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Energy harvesting using piezoelectric cantilever: improved SPICE model, simulations and measurements

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Abstract **– Piezoelectric materials enable the conversion of ambient vibration into electrical energy. The development of piezoelectric energy harvesting systems needs accurate models for system-performance evaluation. Mechanical researchers build distributed models for energy scavengers, simplifying the energy harvesting circuit and using analytical derivation, while electrical researchers focus on the modeling of the energy harvesting circuit and simplifying the structural conditions of the scavenging devices. The challenges for accurate modeling of such electro-mechanical systems remain, when complicated mechanical conditions and practical energy harvesting circuits are considered in system design. This article addresses the above-mentioned problem, employing an equivalent circuit, which bridges the structural modeling and electrical functionality, allowing simulations of complex circuitry. The equivalent circuit has been simulated using SPICE software. Measurements are performed for verification of the proposed model.**

Keywords: energy harvesting, vibrations, piezoelectric, equivalent circuit model, simulations, measurements.

I. INTRODUCTION

Most wireless nodes for sensing applications are designed to include an internal battery, which requires periodical maintenance for long-term operation. With the use of low-power hardware components and optimal power utilization by firmware, self-powered wireless sensing nodes are moving from a concept into practice. Vibration is a promising ambient source for energy harvesting and piezoelectric materials allow the conversion of mechanical energy into electrical energy, providing a medium of turning ambient motion into electrical energy. Analytical modeling is necessary for the design process to understand various, inter-related quantities and to optimize the key design parameters, while studying and implementing power harvesting devices. Nevertheless, the modeling of the piezoelectric cantilever, which is a component located between two worlds - mechanical and electrical - is not an easy task.

Up-to-date Equivalent Circuit Model (ECM), always takes into account a linear relationship between stress (σ) and strain (δ) parameters, which is valid only for low strain conditions and cannot be taken into account for higher vibration values. The objective of this paper is to present an improved electrical ECM that can be used either for low and high vibration conditions. Among electrical simulators we have chosen SPICE [1] since, due to its wide diffusion, allows an easy validation of the model. The SPICE software used for this study is the PSpice enhanced version.

The paper is organized as follows. Section II recalls the theoretical principles for spring mass systems applied to a cantilever beam. Section III describes the selected materials and their properties. In section IV, after a brief description of most common used ECM, we introduce and describe our improved SPICE model. Section V compares simulation and measurements results. Finally, conclusions are presented in section VI.

II. THEORETICAL BACKGROUND

A. Spring-mass system energy harvester

Energy harvesting modeling from vibration is based on spring-mass systems. A generic model for linear vibration energy harvester was first introduced by Williams and Yates [2]. The model is a lumped-parameter second order dynamic system which relates the input vibration to the output relative displacement as represented in Fig. 1.

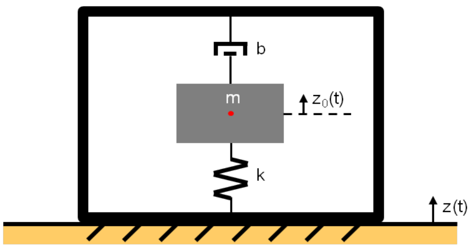

Fig. 1. Model of spring-mass energy harvester.

Applying Lagrange-D'Alembert principle to the system, the dynamic equation describing its motion is given as:

$$m\ddot{z}_0 + b\dot{z}_0 + k z_0 = -m\ddot{z} \qquad (1)$$

where z is the displacement of the housing and z_0 is the relative displacement of the mass with respect to the housing. The natural vibration frequency for such a

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system is given as $\omega_n=\sqrt{(k/m)}$. Electrical energy can be extracted via certain transduction mechanisms by exploiting either relative displacement or strain. Considering the above-mentioned transduction mechanism, the damper will consist of two parts: the mechanical damping (b_m) and the electrical damping (b_e), which removes energy from the mechanical system: $b= b_m + b_e$. Defining the overall damping factor of the system ($\zeta_T=\zeta_m+\zeta_e=b/2m\omega_n$) and assuming a sinusoidal vibration displacement of the housing ($z(t)=A \sin(\omega_n t)$), the instantaneous dissipated power (P) within the damper is the sum of maximum electrical energy extracted by the transduction mechanism, P_e, and mechanical loss, P_m. The maximum electrical power that can be extracted by the kinetic energy harvester, P_e, is given by [3]:

$$P_e=\frac{P}{2}=\frac{mA^2\omega_n^3}{16\zeta_m}=\frac{ma^2}{16\zeta_m\omega_n} \qquad (2)$$

where $a=A\omega_n^2$ is the peak value of the acceleration applied to the base. Eq. (2) shows that the maximum power delivered to the electrical domain is inversely proportional to its resonant frequency and damping.

