

Measurement of Amplitude Ratio and Phase Shift between Sinusoidal Voltages with Superimposed Gaussian Noise

Pawel Rochninski and Marian Kampik

*Institute of Measurement Science, Electronics and Control, Silesian University of Technology,
ul. Akademicka 10, 44-100 Gliwice, Poland, phone +48 32 237 10 64, fax +48 32 237 2034,
pawel.rochninski@polsl.pl*

Abstract- The paper presents basic description and some simulation results of a correlation method which together with the fractional delay sampling technique can be applied to measurements of the complex amplitude ratio of sinusoidal voltages with superimposed Gaussian noise. The method allows reduction of the influence of the white noise on the result of measurement.

I. Introduction

Estimation of amplitude and phase shift between sampled sinusoidal voltages is required to measure impedance, electrical power, and also some diagnostic parameters of electrical insulation. The uncertainty of such measurement depends on the resolution of A/C converter, properties of the processing path, the contents of noise and higher harmonics in the signals and also the selected digital signal processing algorithm. It is also necessary to provide synchronization between sampling frequency and frequency of the analyzed signals [1].

Digital algorithm can transfer errors from input to output, as well as can suppress or strengthen them, and even inserts its own ones. The analysis of the current state of research shows that the most accurate measurement of sinusoidal voltages is carried out by using Digital Fourier Transform DFT algorithm (DFT) [2]. The aim of this paper is to present a digital signal processing algorithm that will help to achieve greater accuracy of measuring sinusoidal voltages than the DFT.

II. Description of the method and simulation results

Figure 1 presents the fractional delay sampling technique applied to sinusoidal signals $u_1(t) = U_{m1}\sin(2\pi ft + \varphi_1)$ and $u_2(t) = U_{m2}\sin(2\pi ft + \varphi_2)$ [3], [4]. The technique requires to set sampling frequency f_s as a multiple of signal frequency f . The DVM internal trigger is used to start data acquisition at the moment when $u_1(t)$ signal crosses zero-level. In the first period of $u_1(t)$ signal N samples at regular time intervals of $T_s=1/f_s$ seconds are acquired. Then N samples of $u_2(t)$ signal are acquired at regular time intervals of $T_s=1/f_s$ seconds. When two N – sample series of each signal are acquired, then the positive zero – level crossing of $u_1(t)$ is detected. When it occurs the acquisition of the next series of samples starts, but the start is delayed by $T_s/2$ seconds. The result is a second N - samples series of $u_1(t)$ signal. Then a second N - samples series of $u_2(t)$ signal is acquired with similar fractional delay. The procedure is repeated M times. At the end of the data acquisition it is possible to build $M \times N$ matrix of samples for each signal, where M (80 in presented simulations) is a number of series and N (40 in presented simulations) is the number of samples acquired during one series. Having the matrix of each signal it is possible to calculate matrixes of autocorrelation and cross-correlation of the signals. Considering relationship between some eigenvalues of the correlation matrixes it is possible to estimate amplitudes and phase shift between the voltages $u_1(t)$ and $u_2(t)$.

