19th Symposium IMEKO TC 4 Symposium and 17th IWADC Workshop
Advances in Instrumentation and Sensors Interoperability
July 18-19, 2013, Barcelona, Spain

The analysis of exponential signals by maximum likelihood estimation

Linus MICHAELI, Marek GODLA, Ján ŠALIGA, Jozef LIPTAK, *István Kollár

Dept. of Electronics and Multimedia Communications, FEI TU of Košice, Slovak Republic,

Email:{linus.michaeli,marek.godla,jan.saliga,jozef.liptak}@tuke.sk

**Budapest University of Technology and Economics, Budapest, Hungary, Email: kollar@mit.bme.hu*

Abstract

A new method to determine the exponential components of a signal using the maximum likelihood method will be presented in this paper. The exponential stimulus signal is assumed to be distorted by superimposed exponential components with longer time constant and smaller peak value. The exponential signal to be identified is acquired by a general purpose DAQ board. The advantage of the proposed method is lower sensitivity to the additive noise and restriction of the signal processed to the full scale range of the ADC. The sensitivity of the proposed method on the measurement and processing conditions is studied theoretically and by simulation.

I. Introduction

The decomposition of the distorted signal represented by the superposition of the known components is a measurement task important for assessment of the periodical signal of the arbitrary shape. Analytical description of signal components is known on the base of the physical study of their generation. Identification based on FFT spectrum analysis is not suitable in the case when component are nonharmonic and not orthogonal. Spectrum in this case is continuous and parameters of components can not be identified.

The exponential stimulus signal is one possible signal suitable for dynamic ADC testing [1]. The main advantage of its implementation is the simplicity of generating RC which significantly suppress the signal distortion. The remaining distortion sources are represented by the dielectric absorption of the capacitor and limited frequency bandwidth of the preprocessing blocks. Distortion is determined by the superimposed exponential components with longer time constants to the basic exponential function [3]. Another superimposed component is the thermal noise of the input buffer. The least squared (LS) method from best fitting of the real exponential stimulus signal is limited because of the quantization noise without ideal symmetrical distribution [2]. An alternative way of analysis is the identification of the superimposed components using Prony's estimation method when sampling is equidistant [4].

Here maximum likelihood (ML) estimation is proposed and studied further under consideration of analog to digital conversion with low nonlinearity [5]. However the ML method is characterized by the huge computational complexity, the low sensitivity to clipping of the signal and independence on the shape of the quantization noise distribution is the main advantage, in contrary to the LS method [6], [7]. The computation time is caused by the complexity of the multidimensional optimization. The selection of the initial values in the optimization algorithm can speed up whole optimization procedure. Optimization ML method studied in the paper utilize the initial values estimated by Prony's method. The proposed method the ML method has been assessed from the data record acquired by the simulated ADC with adjustable sampling frequency, resolution and superimposed thermal noise for the multiexponential input signal with known parameters of the basic and the distorting components. Difference between known generated components and their assessment allows to determine the parameter estimation precision.

II. Mathematical model

The stimulating exponential signal is described by the equation (1)

$$x_{in}(t) = A_1 e^{-B_1 t} + \sum_{i=2}^{n} A_i e^{-B_i t} + C \tag{1}$$

While parameters A_1 and B_1 represent the exponential signal obtained from the ideal generator, the parameters A_i and B_i represent the other exponentials components caused by dielectric absorption for a limited frequency bandwidth. The parameter C describes offset of the whole exponential signal. The components meet the following conditions: $A_1 >> A_i$, $0 < B_1 < B_i$. The distorted multiexponential is measured together with the additional noise generated by the analog components. The parameter n determines number of distorted signal components. The superimposed input noise $\eta(t)$ is assumed to have Gaussian distribution with zero mean and variance σ^2 .

The sampled input signal $x_s(j)$ with period length Ts is represented by (2) before its quantization in the DAQ board.

$$x_S(j) = \sum_{i=1}^{n} A_i . e^{-B_i \cdot jT_S} + C + \eta(jT_S) \tag{2}$$

We consider analog to digital conversion with an ideal ADC. The quantisation levels T(k) are equidistant T(k) = Q(k+0.5), where Q is the ideal code bin width. The recorded digital sample from the ADC output is $k(j)$ corresponding to the j-th sample of input exponential signal $x_s(j)$.