B. Cantilever beam energy harvester

Vibration properties of the cantilever beam are very similar to spring-mass system.

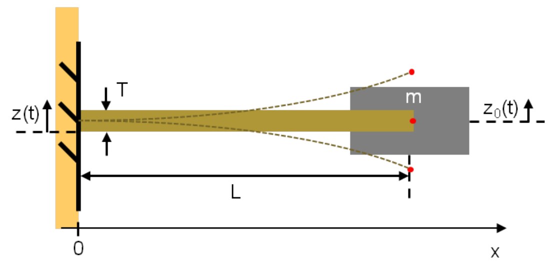

Fig. 2. Model of cantilever beam energy harvester.

Equations governing the behavior of a cantilever beam with rectangular cross section (Fig. 2) are similar to equations describing spring-mass systems, whereas equivalent mass (m_{eq}) and stiffness (k_{eq}) are [4]:

$$m_{eq}=0.23\,\rho(W\cdot L\cdot T)+m\,; \quad k_{eq}=\frac{3YI}{L^3}=\frac{YWT^3}{4L^3} \qquad (3)$$

ρ, W, L, T, Y, I are, respectively, the beam material density, width, length, thickness, Young's modulus and moment of inertia. When the beam is bent, the elongation and compression will generate both strain (δ) and stress (σ) within the beam material. In case of low strain, many materials obey Hooke's law, so that stress is proportional to strain with the constant of proportionality being the modulus of elasticity or Young's modulus, Y ($\sigma=\delta Y$). For a cantilever, the relationship between stress, force, displacement and dimensions, is:

$$\sigma(x,z)=\frac{M(x)\cdot z}{I}=\frac{F\cdot(L-x)\cdot z}{I} \qquad (4)$$

where M is the bending moment, I is the area moment of inertia of the cantilever beam and F is the force applied to its free end. As represented in Fig. 3, when the induced strain increases, many materials (including piezoelectric) deviate from this linear behavior.

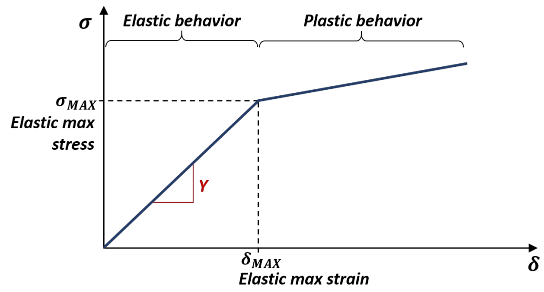

Fig. 3. Stress-strain curve for loaded material.

The point of departure of nonlinearity is termed as elastic limit. Up-to-date, ECMs rely on a linear relationship between stress and strain parameters of the piezoelectric energy harvester; this assumption, being valid only for low strain conditions, cannot account for higher vibration (and strain) values, that can be possible at the price of reduced life expectancy of the piezoelectric energy harvester [5].
Despite piezoelectric energy harvesters are usually designed to operate within the elastic region, ECM taking into account the physical deviation from linear characteristic can help researchers and electrical engineers to simulate piezoelectric energy harvester, by considering its real response for low and high vibrations.

III. MACRO-FIBER COMPOSITE AND THE CANTILEVER STRUCTURE

The Macro Fiber Composite (MFC) is a layered, planar device made up of rectangular cross-section, lead zirconium titanate (PZT) unidirectional ceramic fibers, embedded in a thermosetting polymer matrix. This fiber-reinforced layer is then sandwiched between copper-clad Kapton film layers that have an etched electrode pattern and an additional metal layer. Fig. 4 shows the structure of the MFCs used to implement the piezoelectric cantilever.

