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Optimal Non-parametric Estimation of 1/f Noise Spectrum in Semiconductor Devices

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***Abstract* – In this paper, we discuss the optimal non-parametric estimation of 1/f noise in MOSFET devices. We adopt a simple experimental procedure to evaluate the performance of different methods for the spectrum estimation. In particular, we analyze the variance of the spectrum in the case of averaged periodograms, according to Bartlett and Welch methods. The influence of the adopted window function and of the overlap between segments is investigated. Finally, an optimal power spectrum estimation is identified, allowing to minimize the dispersion of the spectrum.**

***Keywords*: 1/f noise; MOSFET; non-parametric methods; power spectrum; PSD; Welch method.**

I. INTRODUCTION

The study of 1/f noise in electronic devices is important for the low noise design, as in the case of RF or microwave applications. Moreover, it has been also shown that the measurement of 1/f noise is a useful technique for the investigation of material quality in semiconductor devices and in particular in the case of MOSFETs [1-4]. Several physics-based models have been developed to correlate the experimentally measured drain current 1/f noise and the defect density (or other equivalent figures of merit) in the gate oxide [1, 2]. Such a physical quantity typically affects the reliability of MOSFETs and leads to dispersion effects in the devices. Therefore, the accurate estimation of low frequency noise in semiconductor devices is a powerful tool for a technology assessment. Also in the case of power MOSFETs, 1/f noise estimation has been proved to be a useful tool in order to assess a given technology and to give information about the defects in the device [5-9].

In [10] the authors have recently discussed the measurement of 1/f noise in power MOSFETs by means of an experimental set-up which is able to bias the device under test, to amplify the drain voltage fluctuations and to estimate the Power Spectral Density (PSD) of the noise. The considered application requires to evaluate the PSD with high spectral resolution. Typically, the spectrum is estimated close to 1 Hz and a frequency resolution of about 15 mHz is adopted. Hence, a long time window is required for this kind of measurement. Moreover, in the case of random signal, the averaging of periodograms is essential to reduce the variance associated to the PSD estimation. As a result, the measurement of the spectrum is very time consuming. Therefore, the seek of efficient methods for the estimation of 1/f noise spectrum can be beneficial in order to reduce the measurement time and to improve the accuracy of the periodogram estimation.

In this abstract, we improve the experimental set-up reported in [10], by optimizing non-parametric methods for the 1/f noise estimation. To this purpose, we consider the main techniques for the averaging of periodograms (such as Bartlett and Welch methods), allowing to minimize the variance of the spectrum. We propose a simple methodology for the experimental performance evaluation of non-parametric methods, allowing to estimate the dispersion of the spectrum from a single experimental realization. An experimental study of 1/f noise in power MOSFETs is reported. Moreover, the main parameters of the methods, such as the window function and the overlap between segments, are considered and optimized in this work.

II. REVIEW OF PSD NON-PARAMETRIC ESTIMATION

The well-known method based on the so-called periodogram represents the basic resource for the non-parametric estimation of the PSD of a stochastic process. By considering the sequence $x(n)$ obtained from the uniform sampling of a stochastic process $x(t)$ at the N instants $t_n = nT_S$, with n = 0, 1, ... N-1, $f_S = 1/T_S$ being the sampling frequency, the periodogram estimator is defined as [11]:

$$\hat{P}_{xx}^{(P)}(f) = \frac{1}{N}\left|\sum_{n=0}^{N-1} x(n)e^{-j2\pi n f}\right|^2 \qquad (1)$$

In Eq. (1), f represents the normalized (or digital) frequency axis obtained as the ratio between the conventional frequency in hertz and the sampling frequency. According to such a normalization, Eq. (1) is

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usually DFT-like evaluated in the interval $[0,1)$ at the k-th frequency $f_k = k/N$ with $k = 0, 1, \ldots N$-1. Zero-padding is commonly introduced in order to improve the numerical resolution. To refer to the one-sided PSD $G(f)$, it is common to multiply eq. (1) times 2.