The best estimation of the excitation signal is represented by the situation when the code samples $k(j)$ match best the values of the sampled input signal $x(j)=x_s(j)/Q$, normalized by the code bin width Q. The probability of any digital sample $k(j)$ matching the normalized input value $x(j)$ is expressed by one of following expressions ($F(\,)$ denotes the normal cumulative distribution function)

$$\begin{gathered} P(k(j)=0,x(j)) = F([T(0)-x(j)],0,\sigma) - F([-\infty - x(j)],0,\sigma) \\ P(k(j),x(j)) = F([T[k]-x(j)],0,\sigma) - F([T(k-1)-x(j)],0,\sigma) \\ P(k(j)=\left(2^N-1\right),x(j)) = F([\infty - x(j)],0,\sigma) - F([T(2^N-2)-x(j)],0,\sigma) \end{gathered} \tag{3}$$

The optimisation task is to find constants A_i,B_i,C from expression (2) when the product of probabilities P((k(j),x(j)), for all codes k(j), achieves maximum. The maximum of the joint probability P_{final} represents the maximal likelihood

$$P_{final} = \prod_{j=0}^{I} P((k(j),x(j)) \tag{4}$$

In order to avoid the rounding errors in the optimisation algorithm the most suitable optimization strategy is to find minimum of cost function $CF(p)$ represented by the negative logarithm of (4) where p represent set of parameters A_i,B_i,C.

$$CF(p) = -\log P_{final} = -\sum_{j=0}^{l} \log P(k(j), x(j)) \quad (5)$$

III. Experimental results

Tests were performed with the data acquired using simulated ideal ADC with adjustable $INL(k)$ function. The experimental evaluation of the ML method on the acquisition conditions was performed by comparison of the known parameters of generated multiexponential signal and parameters obtained by ML or LMS optimization. The robustness of the proposed ML method on the number of samples and superimposed noise was studied by the simulation tool developed in the LabVIEW environment (figure 1). Signal from the generator of the multiexponential signal $x_s(jT_s)$ is quantized in the ADC model which allows to implement known nonlinearity function INL(k). Three exponential components were chosen in the input shape according to the studies about dielectric absorption [3].

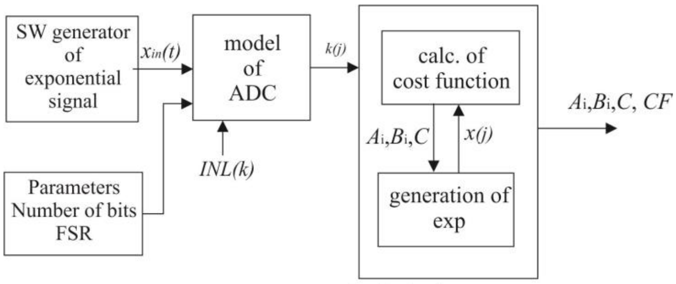


Figure 1. Block scheme of the simulation environment

The selection of the optimization strategy was another topic for the performed study. The basic differential evolution strategy was assessed here. The convergence process was accelerated by adaptive changing of the dispersion of the Gaussian distribution in (3).

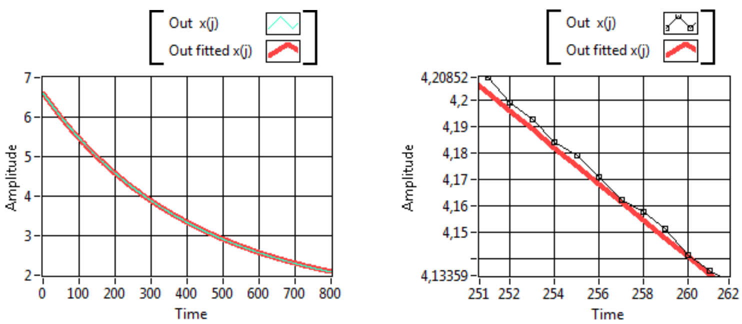


Figure 2. Estimation results based on the ML method for multiexponential signal with and without superimposed noise

The shape of the simulated multiexponential signal with three exponential components without and with noise is shown in Fig. 2. The effective value of the superimposed noise is equal to 0,8 LSB. The ideal ADC with resolution of 12 bit ADC was simulated with the 800 samples in the digital record.