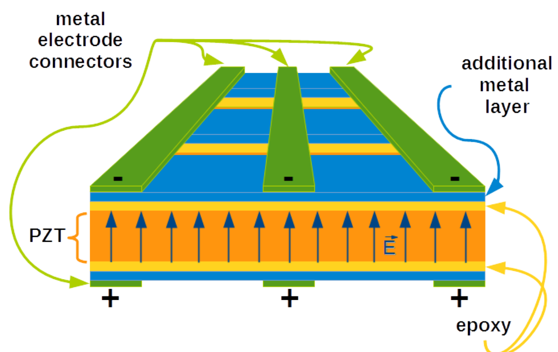

Fig. 4. MFC structure used for energy harvesting applications (no Kapton displayed).

Comparing different piezoelectric materials, an important property is the piezoelectric strain coefficient, d_{31} in the structure we used, where subscripts describe the direction of the electric field and the direction of the dielectric displacement inside the material. Table 1 shows the piezoelectric strain coefficient and other useful physical parameters for the considered material.

Table 1. MFC parameters.

MFC parameters	Value	Units
d_{31}	$1.98 \cdot 10^{-10}$	[m/V]
ε	$1.5 \cdot 10^{-8}$	[F/m]
Cu real area (1 side), A_C	20 x 45= 900	[mm^2]
PZT thickness, T_{PZT}	0.18	[mm]

The implemented cantilever uses a bimorph structure as featured in Fig. 5, with two MFC plies connected in parallel.

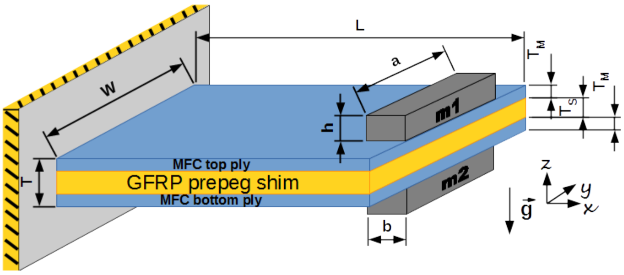

Fig. 5. Implemented piezoelectric cantilever beam.

The main characteristics and dimensions of the materials are described in Table 2.

Table 2. Cantilever materials and dimensions.

Material	Length [mm]	Width [mm]	Thickness [mm]	Y [N/m^2]
MFC	L=49	W=24	T_M=0.35	$Y_M=3\cdot10^{10}$
GFRP	L=49	W=24	T_S=0.11	$Y_S=18\cdot10^9$
TUNGSTEN	b=8	a=18	h=6	/

Considering each material Young's modulus and basing on the "Rule of Mixtures" (Voigt model) [6], [7] it is possible to calculate the Young's modulus related to both MFC plies ($Y_{MFC}=25.9\cdot10^9$ N/m^2) and to the overall bimorph cantilever structure ($Y_C=28.3\cdot10^9$ N/m^2).

IV. EQUIVALENT CIRCUIT MODEL

A. ECM for piezoelectric cantilever beam

Electromechanical models of piezoelectric cantilever are based on analogies between electrical and mechanical domains [8]. Most recent ECMs for piezoelectric cantilever, model both mechanical and electrical portion, as circuit elements, considering the material stress as the input variable acting across the mechanical side [9]. However, practical applications have kinematic base excitation, where the base displacement is easier to quantify with respect to the stress impressed on the piezoelectric beam. Thus, the above-mentioned ECM can be modified by applying the Norton's theorem to the input voltage generator, attaining the functional schematic represented in Fig. 6, where the the base speed displacement $\dot{z}$, has been used. Taking into account the analogies between mechanical and electrical domains and the spring-mass energy harvester of Fig. 1, it is possible to express the electrical parameters as follows: $L_m=m$, $R_m=b$, $C_m=1/k$ ($1/k_{eq}$ for cantilever beam). Moreover, α represents the electromechanical coupling factor, C_P is the capacitance of the piezoelectric element in static conditions, and R_{LOSS} takes into account the dielectric losses.