From a general standpoint, the a-priori expected value of the estimator in eq. (1) is given by

$$E\left\{\hat{P}_{xx}^{(P)}(f)\right\} = \int_{-1/2}^{+1/2} P_{xx}(\xi) W_B(N, f-\xi) d\xi \qquad (2)$$

where $P_{xx}(f)$ is the true spectrum and $W_B(N, f)$ is the spectral characteristic of the N-point Bartlett window. Thus, for a given, finite N-point observation the periodogram provides a (biased) estimate of $P_{xx}(f)$ that is affected by *smoothing* (due to the main lobe of $W_B(N, f)$), which reduces the frequency resolution of the method, and spectral *leakage* (due to the presence of side-lobes in W_B). However, as the data record is made larger, both sources of uncertainty reduce since it is straightforward to show that

$$\lim_{N\to\infty} E\left\{\hat{P}_{xx}^{(P)}(f)\right\} = P_{xx}(f) \qquad (3)$$

In other words, the periodogram is an *asymptotically* unbiased estimator. As far as the variance is concerned, the a-priori value is given by

$$Var\left\{\hat{P}_{xx}^{(P)}(f)\right\} = P_{xx}{}^2(f)\left[1+\left(\frac{\sin 2\pi f N}{N \sin 2\pi f}\right)^2\right] = \qquad (4)$$
$$= P_{xx}{}^2(f)\mathrm{K}(N, f)$$

provided that $x(t)$ can be considered Gaussian. For large records, it holds

$$\lim_{N\to\infty} \mathrm{K}(N, f) = 1 \qquad (5)$$

and the asymptotical value of the variance is given by

$$\lim_{N\to\infty} Var\left\{\hat{P}_{xx}^{(P)}(f)\right\} = P_{xx}{}^2(f) \qquad (6)$$

Thus, the periodogram is an *inconsistent* estimator, since its dispersion does not tend to zero for large values of N. It is worth noticing that $\hat{P}_{xx}^{(P)}(f)$ is clearly a *non-stationary* stochastic process in the frequency domain, even in the asymptotical sense, since its statistical moments are frequency-dependent.

A possible solution in order to reduce the unacceptable variance (4)-(6) is the averaging of periodograms, usually referred to as the Bartlett method [12]. In this case the overall record of length N is subdivided into Q shorter non-overlapping segments of length L, such as $L = N/Q$. Then, the basic periodograms (1) calculated for each segment of length L are averaged at the corresponding frequency points and the improved estimator $\hat{P}_{xx}^{(B)}(f)$ is obtained. It can be shown that, from Eq. (2),

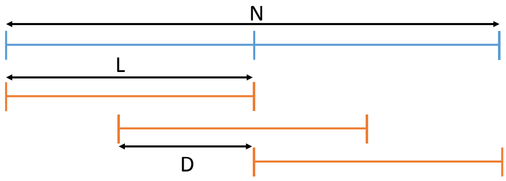

Fig. 1. Schematic of the time window adopted in the case of Welch method. The time window N is divided in smaller segments of length L partially overlapped. The overlap is defined as O_F =D / L.

$$E\left\{\hat{P}_{xx}^{(B)}(f)\right\} = \int_{-1/2}^{+1/2} P_{xx}(\xi) W_B(N/Q, f-\xi) d\xi \qquad (7)$$

and $\hat{P}_{xx}^{(B)}(f)$ is clearly still asymptotically unbiased. However, the resolution is reduced due to the use of shorter segments with respect to the basic method. The advantage inherent in the averaging is represented by a corresponding reduction in the a-priori variance:

$$Var\left\{\hat{P}_{xx}^{(B)}(f)\right\} = \frac{1}{Q} P_{xx}{}^2(f)\mathrm{K}(N/Q, f) \qquad (8)$$

which also holds in the asymptotical sense:

$$\lim_{N\to\infty} Var\left\{\hat{P}_{xx}^{(B)}(f)\right\} = \frac{1}{Q} P_{xx}{}^2(f) \qquad (9)$$

For a given N-point observation record, a trade-off is therefore needed in the choice of Q, in order to effectively reduce the statistical dispersion of the estimator while preserving an adequate resolution: in the practice, the optimal value for Q could not exist and the record must be made longer, with a measurement process becoming more time consuming.