Simulations showed high sensitivity of the optimisation process on the initial conditions. Increasing number of the exponential functions (2) requires good choice of the initial conditions. The suitable way how to estimate the initial values constants A_i,B_i,C is standard LS method from the data record. The drawback is that the estimation of higher components is strongly influenced by the noise level. Another possibility is utilization of the modified Pony's method [4] for the first estimate of exponential components from reduced amount of recorded samples. The final optimization is performed by the minimization of the cost function $CF(p)$ (5). Simulated studies showed high sensitivity of the method on the superimposed noise.

The results of the parameter identification for noisy and noiseless signal are shown in Table 1.

Constants	Real input	Estimated parameters without noise	Estimated parameters with noise σ=0,8LSB
A1	5	5,0013	5,0074
A2	1	0,9729	1,0646
A3	0,5	0,5238	0,3854
B1	5	4,99817	4,9972
B2	0,50	0,5116	0,4046
B3	0,05	0,0658	0,2035
C	0,1	0,0959	0,1405
Cost funct.	-	699,72	1119,8

Table 1. Estimated parameters for known input signal for both cases without noise (third column) and with superimposed noise with effective value of 0,8 LSB.

The sensitivity of the ML method on the additive noise with Gaussian distribution is shown on figure.3. The estimation error decreases with the increasing signal value in comparison with the additive noise.

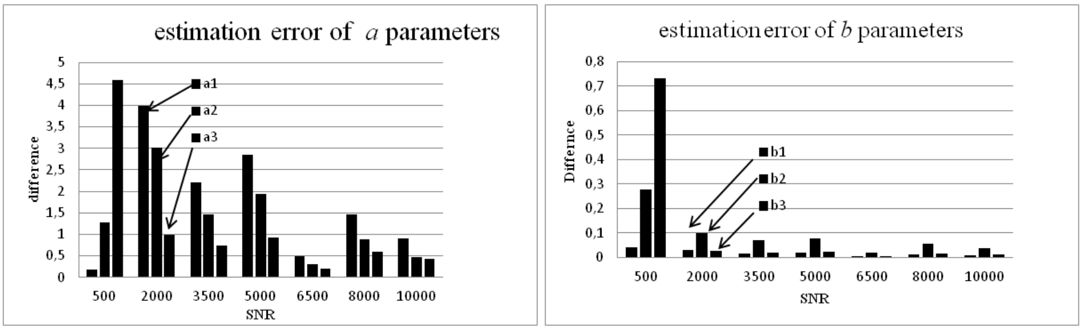


Figure 3. Estimation error for data acquired by the 16 bit ADC

The robustness of ML method on the ADC resolution is shown on the figure 4. The achieved results confirm the theoretical assumption that the estimation by ML is better for lower resolution when the quantization noise is less symmetrical. The increasing resolution of the ADC increases the impact of the additive noise as source of estimation error.

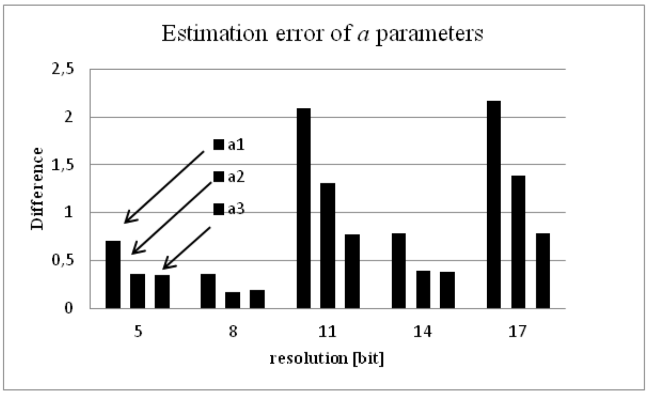
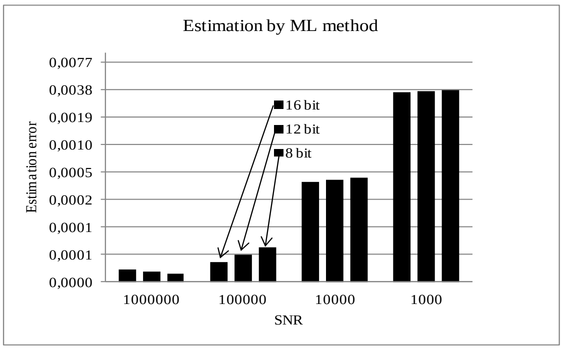
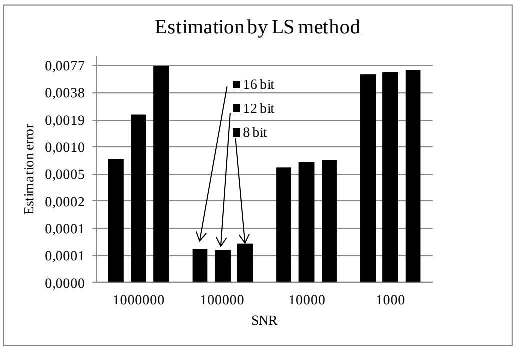


