19th IMEKO TC 4 Symposium and 17th IWADC Workshop
Advances in Instrumentation and Sensors Interoperability
July 18-19, 2013, Barcelona, Spain

Thermal AC voltage standards with calculable AC-DC transfer difference in frequency range from 10 kHz to 1 MHz

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Abstract- The paper presents design and basic metrological properties of three thermal AC voltage standards with nominal input voltages 1.5 V, 3 V and 6 V. Each standard is composed of a range resistor connected in series with a single junction thermal converter (SJTC). The AC-DC transfer differences of these standards are calculated in frequency range from 10 kHz to 1 MHz.

I. Introduction

Despite of advances in quantum AC voltage standards the most accurate standards of AC voltage are still based on thermal voltage converters (TVC) and AC-DC transfer. By means of AC-DC transfer the AC voltage is determined in terms of known DC voltage. The main metrological parameter of the TVC is AC-DC transfer difference, defined as:

$$\delta_{\mathrm{u}} = \left.\frac{U_{\mathrm{AC}} - U_{\mathrm{DC}}}{U_{\mathrm{DC}}}\right|_{P_{\mathrm{AC}} = P_{\mathrm{DC}}} \qquad (1)$$

where U_{AC}, U_{DC} is unknown AC or known DC voltage applied to the TVC input, respectively; P_{AC} and P_{DC} are values of Joule's power dissipated in the TVC heater for AC or DC heater voltage, respectively.

Because Republic of Poland does not have its own primary AC voltage standard, we have been developing a set of standards which in the future can serve as national primary AC voltage standard. The set is composed of three standards with nominal input voltages U_N of 1.5 V, 3 V and 6 V, respectively. The set covers AC voltage range from 0.7 V to 6 V. The voltage range will be extended to 0.5 ... 1000 V by an additional set of working standards based on planar multijunction thermal converters (PMJTC) with appropriate coaxial or planar range resistors. Some of the working standards will be calibrated directly against the 1.5 V, 3 V and 6 V primary standards. Other working standards will be calibrated by the use of step-up or step-down method.

II. Design of the standards

Each calculable standard is composed of the range resistor connected in series with the SJTC. The SJTC has low reversal error (<0.005%), heater resistance $R_H = 90\ \Omega$, nominal input current $I_N = 5$ mA and nominal input voltage $U_N = 0.45$ V. Range resistors are made of a nonmagnetic resistive wire with a diameter of 14 µm or 25 µm. Basic parameter of range resistors are listed in table 1.

Table 1. Parameters of range resistors used in the calculable AC voltage standards

U_N	A resistance of range resistor	Length of heating wire l_R	Diameter of the heating wire d_R
1,5V	210 Ω	78 mm	25 µm
3V	510 Ω	190 mm	25 µm
6V	1110 Ω	130 mm	14 µm

The range resistor with the SJTC is mounted in a common coaxial housing made of DHP copper tube. Such design eliminates one coaxial connector and increases accuracy of the standard. The construction was inspired by the design presented in [1]. Its cross-section is shown on figure 1.

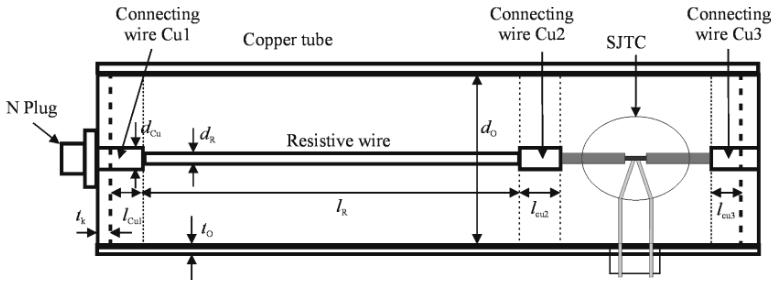

Figure 1. Cross-sectional view of the calculable AC voltage standard

Dimensions of the coaxial housings are listed in table 2. The housings of the three standards are the same, except for the tube length and diameter of the resistive wire. These dimensions were optimized using a mathematical model of the standard, which is described in [2]. The elaboration of the model was simplified due to SJTC geometry and coaxial design of the construction. The tube diameter is the same for all housings because this parameter has small influence on the AC-DC differences of the standards. Using the same diameter of the housing for all AC standards simplified construction and reduced cost of standards.