A further improvement is the Welch method [13], consisting in the averaging of *modified* periodograms calculated over overlapping data segments. In this case, the record of length N is still divided into Q segments of length L, but these are now partially overlapped over D samples (see Fig. 1). The degree of overlapping is defined by the factor $O_F = D/L$. According to such a strategy, in correspondence with an optimal value of Q in terms of variance reduction, longer values for L can be considered by progressively increasing the overlapping, until the corresponding frequency resolution meets the requirements of the application. However, for significant overlapping among adjacent segments, statistical correlation among the corresponding elementary periodograms is introduced, which reduces the effectiveness of averaging large numbers of segments. From a general standpoint, the variance associated with the PSD Welch estimator is not a monotonic decreasing function of the overlap factor. In order to smooth the perturbation due to correlation among adjacent periodograms, each segment is thus weighted by introducing a given time-window $w(t)$ before evaluating the so-called modified periodogram on it. Many possible windows are suggested by the literature, most of them corresponding to those widely exploited for leakage reduction in the digital spectrum analysis of deterministic signals.

For the Welch estimator $\hat{P}_{xx}^{(W)}(f)$ derived as discussed above, it holds

$$E\left\{\hat{P}_{xx}^{(W)}(f)\right\} = \int_{-1/2}^{+1/2} P_{xx}(\xi) W_{conv}(L, f-\xi) d\xi \qquad (10)$$

in which $W_{conv}(L, f)$ is the spectral characteristic of the convolution of the L-point $w(t)$ with itself [11]. Equation (10) shows how the smoothing of the influence of correlation on $\hat{P}_{xx}^{(W)}(f)$, achieved by the use of windowing, leads to a further smoothing of the estimated PSD and a corresponding further reduction in the resolution.

As far as the variance is concerned, its a-priori computation represents a tough issue of the Welch method, since it requires non-straightforward analytical manipulations, which depend on the window used and the value O_F considered. This means that the optimal set (Q, L, O_F, w) that minimizes the dispersion and preserves resolution of $\hat{P}_{xx}^{(W)}$ cannot be easily identified for a given application by means of analytical investigation, thus experimental methods are strongly required for the purpose of performance evaluation and comparison with other non-parametric techniques. It is worth noticing that in all the cases reported by the literature, the asymptotic variance can be expressed as

$$\lim_{N\to\infty} Var\left\{\hat{P}_{xx}^{(W)}(f)\right\} = C(O_F, w)\frac{1}{Q} P_{xx}{}^{2}(f) \qquad (11)$$

in which C is a constant with respect to the frequency that, for a given window, non-monotonically depends upon the overlapping factor. Equation (11) shows the effectiveness of the use of a large number Q of overlapped data segments, provided that the optimal degree of overlapping is chosen for the given window.

III. EXPERIMENTAL PERFORMANCE EVALUATION OF NON-PARAMETRIC METHODS

In this work, we are mainly interested into further improve the experimental set-up reported in [10], which allows to evaluate the 1/f noise in semiconductor devices, by optimizing the signal processing techniques exploited on the acquired noise samples. In particular, the application requires an accurate estimate of PSD at very low frequency with high spectral resolution. From a more general standpoint, however, this paper aims at investigating the optimal non-parametric method and estimating the best associated parameters in the case that 1/f noise in electron devices is the subject of investigation and high resolution is required.

First of all, a reliable and method-independent experimental procedure for the performance evaluation of a given technique is needed. As discussed in the previous section, a-priori analytical investigation can be cumbersome while a simple, empirical methodology for the systematic comparison among different algorithms, each characterized by several parameters, can allow instead for a fast and exhaustive minimization of the measurement uncertainty.