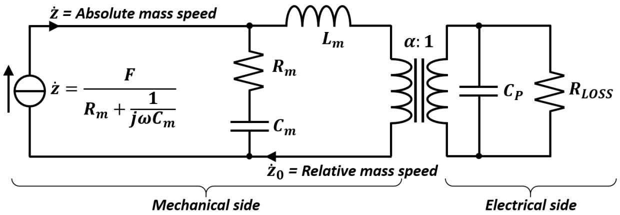

Fig. 6. Functional schematic for piezoelectric cantilever.

The output voltage, through the piezoelectric intrinsic transformer, constitutes a natural negative feedback for the input force. This effect can be treated as electrically induced damping, represented by R_{LOSS} (or $R_{LOSS}//R_{LOAD}$, when a load resistor is connected to the electrical side).

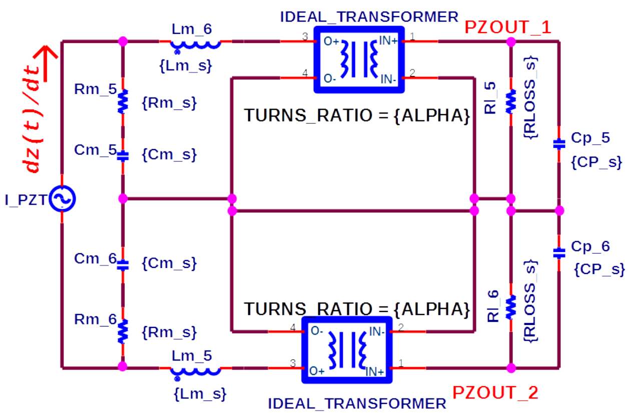

Fig. 7. Piezoelectric cantilever ECM schematic.

The model represented in Fig. 6 is valid only for low strain conditions, and it was implemented (Fig. 7) using a differential schematic to get a differential output voltage.

B. Electrical parameters calculation

The implemented piezoelectric cantilever beam uses a bimorph structure as featured in Fig. 5, with two piezoelectric MFC plies connected in parallel. Taking into account the physical beam dimensions, material properties and the modulus of elasticity Y_C, from equations (3), it is possible to obtain $k_{eq}=707.41$ N/m. Thus, the C_m parameter for the ECM will be $C_m=1/k_{eq}$

$=1.41\ mF$. For a total added seismic mass of 0.0318 kg, then $L_m=m=0.0318\ H$, we will get the beam resonating at the frequency $f_n=(1/2\pi)\cdot\sqrt{(k_{eq}/m_{eq})}=23.75Hz$. Basing on the parameters related in Table 1, it is possible to calculate the overall static capacitance of the cantilever beam: $C_P=2\cdot\varepsilon\cdot A_C/T_{PZT}=154.2\ nF$. To compute R_m, it is necessary to consider the mechanical quality factor $Q_m=f_n/\Delta f_{-3dB}$, which is calculated after measuring the output voltage of the piezoelectric cantilever (open circuit conditions) vs. the excitation frequency. From experimental measurements, we get $Q_m=11.875$; thus, $R_m=(1/Q_m)\cdot\sqrt{(L_m/C_m)}=0.4\Omega$. The resistance R_{LOSS} is related to the dielectric loss $tg(\delta)$ of the material and it is often specified @ 1kHz [10]. From the MFC material data and the expression $R_{LOSS}=1/(C_P\ \omega_t\ tg\delta)$, we obtain $R_{LOSS}=88.64\ k\Omega$. Another important material property is the effective coupling coefficient (K_{31}) which quantifies the material ability to convert mechanical energy into electrical energy, or vice versa. It is functionally-related to the strain coefficient d_{31} and the electromechanical coupling factor (α), by equation (5):

$$\alpha^2=K_{31}^2 k_{eq} C_0;\quad K_{31}=d_{31}\sqrt{\frac{Y_{MFC}}{\varepsilon}} \qquad (5)$$

In equations (5), assuming $C_0=C_m\cdot\alpha^2+C_P\approx C_P$, we extract $\alpha=2.68\cdot10^{-3}\ N/V$.

