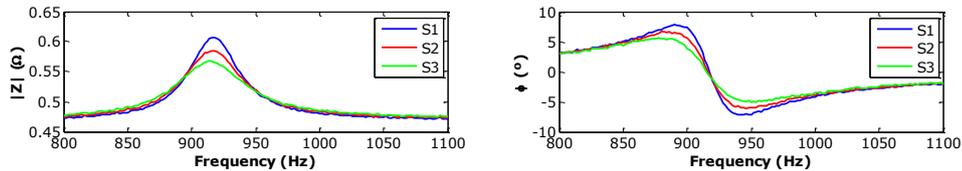


Figure 4. Impedance response of the sensor for three different sample fluids obtained with the 3522-50 LCR HiTESTER from HIOKI at ambient temperature and atmospheric pressure.

The three specified sample fluids were used to measure the sensor's impedance response for both the sweep and MH method at ambient temperature and atmospheric pressure. Fig. 5, Fig. 6 and Fig. 7 present the results obtained when considering 21, 31 and 61 sweep points/harmonics, respectively. All the results were obtained by acquiring 8000 samples at a sampling rate of 8 kS/s for both methods with the developed measurement system.

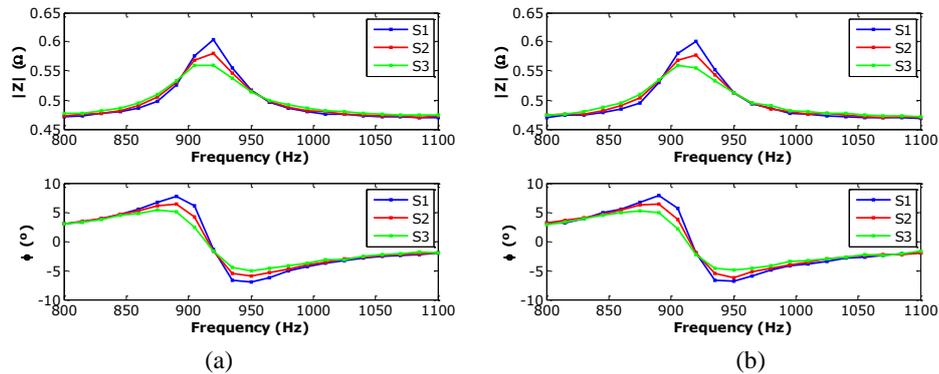


Figure 5. Sensor's impedance response: (a) using sweep method; (b) using MH method, considering 21 sweep points/harmonics at ambient temperature and atmospheric pressure.

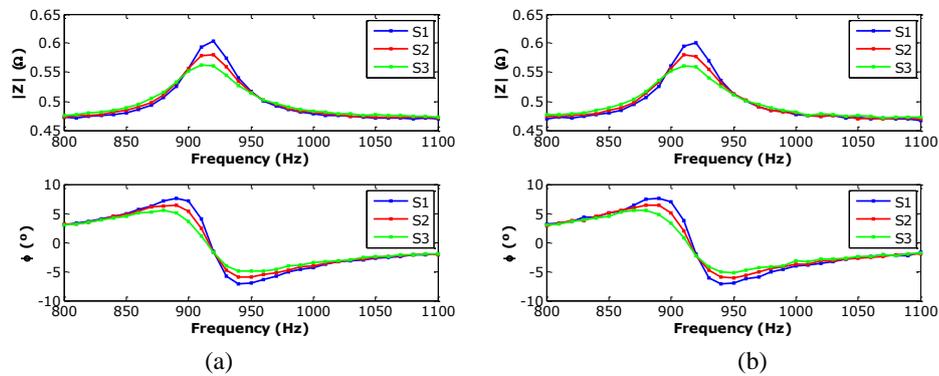


Figure 6. Sensor's impedance response: (a) using sweep method; (b) using MH method, considering 31 sweep points/harmonics at ambient temperature and atmospheric pressure.