To this aim, the following stochastic process in the frequency domain is introduced:

$$\bar{P}_{xx}^{(A)}(f) \triangleq \frac{\hat{P}_{xx}^{(A)}(f)}{P_{xx}(f)} \qquad (12)$$

$\hat{P}_{xx}^{(A)}(f)$ being the given estimator under analysis (A = P,B,W or other possible non-parametric methods). From the results provided in section II, it is straightforward to show that

$$\lim_{N\to\infty} E\left\{\bar{P}_{xx}^{(A)}(f)\right\} = 1 \qquad (13)$$

$$\lim_{N\to\infty} Var\left\{\bar{P}_{xx}^{(A)}(f)\right\} = const \qquad (14)$$

In Eq. (14) the constant value at right-hand side (clearly depending on the method and the associated parameters) is to be regarded as with respect to the frequency. Thus, the process in Eq. (12) is asymptotically *stationary*: its statistical moments (in particular the variance or the standard deviation) can be empirically estimated by suitably processing a *single* experimental realization of $\bar{P}_{xx}^{(A)}(f_r)$ (r = 0,1,…,R-1) (provided that a sufficiently large number N of samples have been obtained and stored in the record) without making use either of complicate a-priori statistical calculations or averaging of large numbers of estimator successive realizations. In the present application, since a high resolution is required, an adequately large number N of samples is supposed to be available from the A/D section of the set-up, independently from the digital method considered, and possible segmentation and overlapping. For this reason, all the estimators in the following are considered and referred to according to the asymptotical sense.

Obviously, for a generic noise $x(t)$, $P_{xx}(f)$ is an unknown quantity and Eq. (12) has no practical use. However, in the present application, the physical sources of fluctuation within the electron device are known and well assessed by both theory and empirical evidence, and the true spectrum $P_{xx}(f)$ can be reliably expressed, for the specific purpose of experimental performance evaluation and comparison of different methods, by the typical 1/f law

$$P_{xx}(f) \cong \frac{1}{2} G_{fit}(f) \triangleq \frac{1}{2}\frac{\beta}{f^{\gamma}} \qquad (15)$$

in the frequency interval of interest, in which the parameters (β, γ) can be estimated as described in the following. Since it holds

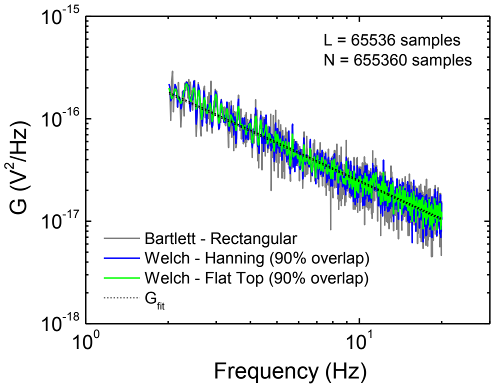

Fig. 2. Comparison of spectrum estimated by means of different methods: Bartlett or Welch (assuming different window functions). The corresponding G_{fit} functions, estimated with Eq. (15), are very similar. N = 655360, L = 65536, and f_s = 1 kHz.

$$StDev\left\{\bar{P}_{xx}^{(A)}(f)\right\} = +\sqrt{\frac{1}{P_{xx}{}^{2}(f)} Var\left\{\hat{P}_{xx}^{(A)}(f)\right\}} \qquad (16)$$

the operative estimate of left-hand side in Eq. (16), given by

$$\hat{\sigma}_{rel} = \sqrt{\frac{1}{R}\sum_{r=0}^{R-1}\left[\frac{G(f_r) - G_{fit}(f_r)}{G_{fit}(f_r)}\right]^2} \qquad (17)$$

is an accurate experimental estimate of the *relative* standard deviation of $\hat{P}_{xx}^{(A)}(f)$. In Eq. (17), $G(f_r) = 2\hat{P}_{xx}^{(A)}(f_r)$ (r = 0,1,…, R-1) is the one-sided PSD obtained through a single realization of the estimator under investigation, while $G_{fit}(f_r)$'s are available once the optimal couple (β, γ) is obtained by means of minimization of (17).

