

## CRACK SIZE IDENTIFICATION AND LOCALIZATION USING ULTRASONIC SENSORS WITH STATIONARY WAVELET TRANSFORM AND HILBERT TRANSFORM

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**Abstract** - This paper presents a method for locating and measuring the size of cracks in beams by time of flight (ToF) of ultrasonic wave. The echo detected was processed using the stationary wavelet transform and the Hilbert transform. The accuracy of the information obtained from the processed signal depends on the selected mother wavelet. To select the best mother wavelet for signal processing were used the Shannon's entropy criterion. The ToFs were obtained from denoising process using the stationary wavelet transform and the envelope of the Hilbert transform. The uncertainties of the ToFs using Fuzzy numbers are also estimated in order to assessment the qualities of the ToF Measurement.

**Keywords:** - ToF, stationary wavelet transform, Hilbert transform, Shannon's entropy

### I. Introduction

Ultrasonic methods of Non-destructive testing NDT use beams of mechanical waves (vibrations) of short wavelength and high-frequency, transmitted from a transducer and detected by the same or other transducer. Such mechanical waves can travel large distances in fine-grain metal, in the form of a divergent wave with progressive attenuation. The frequency is in the range 0.1 to 20 MHz and the wavelength in the range 1 to 10 mm. The velocity depends on the material and is in the range 1000-7000 m/s. Most non-porous, resilient materials used for structural purposes (steel, aluminium, titanium, magnesium and ceramics) can be penetrated. Even large cross-sections can be tested successfully for minute discontinuities. Ultrasonic testing techniques are widely accepted for quality control and materials testing in many industries, including electric power generation, production of steel, aluminium and titanium, in the fabrication of airframes, jet engine manufacture and ship building.

The purpose of this paper is to provide a method to estimate the crack length and the crack depth through the information processing of ultrasonic signals transmitted and received echo returning. The aim of processing is to obtain the transit time or Time of Flight (*ToF*), which is the time difference between the instant that the ultrasonic signal is emitted to the instant when the ultrasonic signal is detected [1], [2]. Numerical results for different positions of a crack in a beam are presented based on simulations. These results show that the stationary wavelet transform and the Hilbert transform are an efficient signal processing approach to extract information from ultrasonic wave propagation for crack localization on beams [2].

### II. Preliminary Definitions

#### A. Crack Detection System Configuration

A usual configuration for locating and measuring the size of cracks in steel beams using ultrasonic transducers is shown in Figure 1 four kinds of waves are generated. The lateral wave travels from the transmitter to the receiver directly along the surface of the specimen. The two diffracted signals from the top and the bottom tips of the crack and the back wall echo travel through the specimen to the receiver.  $H$  is the thickness of the beam. In addition, it is assumed that the crack's orientation is vertical and the two transducers are located symmetrically around the crack [2].

In order to calculate the crack length and the crack depth, the Pythagoras's theorem can be applied. Assuming that the ultrasonic wave velocity in the beam is  $C$ , the following equations are available:

$$2\sqrt{S^2 + D^2} + d = C_0 ToF_0 + C ToF_2 \quad (1)$$

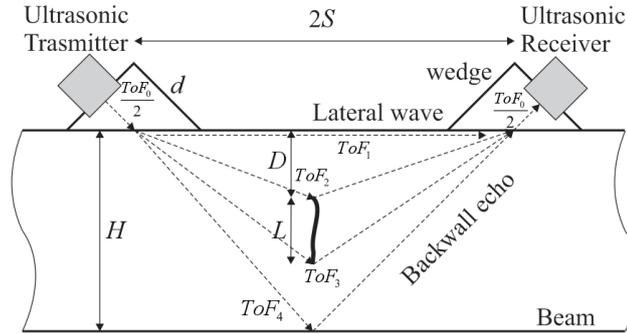


Figure 1. Basic principle of the ToF method.

