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Parameter Extraction in Multi Step Exponential Signal

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Abstract-Multi-exponential function is used to model phenomenon or signal in several applications. The parameter extraction of each exponential is based on the assumption that no constant component is part of the signal. In the paper the superposition of exponential signals and step signals starting in different time instants is taken into consideration, and a procedure based on windowing is pointed out to estimate the parameters of each exponential function in addition to the delay of each step. Numerical tests asses the correctness of the procedure.

I. Introduction

The analysis of multi-exponential function to model phenomenonor system occurs in several applications such as: electronic device testing [1], chemical physic [2], pharmacology [3], applied physics [4].
In the electronic device testing the multi-exponential function models the exponential voltage used as stimulus signal to evaluate the deviation of actual Analog to Digital Converter transfer characteristic from the theoretical one [1]. Due to generation circuit the stimulus signal is distorted by the theoretical one. In order to evaluate the distortion, in [5],[6] is pointed out (i)the logic scheme for the accurate acquisition of the real exponential signal, and (ii)the numerical procedure based on Prony-like method to evaluate the parameters of the multi exponential function modeling the acquired signal. The parameters extraction is based on the assumption that no constant component is part of the signal.
In this paper the multi step exponential signal, i.e. the superposition of exponential signals and step signals starting in different time instants, is taken into consideration. The procedure is proposed to evaluate the delay of each step in addition to the other parameters already evaluated in [5], [6] and [7], as the amplitude and the time constant. This procedure usesboth the sliding observation window and the Prony-like method.
The paper is organized as follows. Initially, the theoretical basis of the Prony-like method is abstracted. Successively, the procedure for the parameter estimation of themulti step exponential signalis discussed. Finally, numerical tests are showed in order to validate the procedure.

II. Prony-like method

The multi exponential signal taken into examination is:

$$v(t) = \sum_{i=1}^{n_p} A_i e^{-\tau_i t} \tag{1}$$

Prony-like method is used to evaluate the parameters A_i and τ_i, i=1, 2,…,n_p of (1) that best fit the acquired samples of the real signal.
The *2n*$_p$ unknown constants are obtained as the solution of the system on $J > 2n_p$ acquired samples:

$$\begin{cases} A_1 + ... + A_{n_p} = v(T_0) \\ A_1\mu_{1,1} + ... + A_{n_p}\mu_{n_p,1} = v(T_1) \\ A_1\mu_{1,2} + ... + A_{n_p}\mu_{n_p,2} = v(T_2) \\ \vdots \\ A_1\mu_{1,L-1} + ... + A_{n_p}\mu_{n_p,L-1} = v(T_{J-1}) \end{cases} \tag{2}$$

with:

$$\begin{cases} \mu_i = e^{-\tau_i} \\ \mu_{i,j} = e^{-\tau_i T_j} = \mu_i^{k(j)} \quad i = 1,2,\dots,n_p; \quad j = 0,1,\dots,J-1 \end{cases} \tag{3}$$

where the sampling time T_j, j=0, 1,…, J-1are the elements of the vector $\boldsymbol{T}$, and the samples $v(T_j)$, j=0, 1,…, J-1 are the elements of the vector v. The μ_i values of (2) are evaluated as the roots of the linear regression equation approximating the exponential trend of the sampled signal:

$$\mu_i^{n_p} + \sigma(n_p)\mu_i^{n_p-1} + \sigma(n_p-1)\mu_i^{n_p-2} + \dots + \sigma(1) = 0 \tag{4}$$

To solve the (4), the first step concerns with the evaluation of the vector $\sigma \in \boldsymbol{R}^{n_p}$. These elements are determined by taking into consideration that the estimation of μ_i is a numerically ill-conditioned problem.
In this paper the method proposed in [6] is used. The vector σ is estimated by linear regression equation on the filtered acquired signal. This method permits to operate with non-uniformly sampled signal.
Starting from the polynomial approximation of the Laplace transform of ν, in [7] is demonstrated that the elements of σ can be obtained as:

$$\sigma = (M^T P^T P M)^{-1} M^T P^T F \tag{5}$$

with:

$$P(i,j) = t(i)^{2n_p+j}, \quad i = 1,2,\dots J; \qquad j = 1,2,\dots,m \tag{6}$$

$$M(i,j) = \begin{cases} (-1)^{n_p} \dfrac{\Gamma(n_p+i)\Gamma(i+j-1)}{\Gamma(i)\Gamma(2n_p+i+1)} a_{i+j-1} \\ \quad j = 1,2,\dots,n_p; \quad i = 1,2,\dots,m+1-j, \\ \quad 0, \quad otherwise \end{cases} \qquad b(i) = \begin{cases} (-1)^{n_p+1} \dfrac{\Gamma(n_p+i)^2}{\Gamma(i)\Gamma(2n_p+i+1)} a_{n_p+i} \\ \quad i = 1,2,\dots, m-n_p, \\ \quad 0, \quad otherwise \end{cases} \tag{7}$$