In Fig. 2 we report an example of power spectrum estimated with different methods, Bartlett and Welch, and considering different window functions. In spite of the different methods, similar $G_{fit}(f_r)$ curves are obtained. This result corroborates the proposed methodology, which is based on the adoption of a $G_{fit}(f_r)$ function to calculate the relative standard deviation in Eq. (17).

IV. OPTIMIZATION OF 1/ f-NOISE PSD NON-PARAMETRIC ESTIMATION

According to the non-parametric methods and to the methodology for performance evaluation discussed in section II and III, in this section we compare the experimental results originating from different approaches for the PSD estimation: Bartlett or Welch methods. Moreover, with the aim of optimizing the non-parametric estimation of 1/f noise, we consider different window functions, having different properties in terms of leakage and resolution, and variable overlap O_F.

In order to generate 1/f noise, we consider a power MOSFET biased in triode region [10]. The open-circuit drain voltage fluctuations of the device are properly sampled by means of the laboratory set-up with very low noise floor, and provided as a digital input to the LabVIEW environment. An example of the one-sided power spectrum is reported in Fig. 2. As expected the spectrum is quite sparse and not exactly representative of 1/f noise, because of the frequency-dependent variance associated with the PSD estimator.

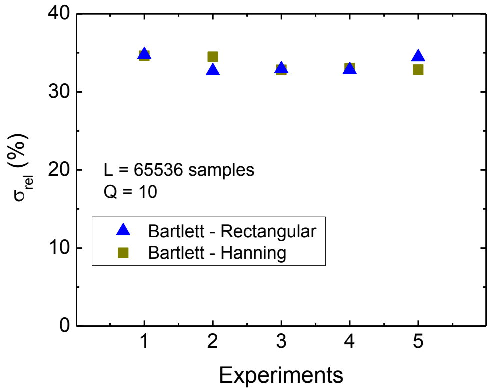

Fig. 3. Repeatability of the σ_{rel} estimation in the case of Bartlett method. We consider a record with N = 655360, L = 65536, and f_s = 1 kHz.

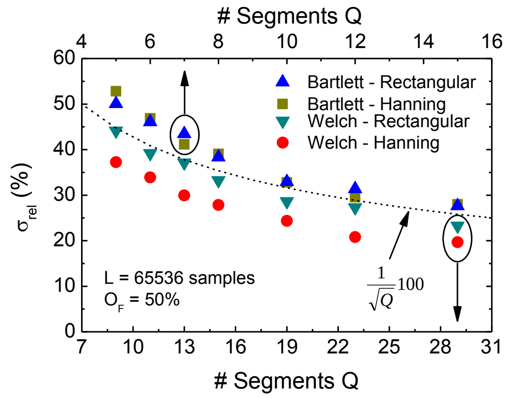

Fig. 4. Relative standard deviation of the spectrum, calculated according to Eq. (17), as a function of the number of segments Q.

In Fig. 3 we report the relative standard deviation, Eq. (17), of the 1/f noise spectrum in the case of Bartlett method. In all the cases, we considered a time window having N = 655360, L = 65536, f_s = 1 kHz. Different experiments were carried out in order to verify the repeatability of the relative standard deviation estimation based on the hypothesis of relative stationarity of the estimator. It is worth noting that applying a Hanning window has basically no impact on the 1/f spectrum evaluation since the same $\hat{\sigma}_{rel}$ is obtained.