Figure 3. Estimation error for data acquired by ADC with different number of bits for SNR=10000

The ML method is more efficient in comparison with LS method when only the quatization noise occurs. Figure 4.a.) show estimation error as difference between assessed function (1) by ML method and stimulation signal at the ADC input with the resolution of 8,12 and 16 bit under various SNR levels. The neighboring figure 4.b.) shows the estimation error when the input signal is estimated by polynomial of 10^{th} order acquired from ADC with the resolution of 8,12 and 16 bit. Quantization noise for ADC with lover resolution is surrendering from the symmetrical distribution with zero mean value. It causes that the estimation by ML is better for ADCs with lower resolution. The superimposed noise confirms the theoretical assumption that the symmetrical additive noise reduces the advantage of the ML method. The number of processed samples in both cases were M=100.





a.) b.)

Figure 4 Estimation error by ML method a.) and by LS using polynomial function of 10^{th} order b.).

Authors assessed proposed method by the experiment where the distorted multiexponential function was generated by the precise DAC with the resolution of 16 bit. The data were acquired by the DAQ board with the resolution of 12 bit. The experiments show sufficient matching of the generated exponentials with identified by the ML method.

The last topic which was studied by the simulation was the estimation of the number n of the exponential components in the considered mathematical form (1). The results of the simulation showed that the values of the parameters A_1,B_1 calculated by the optimization are similar until n in the considered shape (1) is lower than number of exponential in generated signal. When number of supposed exponential is higher than number of components in the generated signal the parameters A_1,B_1 start to change their values in the comparison with

ISBN-10: 84-616-5438-2 | ISBN-13: 978-84-616-5438-3

previous estimates. The table 2 shows results of estimation of multiexponential shape with n=3 components when the parameters of the basic exponential are A_1= 9,8 and B_1= 0,4

No of simulated exponentials	1	2	3	4
A_1	10	9,76	9,79	5,8. 10^{-8}
B_1	0,47	0,38	0,48	1,77

Tab.2. Estimated values of basic exponential shape for different number n of the considered exponentials.

IV. Conclusions

The simulation shows that ML method is more resistant on the influence of the superimposed noise on the identification of the multiexponential signal parameters. Further study will be focused on the influence of number of samples, and the efficiency of the optimization procedure. The proposed method is efficient for the decomposition of any signals with nonorthogonal components described analytically where parameters of any component are unknown.

V. Acknowledgments

The work is a part of the project supported by the Science Grant Agency of the Slovak Republic (No. 1/0555/11).

VI. References

[1] Saliga,J, Michaeli,L., Holcer,R. :”Noise sensitivity of the exponential histogram ADC test”, Measurement. Vol. 39, no. 3 (2006), p. 238-244. - ISSN 0263-2241

[2] Saliga,J, Michaeli,L., Sakmar,M, Busa,J.: “Processing of bidirectional exponential stimulus in ADC testing” Measurement. Vol. 43, no. 8 (2010), p. 1061-1068. - ISSN 0263-2241

[3] Kundert, K.:“Modeling Dielectric Absorption in Capacitors”, www.designers-guide.org ,2002.

[4] Carni,D.L. et al.: “Measurement of the exponential signal distortion” IEEE Procedings of I2MTC 2012 : May 13-16, Graz, Austria. - Graz : P. 1773-4. - ISBN 978-1-4577-

[5] Balogh,L., Kollár,I., Sárhegyi, A:”Maximum Likelihood Estimation of ADC Parameters.” IEEE Procedings of IMTC 2010. Austin, United States of America, 03/05/2010-06/05/2010. pp. 24-29.

[6] Gendai,Y.: The Maximum-Liklehood Noise Magnitude Estimation in ADC Linearity Measurement, IEEE Trans.on IM, Vol.59, No.7.July 2010,pp.1746-1754

[7] Kollar,I.,Blair,J.J.: Improved Determination of the Best Fitting Sine Wave in ADC Testing, IEEE Trans.on IM, Vol.54, No.5. 2005,pp.1978-1983