$$2\sqrt{S^2 + (D+L)^2} + d = C_0 ToF_0 + C ToF_3 \quad (2)$$

From the equations (1) and (2), the crack depth  $D$  and the crack length  $L$  can be expressed as:

$$D = \sqrt{\frac{1}{4}(C_0 ToF_0 + C ToF_2 - d)^2 - S^2} \quad (3)$$

$$L = \sqrt{\frac{1}{4}(C_0 ToF_0 + C ToF_3 - d)^2 - S^2} - D \quad (4)$$

where:  $D$  is the depth of the crack from the top surface of the beam ;  $L$  the length of the crack,  $2S$  the distance between the beam index positions of the two transducers,  $d$  the distance of the wedge's side,  $ToF_0$  is the tof ultrasonic in the wedge,  $ToF_2$  is the second tof ultrasonic detected in the beam,  $ToF_3$  is the third tof ultrasonic detected in the beam, and  $C_0 = 6300$  m/s and  $C = 4500$  m/s are the sound speed, for practical purposes we can consider an aluminum wedge and an steel beam respectively.

Considering a transmitted signal of 2.25 MHz with amplitude of 1 volt, a sample rate of 250 MHz, distance  $d$  of 0.04 m, distance  $H$  of 0.04m, distance  $S$  of 0.02m, distance  $D$  of 0.02m, distance  $L$  of 0.0055m and a Gaussian noise of 5% amplitude of transmitted signal, Fig. 2 shows the received signal.

Based on Fig. 2, the two waves diffracted by the crack tips are expected to appear in between the lateral wave and the back wall echo. In order to get the ToF of the ultrasonic signal diffracted by the crack tip, the wedge delay time  $ToF_0$  must be calculated. It can be determined if the transmitter and the receiver are placed face to face as show in Fig. 3. With the parameters of frequency and amplitude above, the  $ToF_0$  value is 6.3760  $\mu$ s.

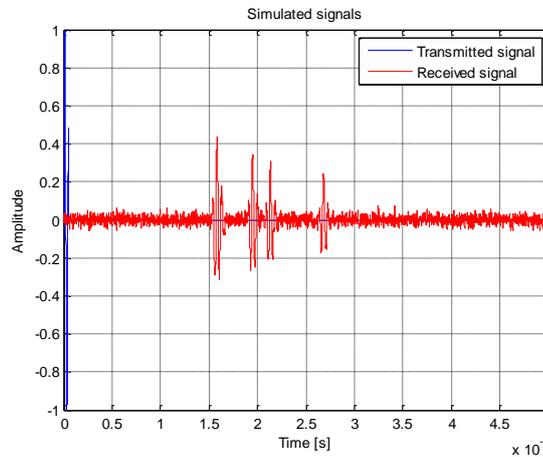


Figure 2. Expected signals from a ToF setup.

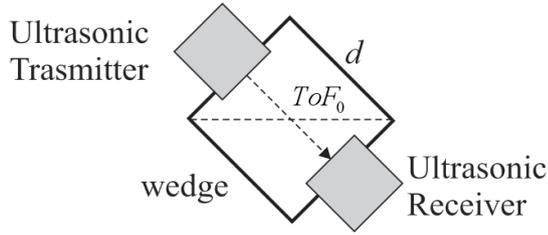


Figure 3. Signal collected setup from sensor to sensor.

### III. Measurement Model of the Time of Flight

#### A. Stationary Wavelet Transform

The SWT is also known as non-decimated wavelet transform, wavelet transform time-invariant, maximum overlap wavelet transform. It has a structure similar to DWT but is not performed by the decimation process. In the DWT, the decimation stage which is after the filter makes DWT time-varying, while SWT modified filters interpolating zeros depending on the level of decomposition in the low-pass filters and high pass. The implementation of SWT structure is shown in Fig. 4 where the signal  $x(n)$  is decomposed by low-pass filter  $g_j(n)$  and high-pass filter  $h_j(n)$ , only the filters of each level are as shown in Figure 5 and have the reverse process of decimation, called "up-sampling" which inserts zeros between every two samples [3]. SWT is important in many applications such as change detection and denoising which is part of our objective.