Table 2. Nominal values of construction and material parameters

Parameter	Nominal value	Error bounds
Length of Cu_1	8 mm	±1 mm
Length of Cu_2	8 mm	±1 mm
Length of Cu_3	5 mm	±1 mm
Diameter of Cu_1, Cu_2 and Cu_3	2.0 mm	±0.1 mm
Internal diameter of housing	60.0 mm	±0.2 mm
DHP tube wall thickness	2.0 mm	±0.3 mm
Electric conductivity of the housing	$4.9\cdot10^{7}$ S/m	$\pm0.4\cdot10^{7}$ S/m
Resistivity of resistive wire	$1.32\cdot 10^{-6}\ \Omega\cdot$m	Depends on wire diameter

III. AC-DC transfer difference of the standards

The AC-DC transfer difference of a thermal voltage converter is determined by many effects. The determination of δ_U is usually performed in three frequency ranges:

1. At low frequencies (10 ... 100 Hz), due to insufficient averaging of SJTC heater temperature, the AC-DC difference is determined mainly by nonlinear heat-transport effects from TVC heater to ambient. It is very difficult to calculate δ_U with appropriate accuracy in this frequency range, but it is possible to measure it.
2. At frequencies between approximately 100 Hz and 10 kHz the AC-DC transfer difference has usually the lowest value and is determined by thermoelectric phenomena in the heater and its connections. It is possible to calculate the effect of these phenomena on the AC-DC transfer difference with moderate accuracy, but nowadays it is easier to measure it as well.
3. At frequencies above approximately 10 kHz the most accurate method of determination the AC-DC transfer difference is its calculation. The calculation is based on a mathematical model which includes residual parameters of all elements of the standard (range resistor, housing, N connector, SJTC and others) [2]. The model requires input parameters, like material constants and geometrical dimensions which must be determined empirically.

To calculate the AC-DC transfer difference in 10 kHz – 1 MHz frequency range an appropriate mathematical model of the AC standard was developed in [1] with several enhancements. The calculable AC voltage standard shown in Figure 1 was in divided into 11 section. Each section was modeled as independent two port network. The method used allows for easy expansion of the model, for example by adding new sections to it. The chain (ABCD) matrix of the of the whole AC voltage standard is equal to:

$$\mathbf{A}_\mathrm{t} = \begin{bmatrix} A_\mathrm{t} & B_\mathrm{t} \\ C_\mathrm{t} & D_\mathrm{t} \end{bmatrix} = \prod_{i=1}^{11} \mathbf{A}_i , \qquad (2)$$

where $\mathbf{A}_i$ is the chain matrix modeling the *i*-th section.

AC-DC transfer difference of the AC standard is equal to:

$$\delta_\mathrm{u} = \left| \frac{A_\mathrm{bf} D_\mathrm{bf} - B_\mathrm{bf} C_\mathrm{bf}}{\frac{D_\mathrm{t}}{B_\mathrm{t}} A_\mathrm{bf} - C_\mathrm{bf}} \right| \sqrt{\frac{2R_\mathrm{H}}{k_\mathrm{I} R_\mathrm{HAC}}} \frac{1}{R_\mathrm{DC}} - 1, \qquad (3)$$

where: B_t, D_t are elements of the two-port network matrix (2) representing the all sections, A_{bf}, ..., D_{bf} are elements of the two-port network matrix (2) modeling the section from N plug to SJTC heater, coefficient k_I describes which part of SJTC current generates Joule heat in SJTC heater; R_H and R_{HAC} represent SJTC heater resistance for DC current and AC current, respectively.

In comparison to the model presented in [1] the developed model was extended to model section using Bessel functions and theirs approximations [3]. Because SJTCs used in our standards have leads made of ferromagnetic wires, it was necessary model their effect on AC-DC transfer difference as well. This enhanced model is presented in [2]. It was extended also to calculate influence of a Tee connector used to connect compared AC standard with other standard and AC/DC voltage source. As usual, the reference plane was assumed to be located in the middle of the Tee connector. Because of backward compatibility and lack of appropriate commercially available Tee connectors, we decided to build our own Tee connector. Its characteristic impedance is equal to 50 Ω. The Tee connector was modeled as a serial connection of sections with different geometrical dimensions. The model of the Tee connector was combined with the developed model of the AC voltage standard, what increased slightly the AC-DC transfer difference. The highest increase is for AC standard with U_N = 1.5 V, but is still lower than 2 μV/V at 1 MHz.

The AC-DC differences of the three standards, calculated from the developed mathematical model are presented in Table 3.