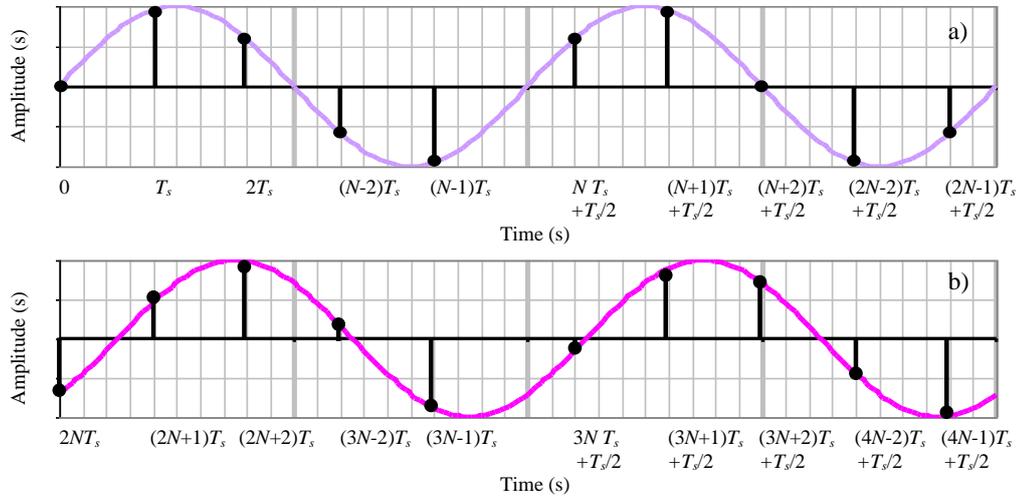


Figure 1. The fractional delay sampling technique applied to a digital sampling of sinusoidal signals
 a) $u_1(t) = U_{m1}\sin(2\pi ft + \varphi_1)$, b) $u_2(t) = U_{m2}\sin(2\pi ft + \varphi_2)$

Figure 2 shows the standard deviation of amplitude ratio of $u_1(t)$ and $u_2(t)$ for DFT method and the correlation method combined with the fractional delay sampling technique as a function of standard deviation $s(U)$ of the Gaussian noise superimposed on the $u_1(t)$ and $u_2(t)$ signals. From Figure 2 a minimal reduction of the signal noise on the standard deviation of the result of amplitude ratio measurement may be observed.

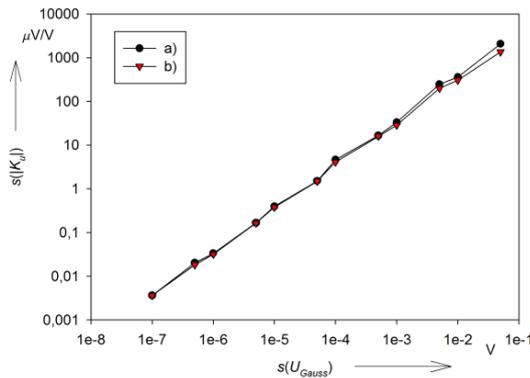


Figure 2. The standard deviation of amplitude ratio estimation of $u_1(t)$ and $u_2(t)$ as a function of standard deviation of Gaussian noise for: a) DFT method, b) the correlation method with fractional delay sampling

Figure 3 shows the standard deviation $s(\varphi)$ of phase shift φ between sinusoidal voltages $u_1(t)$ and $u_2(t)$ for DFT method and the correlation method with fractional delay sampling technique as a function of standard deviation of Gaussian noise. From Figure 3 it is seen a minimal reduction of the signal noise on the standard deviation of the result of phase shift measurement.

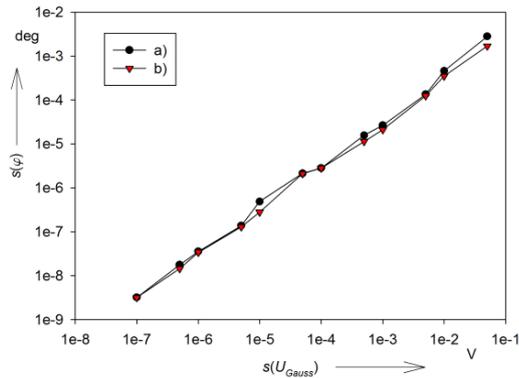


Figure 3. The standard deviation of phase shift between $u_1(t)$ and $u_2(t)$ as a function of standard deviation of Gaussian noise for: a) DFT method, b) the correlation method with fractional delay sampling

During the preliminary simulation studies it was found that for sinusoidal signals $u_1(t)$ and $u_2(t)$ with superimposed Gaussian noise the maximal relative error of amplitude ratio estimation depends on the phase shift between the signals. Figure 4 shows the maximum relative error of amplitude ratio estimation of signals $u_1(t)$ and $u_2(t)$ for DFT method and the correlation method with fractional delay sampling technique as a function of standard deviation of Gaussian noise. In order to determine the maximum relative error, the phase shift between signals $u_1(t)$ and $u_2(t)$ was changed in the range $\pm\pi$. In the figure 4 it is seen a minimal reduction of the maximal relative error of amplitude ratio measurement.