#### B. Criteria for Selecting the Mother Wavelet Using Shannon's Entropy

Shannon's entropy measures the energy dispersion or randomness within a process. The energy concentration implies entropy lower values. This criterion may be used to choose the best mother wavelet among a group of orthogonal mother wavelet which can be used to transform the signals. For the SWT of a signal  $x(t)$ , an orthogonal mother wavelet is selected among several possibilities previously chosen for compatibility with the features to be extracted from the signal, for example, Biorthogonals, Coiflets, Daubechies, Symlets, discrete Meyer and others. Whereas  $c_{d_0,i}$  are coefficients of the SWT of  $x(t)$ , for a mother wavelet chosen arbitrarily the Shannon entropy of detail level  $d_0$  is given by [4]:

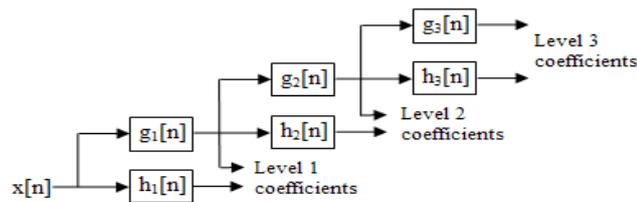


Figure 4. A 3 level SWT filter bank.

$$S(d_0) = \sum_{i=1}^n \left( \frac{c_{d_0,i}^2}{A} \cdot \ln \left( \frac{c_{d_0,i}^2}{A} \right) \right) \quad (5)$$

$$A = \sum_{d_0=1}^{D_0} \sum_{i=1}^n c_{d_0,i}^2 \quad (6)$$

where:  $D_0$  is the maximum level of detail used in the transform and  $n$  the number of details.

Fig. 5 shows how the method can be used to identify the best mother wavelet and the level of detail appropriate to separate the information contained in the echo signal mixed with random noise of Gaussian kind. The mother wavelets used are Daubechies 40 (db40), Symlet 20 (sym20), Biothogonal 6.8 (bior6.8)

and Coiflets 5 (coif5) with a maximum level of detail equal to 3. The best mother wavelet to be used is the one that presents the lowest Shannon Entropy.



Figure 5. Up-sampling SWT filter.

For this signal, the lowest value of the entropy was obtained for mother wavelet discrete Daubechies (db40) and the level of detail 3, as showed in Fig. 6.

### C. Hilbert Transform

The Hilbert transform of a function  $f(x)$  is defined by:

$$F(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(x)}{t-x} dx \quad (7)$$

The Hilbert transform can be considered to be a filter which simply shifts phases of all frequency components of its input by  $-\pi/2$  radians. An analytic signal  $Y(t)$  can be constructed from a real-value input signal  $y(t)$ :

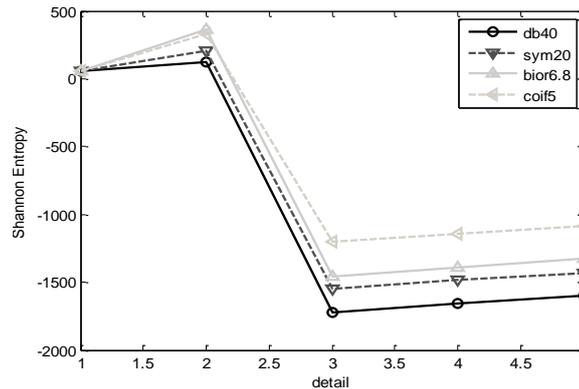


Figure 6. Shannon entropy of SWT coefficients for mother wavelet db40, Symlet 20, Biorthogonal 6.8 and Coiflents 5.

$$Y(t) = y(t) + jh(t) \quad (8)$$

where:  $Y(t)$  is the analytic signal constructed from  $y(t)$  and its Hilbert Transform,  $y(t)$  is the input signal,  $h(t)$  is the Hilbert Transform of the input signal. The real and imaginary parts can be expressed in polar coordinates as:

$$Y(t) = A(t) \exp(j\psi(t)) \quad (9)$$

where:  $A(t)$  is the “envelope” or amplitude of the analytic signal,  $\Psi$  is the phase of the analytic signal.