In Fig. 4 we analyze the relative standard deviation as a function of the record length N. Since a minimum resolution is required by the considered application, a constant (and large enough) segment length L is chosen. Hence, the number of segments Q are changed accordingly. By increasing the number of segments $\hat{\sigma}_{rel}$ reduces in all the four cases, as expected. If we consider a

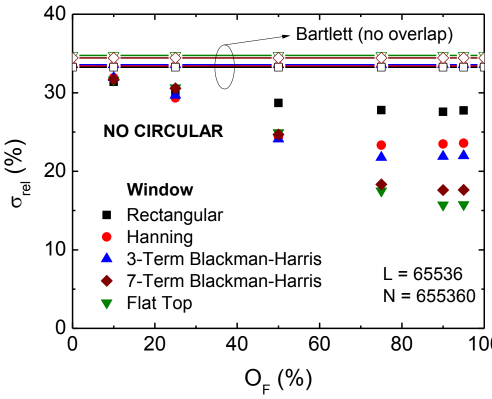

Fig. 5. Relative standard deviation of the spectrum as a function of the overlap in the case of different window functions (solid symbols). We also report Bartlett periodogram estimator (continuous lines with open symbols).

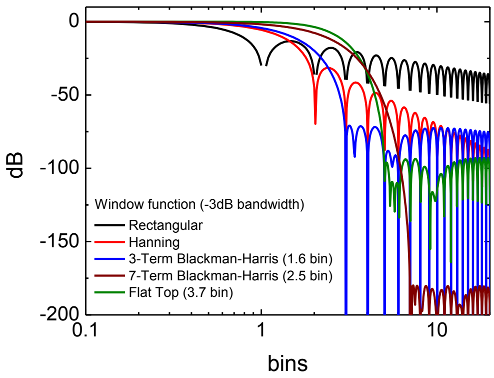

Fig. 6. Frequency-domain response of the window functions.

value Q large enough, σ_{rel} can be approximated as $1 / \sqrt{Q}$. On the other hand, for a given record length, the $\hat{\sigma}_{rel}$ resulting from the application of the basic Welch method is always lower with respect to the Bartlett method, when $O_F = 50\%$. This is a preliminary prove of the effectiveness of the Welch method for the estimation of the spectrum in the case of 1/f noise, at least with the overlap factor above. Moreover, the advantage of Welch method is even more highlighted if a simple Hanning window is considered.

An important parameter to consider when implementing the Welch method is the overlap factor O_F between adjacent segments. In Fig. 5 we report the σ_{rel} as a function of O_F. The total record length is kept constant to $N = 655360$, while single segments have length $L = 65536$. Hence, the number of segments in the case of no overlap is 10. The relative standard deviation for the Bartlett method is also reported in Fig. 5 ($\hat{\sigma}_{rel}$ = 33.2% in the case of rectangular window). The application of Welch method leads to a reduction of $\hat{\sigma}_{rel}$, which is strongly related to the considered O_F and to the window function. While for small overlaps the adopted window does not significantly affect the spectrum dispersion, for large overlaps $\hat{\sigma}_{rel}$ is window dependent and the best

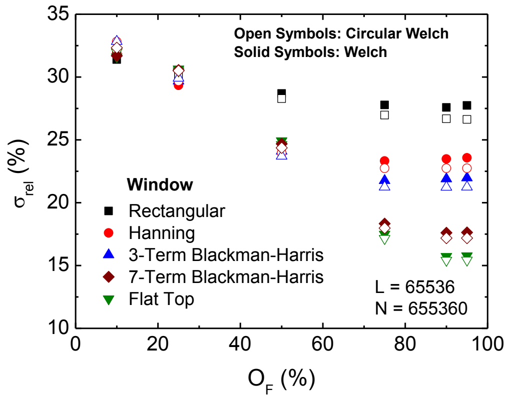

Fig. 7. Relative standard deviation of the spectrum as a function of the overlap. Open symbols represents the case of circular overlap [14].

performance is achieved in the case of a flat top window. In fact, for large overlaps, the correlation among adjacent periodograms reduces the effectiveness of averaging large numbers of segments. In this case, the smoothing of the correlation due to the window function plays an important role. In order to understand the properties of the window functions, which allows to optimize the non-parametric estimation of the spectrum, let us consider their frequency-domain response in Fig. 6. Among the considered windows, the minimum spectral leakage is obtained in the case of 7-term Blackmann-Harris window. On the other hand, the maximum main lobe -3dB bandwidth, and hence the maximum flatness, is obtained in the case of flat top window. Based on this observation, we can assert that the smoothing of the correlation is mainly due to the main lobe bandwidth, rather than the spectral leakage.