Γ is the gamma function available in Matlab according to [8] and [9], a_r, $r=n_p+i$ and $r=i+j-1$ are the coefficients of the polynomial of degree m-1 which best fits in the least-squares sense [10] the J input samples of the vectors v and T. J is chosen to optimize the estimation, and m is lower than 2.5 $J^{1/2}$, in order to avoid the oscillation between the samples [7].
Successively, to calculate the parameters A_i the linear system obtained by substituting μ_i into (2) is solved.

III. Parameters estimation of the multi step exponential signal

The multi step exponential signal $v(t)$ taken into examination is different from (1). It is:

$$v(t) = \sum_{i=1}^{n_p} A_i \left[u(t) + (e^{-\tau_i(t-t_i)} - 1)u(t-t_i) \right] \tag{7}$$

Where $u(t)$ is the step function, A_i and τ_i are the amplitude and the time constant of the i-th component, respectively, n_p is the number of components, and t_i is the delay of the i-th step. It is assumed $t_1=0$, and $0<\tau_1<\tau_i$, $A_1>A_i$ for $i=2, 3,\dots,n_p$. Example of multi step exponential signalis shown in Fig.1, in the case $n_p=2$ and one step.
In the case $t_i=0, i=1, 2,\dots,n_p$, the Prony-like method is used to evaluate the parameters A_i and τ_i. In the scenario under examination, characterized by superposition of exponential signals and step signals starting in different time instants t_i, the method must to be modified. In particular, the iterative procedure based on the sliding observation window on the acquire signal is proposed. Prony-like method is used to estimate the parameters of the multi-exponential function modelling the signal inside the window. These

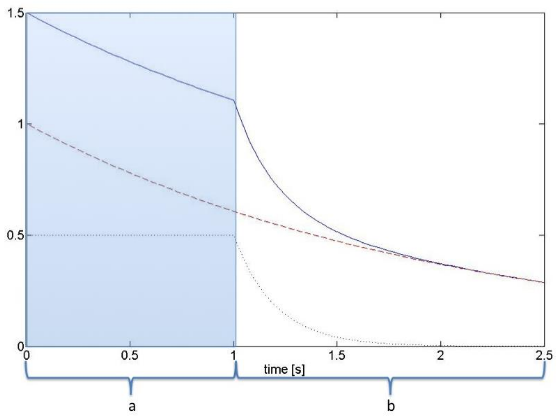

Figure 1. Multi step exponential signal composed by two components (dashed lines) and one step.

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parameters are stored into a vector and are used to estimate the delay t_i. By operating in this manner, two different cases arise:

1. there are some signal components that have constant trend, area ***a*** in Fig.1. Because the Prony-like method refers to exponential trend and the signal under examination has constant component, the estimated time constant and amplitude are not coherent with their waited values. By sliding the observation window they are highly changeable.
2. all the signal components have exponential trend, area ***b*** in Fig.1. Thetime constant and amplitude estimated by the Prony-like method are coherent with their waited value, and by sliding the observation window they are not highly changeable.

The result of this analysis, shown in Fig.2, is used to determine the greater delay t_{iMAX} among the different steps. In particular, t_{iMAX} is estimated in correspondence of the beginning of the window after that all the multi-exponential parametersare not highly changeable. To evaluate the other step delay it is assumed that step under examination is wider than the observation window used in the estimation. Under this hypothesis the same analysis can be done on the part of the signal included in the time interval [0, t_{iMAX}], iteratively.

IV. Numericaltests

Numerical tests are performed in Matlab environment. The multi step exponential signalwas generated mathematically according to (7), with $n_p=2$. The amplitude are A_1=16V and A_2=0.5V, respectively. The time constants are τ_1= -0.037s^{-1} and τ_2= -5.000 s^{-1}, respectively. Only one step is considered, and it is t_2=0.250 s. The signal is sampled by the scheme and procedure proposed in [5]. The length of the observation window is equal to 50 samples. The window is sliding in the range [0, 1.000] s with step of 0.025 s. If the observation window begins before the step, it is possible to observe highly changeable of the estimated parameters, as shown in the time interval [0, 0.250] s of Fig.2. This effect is caused by the constant component of the signal. If the observation window begins after the step, it is possible to observe not highly changeable of the estimated parameters as shown in the time interval [0.250, 1.000] s of Fig.3. This effect is caused by the absence of the constant component of the signal.
In order to evaluate the step delay,thetwo threshold$T_{A,i}$and$T_{\tau,i}$ are established as:

$$T_{Ai} = mean(A_i) \pm 0.5 * std(A_i) \qquad T_{\tau i} = mean(\tau_i) \pm 0.5 * std(\tau_i) \tag{9}$$