### IV. Results of simulations

SNR (signal-to-noise ratio) values show that level noise of the de-noising signal is lower than original signal. Figures 7 and 8 illustrated the waveforms simulates for the ultrasonic echoes.

$$SNR = 10 \log \left( \frac{P_{signal}}{P_{noise}} \right) dB \quad (10)$$

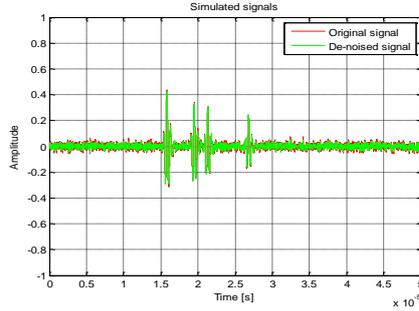


Figure 7. Original signal and De-noised signal.

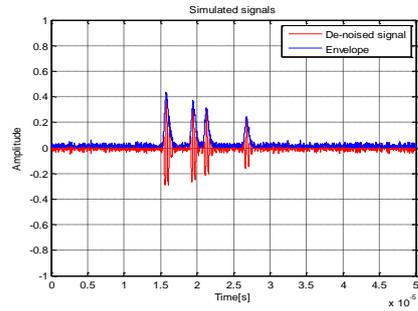


Figure 8. De-noised signal and its Envelope

$$SNR_{os} = 1.4218, \quad SNR_{ds} = 1.7180 \quad (11)$$

### V. Uncertainties Assessment of the ToF

Since there is always margin for doubt in any measurement, in this section, the estimate the uncertainty of ToF measurement is carried out. The uncertainty is the parameter associated with the result of a measurement that characterizes the dispersion of the values; the uncertainty of a measurement is usually associated with quality [5]. The normality tests applied to sets of data to determine its similarity to a normal distribution. The null hypothesis is, in these cases, if the data set is similar to a normal distribution, so a sufficiently small  $P$ -value indicates non-normal data. The empirical distribution function  $F_n$  for  $n$  independent and identically distributed random variables observations  $X_i$  is defined as:

$$F_n(x) = \frac{1}{n} \sum_{i=1}^n I_{X_i \leq x} \quad (12)$$

where:  $I_{X_i \leq x}$  is the indicator function, equal to 1 if  $X_i \leq x$  and equal to 0 otherwise. The Kolmogorov-Smirnov statistic for a given cumulative distribution function  $F(x)$  is:

$$D_n = \sup_x |F_n(x) - F(x)| \quad (13)$$

where:  $D_n$  is the Kolmogorov-Smirnov statistic. The  $P$ -value depends on the level of significance  $\alpha = 0.05$  and can be found from the condition.

$$\alpha = 0.05 = P(D_n \geq P\text{-value} | H_0) \quad (14)$$

where:  $H_0$  is the null hypothesis. For 100 samples, the normality test values using (12), (13) and (14) the  $P$ -value of the test Kolmogorov-Smirnov normality, is less than 0.01. Therefore, the ToFs do not follow a Normal distribution. But if the ToFs had followed a normal distribution then an analysis of performed of type A had been made, the uncertainties associated of the distances  $D$  and  $L$  can be calculated as follow [5]:

$$(uD)^2 = \left( \frac{\partial D}{\partial ToF_2} uToF_2 \right)^2 \quad (15)$$

$$(uL)^2 = \left( \frac{\partial L}{\partial ToF_2} uToF_2 \right)^2 + \left( \frac{\partial L}{\partial ToF_3} uToF_3 \right)^2 + 2 \left( \frac{\partial L}{\partial ToF_2} \frac{\partial L}{\partial ToF_3} u(ToF_2, ToF_3) \right) \quad (16)$$

$uToF_2$  and  $uToF_3$  are the uncertainties associated of the  $ToF_2$  and  $ToF_3$  respectability calculated from its mean and standard deviation as follows:

$$uToF_2 = \frac{\sigma(ToF_2)}{\sqrt{N}}, \quad uToF_3 = \frac{\sigma(ToF_3)}{\sqrt{N}}, \quad (17)$$

$$u(ToF_2, ToF_3) = corr(ToF_2, ToF_3) \times uToF_2 \times uToF_3 \quad (18)$$

where:  $\sigma(ToF_2)$  and  $\sigma(ToF_3)$  are the standard deviation,  $N$  the number of samples, and  $corr(ToF_2, ToF_3)$  the correlation between  $ToF_2$  and  $ToF_3$ .