C. Improved ECM for piezoelectric cantilever beam

To take into account the nonlinear behavior of the stress-strain curve, the proposed ECM has been implemented by replacing the input current generator with analog blocks, monitoring the $F\cdot z$ physical quantity (kinetic energy transfer from vibration to material). Elastic and plastic characteristics of the piezoelectric cantilever beam are modeled through the following relationship [11]:

$$\begin{gathered}\sigma=\delta Y_C \quad (if\ \sigma\leq\sigma_{MAX})\\ \sigma=(1-\gamma)\sigma_{MAX}+\gamma\delta Y_C \quad (if\ \sigma\geq\sigma_{MAX})\end{gathered} \qquad (6)$$

where γ represents the slope-changing coefficient in the stress-strain curve once the elastic limit is reached ($\sigma=\sigma_{MAX}$ or $\delta=\delta_{MAX}$). We also assume $Y_C\approx Y_{MFC}$. Calculating the z variable from equation (4) and deriving the result, we can get the absolute mass speed when the beam is moving under elastic conditions:

$$\dot{z}_{EL}(t)=\frac{d}{dt}\left(\frac{\delta Y_{MFC} I}{F(t)\cdot L}\right)=\frac{d}{dt}\left(\frac{\sigma I}{F(t)\cdot L}\right)\quad(\sigma\leq\sigma_{MAX}) \qquad (7)$$

Exploiting equations (6) and (7), it is possible to compute the velocity variable when the beam is moving under plastic conditions:

$$\dot{z}_{PL}(t)=\beta\frac{d}{dt}\left(\frac{1}{F(t)}\right)+\gamma\cdot\dot{z}_{EL}(t)\quad(\sigma\geq\sigma_{MAX}) \qquad (8)$$

In equation (8), $\beta=(\sigma_{MAX}-\gamma\cdot\sigma_{MAX})\cdot(I/L)$ is a constant coefficient. For MFC material $\sigma_{MAX}\approx 36.43\cdot10^6\ N/m^2$ and $\gamma=0.157$ [11]. Taking the overall physical beam dimensions (2) into account, we get $\beta=666.2\cdot10^{-6}\ N/m$.

Fig. 8 shows the functional schematic of the proposed input current generator for the piezoelectric cantilever.

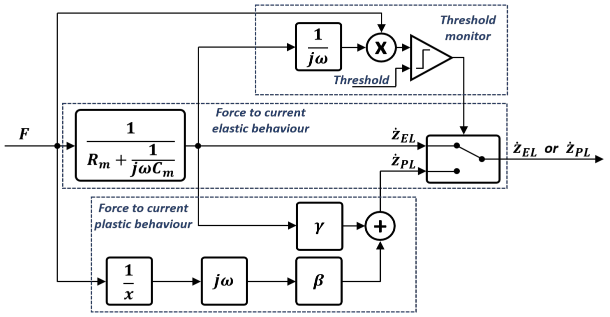

Fig. 8. Functional schematic of the input current generator for the improved piezoelectric cantilever ECM.

Within the threshold monitor block, it is possible to set the value of the $F\cdot z$ product, that allows the input current generator to switch from elastic behavior current to plastic behavior current. From equation (4), using $x=0$ (max strain section), $\sigma_{MAX}\approx 36.43\cdot10^6\ N/m^2$ and overall physical beam dimensions (Table 1), the threshold value is calculated to be $|F\cdot z|_{MAX}=790\cdot10^{-6}\ N\cdot m$. It is interesting to note that from equation (4), at $x=0$, the coordinate z_{MAX}, corresponding to the maximum allowable strain, is:

$$z_{MAX}=\frac{\delta_{MAX} Y_C W T^3}{12 F L} \qquad (9)$$

Considering the law governing the cantilever deflection respect to the equilibrium position z_0, given by:

$$\Delta z_0=\frac{F}{k_{eq}}=\frac{4 F L^3}{Y_C W T^3} \qquad (10)$$

and equating relations (9) and (10), it is possible to express the maximum square force, corresponding to the elastic limit:

$$F^2=\frac{\delta_{MAX} Y_C^2 W^2 T^6}{48 L^4} \qquad (11)$$

From equation (11), using $\sigma_{MAX}=\delta_{MAX}\cdot Y_C$, we obtain the maximum applicable force $F=0.793\ N$.