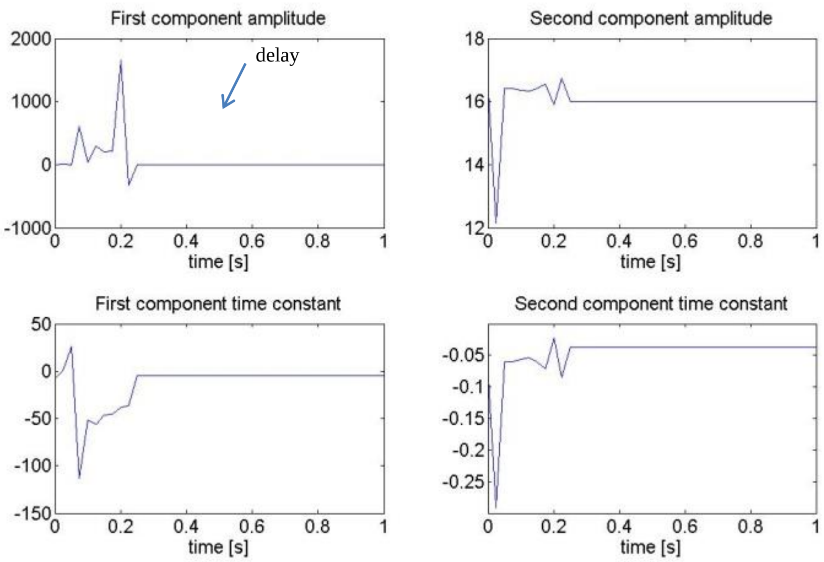

Figure 2.Trend of parameters versus the sliding of the observation window. Step at 0.250 s.

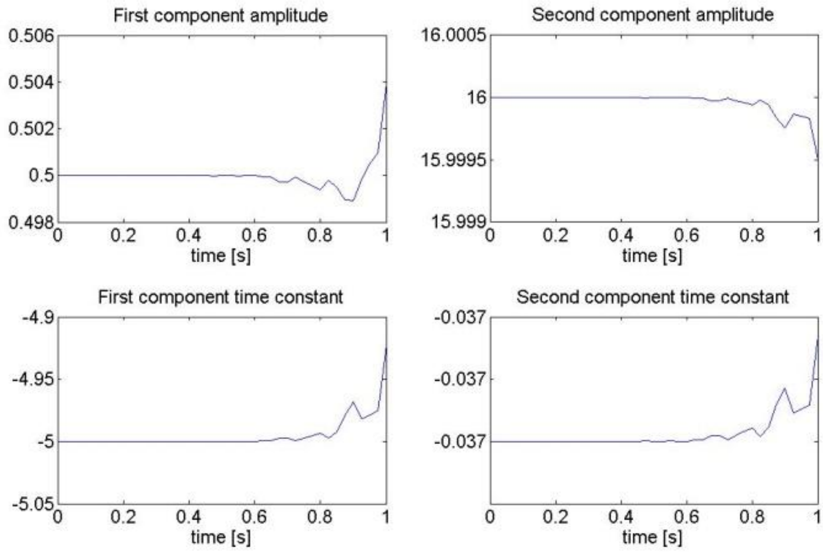

Figure 3.Trend of parameters versus the sliding of the observation window.Without step.

where *mean*(A_i) and *std*(A_i) are the mean value and the standard deviation of A_i evaluated in each sliding window, *mean*(τ_i) and *std*(τ_i) are the mean value and the standard deviation of τ_i evaluated in each sliding window. The step delay t_i is evaluated as the beginning of the sliding window so as all the estimated parameters are included in the corresponding threshold range.

Other tests are performed by using multi step exponential signalgenerated with $n_p=3$. The amplitude are A_1=16V, A_2=0.5V and A_3=0.1V, respectively. The time constants of the signal are τ_1= -0.037s^{-1}, τ_2= -5.000 s^{-1}, and τ_3= -15.000 s^{-1}, respectively. The step delays are t_2=0.00458 s and t_3=0.01100 s. The length of the observation window is equal to 100 samples. The increasing of the number of samples is necessary to estimate the slowest component in the signal with adequate accuracy. The window is sliding in the range [0, 0.010] s with step of 50*10^{-6} s. The assumption that step under examination is wider than the observation window used in the estimation is respected. The trend of the estimated parameters is shown in Fig.4 and by using the (9) the step delay t_3 is evaluated.