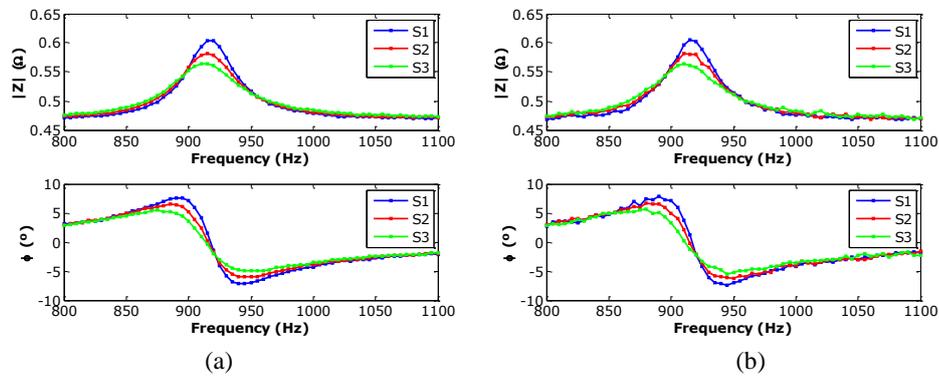


Figure 7. Sensor's impedance response: (a) using sweep method; (b) using MH method, considering 61 sweep points/harmonics at ambient temperature and atmospheric pressure.

The results presented show that both methods correctly estimate the impedance response of the sensor. However, one can see that those obtained with the MH method show more variations than those obtained with the sweep method. This is more evident in the case of Fig. 7. Bear in mind that the harmonic amplitudes of the multi-harmonic signals used in the MH method are quite lower than the amplitude of the sine signal used in the sweep method, and that they decrease as the number of harmonics increases. This associated with the fact that the amplitude of the signal across the sensor is very small, makes it more difficult to obtain an accurate estimate of the amplitudes and phases of the harmonics of the signals.

The total measurement time, along with the acquisition time of both methods as a function of the number of sweep points/harmonics considered is presented in Fig. 8. The total measurement time accounts for the acquisition time and for the processing time, which includes the time spent in the execution of the fitting algorithms and in the determination of the impedance of the sensor.

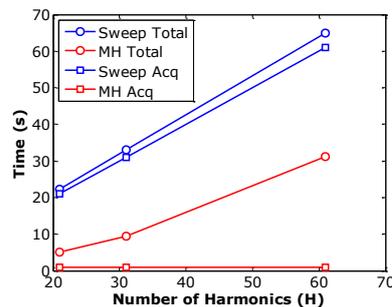


Figure 8. Total measurement times and acquisition times for the sweep and MH methods as a function of the number of sweep points/harmonics considered (21, 31, 61).

The results presented in Fig. 8 show that the MH method has the lowest measurement time. The reason for this lies in the great difference between the acquisition times of both methods. The MH method requires only a single acquisition to obtain the frequency response of the sensor, while in the sweep method one must perform as many acquisitions as the number of frequency sweep points. This makes the MH method advantageous in applications where the measurand depends on the environmental conditions, like in this work, because it avoids stabilizing the measurement conditions for a longer period of time, which may simplify the measurement procedure.

VII. Conclusions

This paper presents a method to estimate the impedance of a vibrating wire viscosity sensor based on the application of multi-harmonic signals. The proposed MH method was tested with multi-harmonic signals composed by the harmonics of interest for the estimation of the impedance of the sensor, and compared to the traditional sweep method. The presented results show that the MH method successfully obtains the impedance response of the sensor in a single acquisition/measurement, unlike the sweep method which requires many acquisitions and presents a longer measurement time. The purpose of this paper wasn't to obtain the viscosity of the tested fluids, but rather to ascertain the validity of the proposed MH method. Eventually it will be necessary to assess the influence of the number of harmonics in the estimation uncertainty of the fluids' viscosity.

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