We also observe in Fig. 5 that, according to the considered window function, an optimum value of O_F can be detected before $\hat{\sigma}_{rel}$ starts raising again. In particular in the case of flap top window the optimum O_F is 90% and leads to $\hat{\sigma}_{rel} = 15.7\%$.

A modification of the Welch method, proposed in [14], is based on a circular overlap, in which the end of the record is concatenated with the beginning. The results, arising from the abovementioned methodology, are reported in Fig. 7 and referred to as Circular Welch. With respect to the conventional Welch method, we observed a saturation of $\hat{\sigma}_{rel}$ for large overlaps, rather than a small increase. Overall, slightly lower $\hat{\sigma}_{rel}$ are achieved.

In the case of Welch method, we can consider the equivalent number of averages (Q_e), that is calculated from the estimated σ_{rel} as:

$$\sigma_{rel} = \frac{1}{\sqrt{Q_e}} \tag{18}$$

In this case, we can estimate the relative increase in Q, which is denoted by:

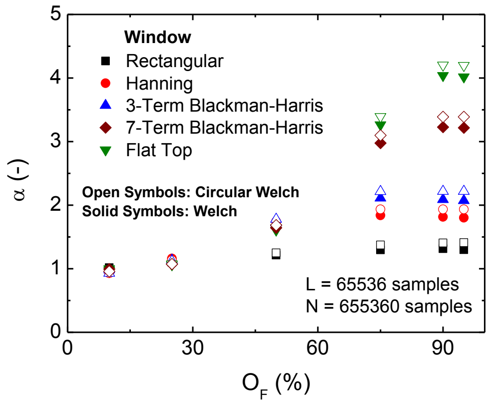

Fig. 8. Relative increase of number of averages, Eq. (19), in the case of Welch method as a function of the overlap and of the considered window function.

$$\alpha = \frac{Q_e}{Q} \tag{19}$$

where Q represents the number of average in the case of Bartlett method, that is calculated as N / L. As reported in Fig. 8, we found out that a maximum α larger than 4 is achieved for 90% overlap when the flat top window is applied. In other words, in order to match the $\hat{\sigma}_{rel}$ obtained with Welch method under these conditions (flat top window and O_F = 90%), the adoption of Bartlett method would require a 4 times longer record.

V. CONCLUSIONS

In this paper, we have analyzed the application of non-parametric methods for the optimal 1/f noise spectrum estimation. We have reported a simple experimental procedure allowing to calculate the variance of the spectrum from a single realization and hence to evaluate the performance of a given method. The experimental set-up discussed in [10] is adopted for the characterization of 1/f noise in a power MOSFET. The experimental results show that the Welch method allows to significantly reduce the relative standard deviation of the spectrum, with respect to the Bartlett method. Moreover, while the application of a window function has no effect in the case of Bartlett method, it results in a further decrease of $\hat{\sigma}_{rel}$ when the Welch method is concerned. The best performance of the Welch method are achieved when a flat top window with 90% of overlap is considered. We found out that the smoothing of the correlation, allowing to improve the effectiveness of averaging large numbers of segments in the case of large overlap, is mainly due to the main lobe bandwidth, rather than the spectral leakage.

Overall, the application of the Welch method (with respect to the Bartlett method), by weighing each segment with a flat top window, allows to achieve a remarkable reduction of $\hat{\sigma}_{rel}$ of about 17.8% in the case of N = 655360 and L = 65536. As an alternative, in the case of Bartlett method, more than 4 times longer record would be necessary to achieve the same $\hat{\sigma}_{rel}$ of Welch method.

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