Table 1 shows some results about distances estimated and its associated uncertainty. Note that the values obtained are very close to the actual value so the proposed method can be considered as an acceptable technique for locating and measuring the size of cracks in beams.

In order to calculate the uncertainties associated with the distances  $D$  and  $L$ , an analysis of performed of type B is used. It by using fuzzy numbers as showed in Fig. 9.

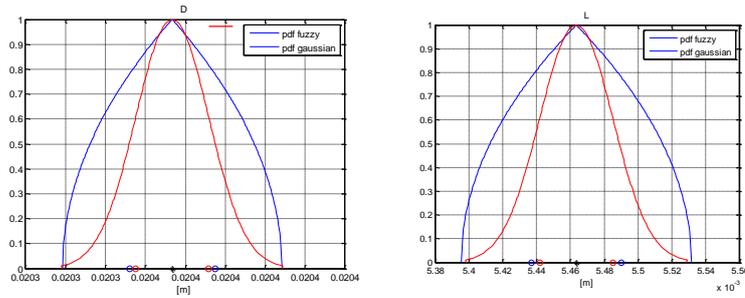


Figure 9. Fuzzy representation for the distances  $D$  and  $L$ .

**TABLE I: ANALYZED RESULT USING THE PROPOSED METHOD.**

Distance $d$ (0.04m), distance $H$ (0.04m), distance $S$ (0.02m)								
$n$	$ToF_2(\mu s)$	$ToF_3(\mu s)$	$D(m)$	$L(m)$	$De(m)$	$Le(m)$	$uDe(\mu m)$	$uLe(\mu m)$
1	15.272	20.620	0.000	0.005	0.0001	0.0049	9.5455	31.616
2	16.324	18.952	0.010	0.010	0.0101	0.0099	12.3666	24.415
3	16.324	20.620	0.010	0.015	0.0101	0.0149	9.9675	34.617
4	16.324	22.496	0.010	0.020	0.0101	0.0199	11.1400	47.516
5	16.324	24.300	0.010	0.025	0.0101	0.0249	10.0444	25.710
6	16.324	26.312	0.010	0.030	0.0101	0.0299	8.5885	31.333
7	18.952	20.620	0.020	0.005	0.0201	0.0049	13.6466	43.010
8	18.952	22.496	0.020	0.010	0.0201	0.0099	9.3433	29.557
9	18.952	24.300	0.020	0.015	0.0201	0.0149	10.2256	31.113
10	18.952	26.312	0.020	0.002	0.0201	0.0199	9.1499	26.825
11	22.496	24.300	0.030	0.050	0.0301	0.0049	11.5755	37.599
12	22.496	26.312	0.030	0.010	0.0301	0.0099	14.6678	23.377

## VI. Conclusions

In this work, analyses of the Stationary Wavelet Transform SWT, the Hilbert Transform and the method to identify the best mother wavelet using Shannon Entropy were carried out in order to define a new procedure for locating and measuring the size of cracks in beams. From uncertainty propagation analysis, it was calculated that the flight-of-times ToFs do not follow a normal distribution so the technique type B with Fuzzy numbers was used in order to calculate their uncertainties associated. The use of the SWT as a filtration system for ultrasonic signals definitely has advantages over the DWT due DWT is sensitive to shift. Besides it does not contain information about the phase, however SWT has a high computational cost. According the results, the use of the Hilbert Transform to determinate the envelope signal is certainly a good tool to calculate the ToFs with a good accuracy.

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