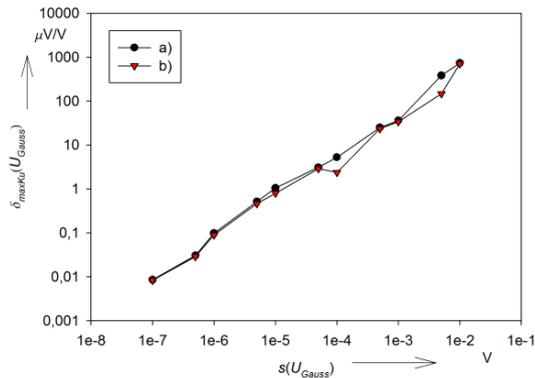


Figure 4. The maximal relative error of amplitude ratio estimation of $u_1(t)$ and $u_2(t)$ as a function of standard deviation of Gaussian noise for: a) DFT method, b) the correlation method with fractional delay sampling

During the preliminary simulation studies of signals $u_1(t)$ and $u_2(t)$ with Gaussian noise it was also found that the error of phase shift estimation between the signals was different for various phase shifts. Figure 5 shows the maximum relative error of phase shift estimation of voltages $u_1(t)$ and $u_2(t)$ for DFT method and the correlation method with fractional delay sampling technique as a function of standard deviation of Gaussian noise. In order to determine the maximum relative error, the phase shift between signals $u_1(t)$ and $u_2(t)$ was changed in the range $\pm\pi$. In the figure 5 it is seen a minimal reduction of the maximal relative error of phase shift between signals $u_1(t)$ and $u_2(t)$.

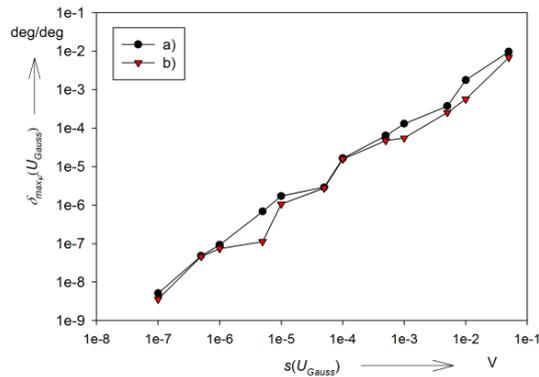


Figure 5. The maximum relative error of phase shift between $u_1(t)$ and $u_2(t)$ as a function of standard deviation of Gaussian noise for: a) DFT method, b) the correlation method with fractional delay sampling

It was also found that when sampling frequency is not synchronized with the frequency of the sampled signal then the error of amplitude ratio estimation depends on the phase shift between signals $u_1(t)$ and $u_2(t)$. Figure 5 shows the maximal relative error of amplitude ratio estimation of sinusoidal voltages $u_1(t)$ and $u_2(t)$ for DFT method and the correlation method with fractional delay sampling technique as a function of frequency error. The error is a difference between the frequency of signals $u_1(t)$, $u_2(t)$ and the assumed one (50 Hz). In order to determine the maximal relative error, the phase shift between signals $u_1(t)$ and $u_2(t)$ was changed within the range $\pm\pi$. Figure 5 shows, that the algorithm using correlation methods is insensitive to the lack of synchronization between the sampling frequency and the frequency of the analyzed signals.

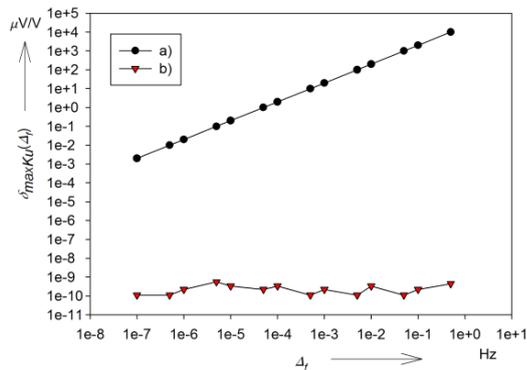


Figure 5. The maximum relative error of amplitude ratio estimation of $u_1(t)$ and $u_2(t)$ as a function of the frequency error for: a) DFT method, b) the correlation method with fractional delay sampling

Figure 6 shows the maximal relative error of estimation of phase shift between sinusoidal voltages $u_1(t)$ and $u_2(t)$ for DFT method and the correlation method with fractional delay sampling technique. This error was plotted as a function of frequency error. In order to determine the maximum relative error, the phase shift between signals $u_1(t)$ and $u_2(t)$ was changed in the range $\pm\pi$. Figure 6 shows that also in the case of phase estimation the algorithm using correlation method is insensitive to the lack of synchronization between sampling frequency and frequency of the analyzed signals.