In order to evaluate the step delay $t_2$the same procedure is followed with n_p equal to 2 and window size equal to 50 samples, on the signal subset included in the time interval [0, t_3]. The result is shown in the Fig.5.

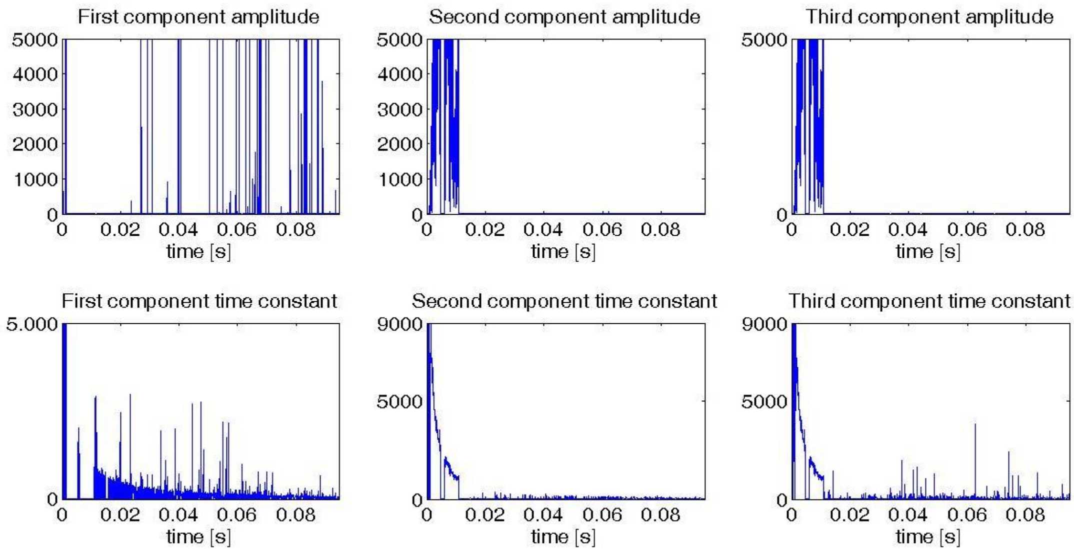

Figure 4. Trend of parameters versus the sliding of the observation window with n_p=3.

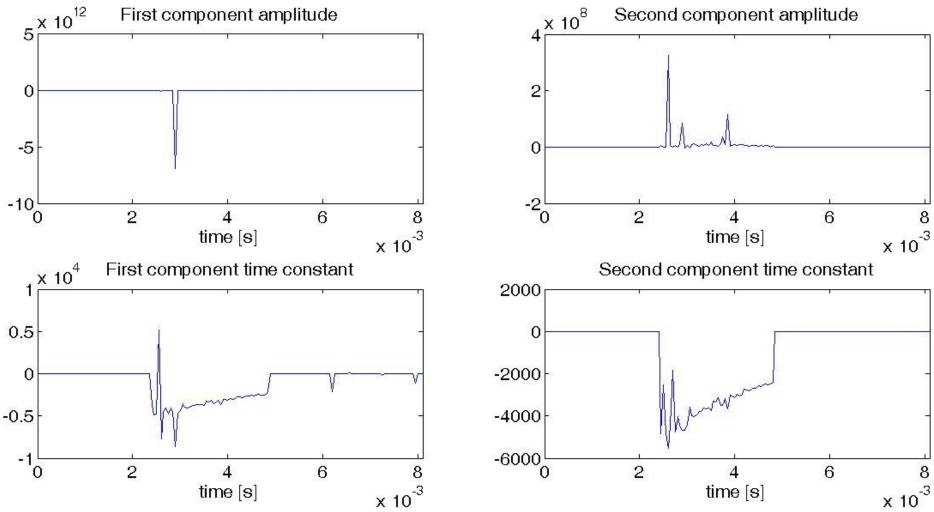

Figure 5.Trend of parameters versus the sliding of the observation window for the estimation of the step delay t_2.

V. Conclusions

In the paper is pointed out the numerical procedure to evaluate the parameters of each exponential function in the case of the multi step exponential signal defined as the superposition of exponential signals and step signals starting in different time instants.

The numerical procedure is based on the use of sliding observation window and the Prony-like method. The changeable of the parameters estimated by the Prony-like method in the observation window is used to infer the step delay. The iterative procedure is pointed out and numerically tested.

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