Table 3. Calculated AC- DC transfer differences δ_U of the standards with their standard uncertainties $u(\delta_U)$

f	1.5 V standard		3 V standard		6 V standard	
	δ_U	$u(\delta_U)$	δ_U	$u(\delta_U)$	δ_U	$u(\delta_U)$
kHz	μV/V	μV/V	μV/V	μV/V	μV/V	μV/V
10	0.45	0.03	0.24	0.03	0.11	0.02
20	0.90	0.07	0.48	0.05	0.23	0.03
50	1.90	0.15	1.00	0.10	0.48	0.05
70	2.43	0.19	1.28	0.12	0.61	0.06
100	3.15	0.25	1.65	0.16	0.78	0.08
200	5.37	0.44	2.80	0.30	1.26	0.14
500	11.8	1.1	6.09	0.82	2.30	0.31
700	15.8	1.5	8.17	1.25	2.68	0.42
1000	22.0	2.3	11.32	2.05	2.85	0.59

Standard uncertainty of δ_U was calculated according to [4] as

$$u_\mathrm{c}(\delta_\mathrm{U}) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial \delta_\mathrm{U}}{\partial x_i} \right)^2 u^2(x_i)}, \qquad (4)$$

where $u(x_i)$ are uncertainties of material constants and geometrical dimensions.

It was assumed that uncertainty of all the input parameters have uniform distribution. The standard uncertainty of the *i*-th material constant is equal to:

$$u(x_i) = \Delta x_i / \sqrt{3} , \qquad (5)$$

where Δx_i is the maximum error of parameter x_i assuming values of these errors listed in table 2. Values of these errors were taken from manufacturers data [5,6,7].
Due to complexity of equation for δ_U it was calculated numerically:

$$\frac{\partial \delta_{\mathrm{U}}}{\partial x_i} = \frac{\delta_{\mathrm{U}}[x_1, \ldots, x_i + u(x_i), \ldots, x_N] - \delta_{\mathrm{U}}[x_1, \ldots, x - u(x_i), \ldots, x_N]}{2u(x_i)} . \qquad (6)$$

Putting (5) and (6) to (4) the standard uncertainty of δ_{U} may be calculated:

$$u(\delta_{\mathrm{U}}) = \sqrt{\sum_{i=1}^{N} \left(\frac{1}{2} \left\{ \begin{array}{l} \delta_{\mathrm{U}}[x_1, \ldots, x_i + u(x_i), \ldots, x_N] - \\ \delta_{\mathrm{U}}[x_1, \ldots, x - u(x_i), \ldots, x_N] \end{array} \right\} \right)^2} . \qquad (7)$$

Tolerance of resistivity of the resistive wire depends of its diameter. For the wire of diameter equal to 14 µm it is ±10% of nominal value. For thicker wires (diameter 25 µm) used in 1.5 V and 3 V standards its tolerance is lower (±8% of nominal value) [5]. Deformation of the heating wire (cause by for example transportation of the standard) can also reduce resistivity of the wire up to 10% nominal value. The influence of this parameter on δ_{U} was calculated by increasing heating wire resistance tolerance for 10% of nominal value [6].

IV. Future comparison of the standards

To validate the results of calculation of AC-DC transfer differences the 1.5 V standard will be calibrated against 3 V standard at 1.5V. The 3 V standard will be calibrated against 6 V standard at 3 V. Results of these two calibrations are differences of AC-DC transfer differences defined as:

$$\Delta\delta_{\mathrm{u}}(1.5\mathrm{V}\text{-}3\mathrm{V}) = \delta_{\mathrm{u}}(1.5\mathrm{V}) - \delta_{\mathrm{u}}(3\mathrm{V}) \quad \text{and} \quad \Delta\delta_{\mathrm{u}}(3\mathrm{V}\text{-}6\mathrm{V}) = \delta_{\mathrm{u}}(3\mathrm{V}) - \delta_{\mathrm{u}}(6\mathrm{V})$$

The relation between the both calculated differences and frequency is shown in Figure 2.

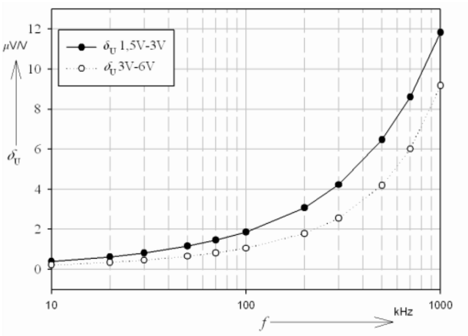

Figure 2. Calculated differences of AC-DC transfer differences of the compared standards

V. Conclusion and future work

The design and basic metrological properties of thermal AC voltage standards with calculable AC-DC transfer difference in frequency range from 10 kHz to 1 MHz were presented. We are putting a lot of efforts to build prototypes of the standards and experimentally verify the calculated AC-DC transfer differences. We hope to present these results at the conference.

The work was supported by the Polish National Research and Development Center (Development Project N R01 0030 10)

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