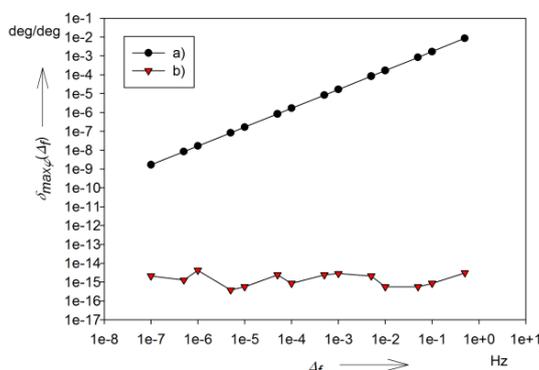


Figure 6. The maximum relative error of phase shift estimation between $u_1(t)$ and $u_2(t)$ as a function of frequency error for: a) DFT method, b) the correlation method with fractional delay sampling

The errors obtained for the correlation algorithm, shown in figures 5 and 6 are very small and can result from computing inaccuracy. In all calculations it was assumed that the A/D converter has resolution of 28-bits and sampling frequency is equal to 2 kHz. It was also assumed that $U_{m1} = U_{m2} = 1$ V and $f = 50$ Hz. During simulations which results are shown in figures 2 and 3 it was assumed that the phase shift between voltages $u_1(t)$ and $u_2(t)$ is $\varphi = \varphi_1 - \varphi_2 = 45^\circ$. Another calculations (not presented here) show that the calculated standard deviation of phase shift does not depend on the value of phase shift between voltages. It was also found that the presented algorithm is insensitive to presence of harmonics and DC-offset. In these simulations it was assumed that the signals $u_1(t)$ and $u_2(t)$ contain nine extra harmonics of equal amplitudes. First, it was assumed that higher harmonics in the signals $u_1(t)$ and $u_2(t)$ are in phase with the fundamental component. The simulation was carried out by changing the amplitude of the harmonics in range from 10 nV to 50 mV. Simulations were also repeated assuming that the harmonics have different phase than sinusoidal signals $u_1(t)$ and $u_2(t)$. Both for the DFT method and the presented algorithm the maximum relative errors did not exceed a value of $4 \cdot 10^{-16}$, i.e. insignificant. Similarly, DC component added to the signals $u_1(t)$ and $u_2(t)$ does not cause any significant errors of amplitude ratio and phase shift estimation. Concluding, simulations showed that the presented algorithm (like the DFT algorithm) is selective, which means that the presence of DC voltage and higher harmonics in the signals $u_1(t)$ and $u_2(t)$ do not affect the error of estimation of amplitude ratio and phase shift of $u_1(t)$ and $u_2(t)$. Simulations show, that when signals $u_1(t)$ and $u_2(t)$ are sampled with only one voltmeter (by the use of an multiplexer) then trigger jitter of this voltmeter equally affects both phases of the signals. That's why the jitter of voltmeter time trigger does not affect the phase shift between signals.

III. Conclusion and future work

Applying of fractional delay sampling technique and correlation method to estimation of amplitude ratio and phase shift of sinusoidal voltages reduces influence of the Gaussian noise superimposed on the measured signals. In addition, the prepared algorithm, in contrast to the DFT method, eliminates the measurement errors related to the lack of synchronization between the sampling frequency and the frequency of sampled signals. An experimental verification of the described method is planned in the future work.

References

- [1] Ramm G., Moser H., Braun A., A new scheme for generating and measuring active, reactive and apparent power at power frequencies with uncertainties of $2.5 \cdot 10^{-6}$, IEEE Trans. Instrum Meas., vol. 48, no.2, April 1999, pp. 422-426
- [2] Krajewski M., Properties analysis of selected digital signal processing algorithms in complex voltage ratio measurement, House of the University of Zielona Gora, APRIL 2010
- [3] Gregory A. Kyriazis, Marcello L. R. de Campos, High-accuracy electrical measurements using fractional delay and PCA, XIX IMEKO World Congress, Lisbon, April 2009, pp. 668-690.
- [4] Valimaki, T. I. Laakso, M. Karjalainen, U. K. Laine. Tools for Fractional Delay Filters Design. IEEE SIGNAL PROCESSING MAGAZINE, pp. 30–60, JANUARY 2005