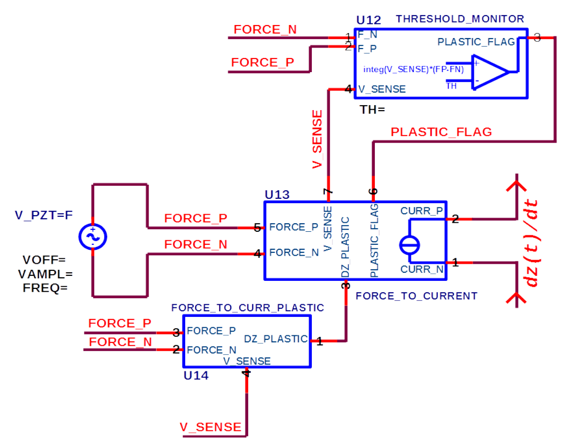

Fig. 9. Schematic of the input current generator for the improved piezoelectric cantilever ECM.

Accounting that the total added seismic mass is 0.0318 kg, then the theoretical maximum acceleration that can be applied to the cantilever beam (base excitation along z direction) is *2.54g*. The functional schematic represented in Fig. 8 can be implemented in the PSpice environment using mathematical and analog library blocks. Fig. 9 shows the schematic design of the input current generator for improved piezoelectric ECM, used for PSpice simulations.

V. SIMULATIONS AND MEASUREMENTS

The experimental setup consists of a bimorph cantilever beam clamped at one end on a rigid base and moved by a shaker that applies an acceleration along the *z* direction. An accelerometer is placed very close to the cantilever in order to measure the imposed acceleration as described in Fig. 10.

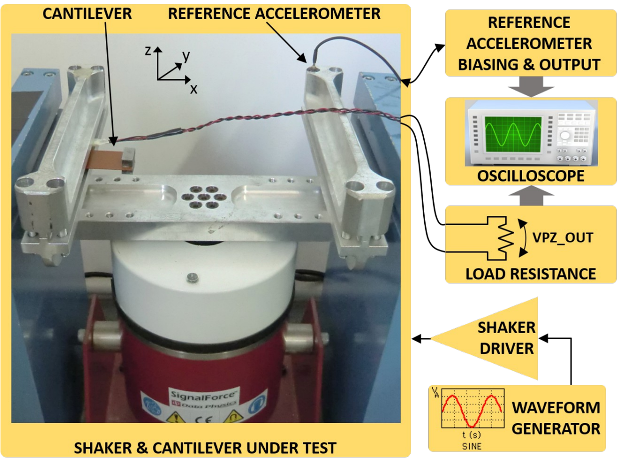

Fig. 10. Experimental setup.

We have measured the output power generated by the piezoelectric cantilever during vibrations across a set of 16 different resistive loads. The same setup has been used to simulate the proposed ECM, with the resistive load as variable parameter. The simulation results and experimental measurements are shown in Fig. 11.

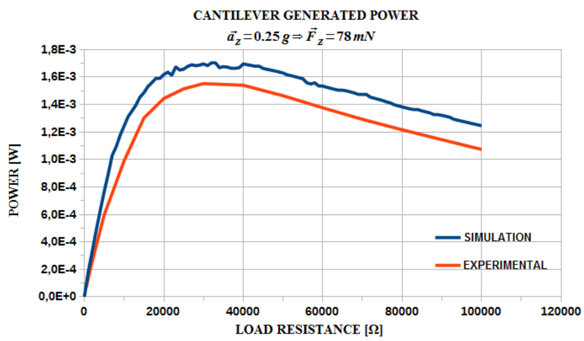

Fig. 11. PSpice simulation and experimental data.

Fig. 12 shows PSpice simulation results, which were achieved by sweeping the input acceleration beyond the elastic limit. The output voltage and power are observed across a load resistor (which equals the optimum resistance value, $R_{LOAD}=R_{OPT}=30k\Omega$) and presents a saturation effect approaching the theoretical value of *2.54g*.

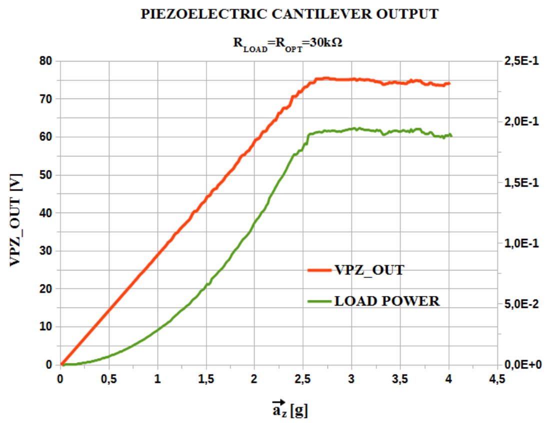

Fig. 12. PSpice electrical model simulation: load power and voltage at different acceleration conditions.

Simulations results have been validated through the experimental measurements, calculating the power transferred to the load for different applied accelerations, up to *2.75g*. The experimental set-up does not allow to reach acceleration levels higher than the above-mentioned value, due to the max allowable displacement of the shaker base, using a test frequency as low as *23.75Hz*. Fig. 13 shows the power transferred to the load vs. the load resistance value, for different applied accelerations. In the graph, it is possible to note that the power transferred to the load at *2.75g* (dashed line), is constantly lower than the one related to the previous value (*2.5g*). Basing on the same results and keeping the load resistance value fixed ($R_{LOAD}=R_{OPT}=30k\Omega$) at different excitation accelerations (Fig. 14), the generated power undergoes a slope change between *2.5g* and *2.75g* starting to decrease approaching the calculated theoretical value of *2.54g*. This behaviour can be explained through the strain hardening theory, according to which a higher stress must be applied to further deform the material, when the yield point is passed and the cantilever stiffness has been reduced.

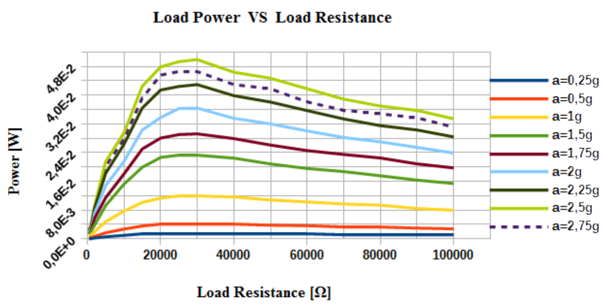

Fig. 13. Load power measurements for different acceleration conditions.

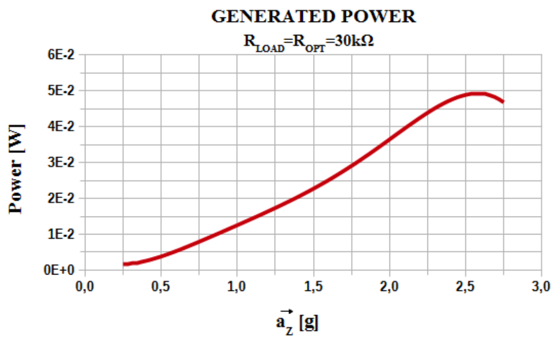

Fig. 14. Measurement of the generated power behaviour, approaching the theoretical yield limit.

VI. CONCLUSIONS

This work discusses the modeling of a piezoelectric cantilever and proposes an improved ECM that can be used for both low and high vibrations. The proposed model takes into account the real behaviour of a piezoelectric cantilever beam, whose stress-strain characteristic can be considered as composed of two separated straight line sections: the first one for the elastic behaviour and the second one for the plastic behaviour. The theoretical expressions and simulation results of the proposed ECM, implemented using SPICE software, were presented. Both show a good match with the experimental results for low vibration conditions (elastic behaviour). Simulation results and measurements for high vibration values indicate that the yield point of a piezoelectric cantilever can be predicted during the energy scavenger design (providing physical boundaries to avoid the damage of the cantilever) and exploited during simulation phases to get more reliable results.

Further comparison between simulation results and measurements for high vibrational values can be proven using a more precise experimental setup. In any case, taking into account the material physical deviation from the elastic behaviour, the proposed ECM can help to simulate a piezoelectric energy harvester, accounting its real response for low and high vibration conditions.

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