

# Coaxial-Cage Line for Geo-materials Electromagnetic Characterization

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**Abstract** – We present the results obtained with a custom coaxial-cage line built to measure the complex dielectric permittivity and magnetic permeability of granular and/or liquid materials. The coaxial-cage line presents an open structure to facilitate the insertion and to control the compactness of granular materials. The measurements have been performed using a Vector Network Analyzer and the electromagnetic parameters of the samples have been retrieved through the Nicolson-Ross-Weir algorithm (in case of magnetic materials) or the Boughriet algorithm (in case of non-magnetic ones). The cell has been used to characterize the electromagnetic properties of some geo-materials (clay soils and magnetite samples). The electromagnetic parameters are shown as a function of frequency (1MHz-1GHz) and temperature (about 200-298K).

## I. INTRODUCTION

In the last twenty years, the use of Ground Penetrating Radar (GPR) has found increasing application in many fields like hydrology, environmental sciences, precision agriculture, geology and glaciology [1]. In order to accurately interpret the GPR data, the knowledge of the electromagnetic properties of the soils, for different mineralogical compositions and physical conditions (temperature, water content), is fundamental. The constitutive parameters of the geo-materials can be measured, in the megahertz-gigahertz range, with various methods based on resonant or non-resonant techniques [2]. One of the most common method is to measure the scattering parameters (S-parameters) of a transmission line filled with the test material (probe-line) through a Vector Network Analyzer (VNA). The constitutive parameters can be determined from the S-parameters using the Nicolson-Ross-Weir algorithm (NRW) [3-4]. This approach is generally employed using small dimension probe-lines, which imply the evaluation of

the electromagnetic (em) parameters of a small volume of material which, in case of geo-materials, could not be fully representative of the bulk em properties of the material [5]. To overcome this problem, we designed a coaxial line with a multi-wires cage shield, which facilitates the insertion of granular samples. The probe, coupled to a VNA in the 1MHz-1GHz frequency band, has been calibrated in air and tested by using reference materials with well-known electromagnetic properties [6]. As an example, we report here the results obtained using the coaxial-cage line to measure the constitutive parameters of a non-magnetic clay soil and a magnetite sample as a function of frequency and temperature (down to about 200K).

## II. MATERIALS AND METHODS

The experimental set-up consists of a two-port Vector Network Analyzer (Agilent E5071C) connected through two coaxial cables to the coaxial-cage line filled with the material under test [6]. To investigate a much larger volume of material, we modified the original NRW setup design, building a transmission line with a section larger than that of the standard cables. The coaxial-cage line consists of a stainless steel cage housed in a plexiglass box, open at the top, and sealed at the two lateral connectors. The cage is made by a central conductor with a diameter of  $2r = 3.00 \pm 0.03$  mm and a length of  $189.85 \pm 0.03$  mm. The outer structure is made by eight equally spaced rods (arranged on a circular pattern) with the same diameter of the inner conductor and with a distance between the inner and the outer rods of  $d = 13.10 \pm 0.06$  mm.

The equations, which allow to determine the relative complex electric permittivity ( $\epsilon_r$ ) and magnetic permeability ( $\mu_r$ ) of the material, are a modified version of the original Nicolson-Ross-Weir algorithm [6]:

$$\sqrt{\frac{\varepsilon_r}{\mu_r}} F_g = \frac{1-\Gamma}{1+\Gamma} \quad (1)$$

$$\sqrt{\varepsilon_r \mu_r} l = j \frac{c}{2\pi f} \ln(T) \quad (2)$$

where  $\Gamma$  and  $T$  are the reflection coefficient and the propagation term, respectively,  $F_g = Z_c / Z_p$  is the ratio of the characteristic impedances of the coaxial cables ( $Z_c$ ) to that of the probe-line in air ( $Z_p$ ),  $l$  is the effective length of the coaxial-cage probe-line,  $f$  is the frequency,  $c$  is the velocity of the light in a vacuum. The terms  $\Gamma$  and  $T$  are determined from the scattering parameters at the beginning and the end of the coaxial-cage line removing the attenuation and the delay due to the cables and connectors with a calibration procedure [6]. The correct computation of Eq. (2) requires that the phase of  $T$  is unwrapped in order to avoid the phase ambiguity. The terms  $l$  and  $F_g$  ( $0.187 \pm 0.003m$  and  $0.400 \pm 0.005$ ) have been estimated measuring the scattering parameters of the probe in air ( $\varepsilon_r = 1$  and  $\mu_r = 1$ ) and applying Eqs.(1) and (2).

The explicit formulas for the electromagnetic parameters  $\varepsilon_r$  and  $\mu_r$  can be retrieved combining Eqs.(1) and (2) as follows:

$$\varepsilon_r = j \frac{c}{2\pi f l} \ln(T) \frac{1-\Gamma}{1+\Gamma} \frac{1}{F_g} \quad (3)$$

$$\mu_r = j \frac{c}{2\pi f l} \ln(T) \frac{1+\Gamma}{1-\Gamma} F_g \quad (4)$$

In general, the solution of the NRW algorithm tends to diverge at frequencies  $f_m$  multiple of the correspondent probe line half-wavelength [13], where  $f_m$  is given by:

$$f_m \approx mc / \left( 2l \operatorname{Re} \left\{ \sqrt{\varepsilon_r \mu_r} \right\} \right) \quad m = 0, 1, 2, \dots \quad (5)$$

In fact, at these frequencies, the reflection coefficient  $\Gamma$  is poorly determined due to the VNA limited dynamics and the presence of electronic noise (see [8] and [9]).

To overcome this limit, Boughriet et al. [7] proposed, in case of non-magnetic materials, a formula for the determination of the permittivity which, can be directly derived from Eq. (2) assuming  $\mu_r = 1$ :

$$\varepsilon_r = \left[ j \frac{c}{2\pi f l} \ln(T) \right]^2 \quad (6)$$

Eq. (6) allows to significantly improve the accuracy around the resonance frequencies since, in contrast to Eq.(3), it does not contain the term  $\Gamma$  (see for more details [6] and [7]). However, the retrieved permittivity is still affected (but to a lesser degree) by some

inaccuracy, being  $\ln(T)$  still dependent on  $\Gamma$  (see [6] and [7]).

It is worth to note that, the presented methodologies implicitly assume that the probe only allows the propagation of the fundamental mode (TEM mode). This assumption is valid below the cutoff frequencies of the transmission line; indeed, above such frequencies, it is possible the excitation of higher modes (TE and TM modes). This implies an upper limit of the measurable frequency range where it is applicable Eqs. (3)-(4) and (6). Considering, at a first order of analysis, the coaxial-cage probe equivalent to a continuous shield coaxial line, the lowest cutoff frequency is given by  $f_c \approx c / \left[ \pi \sqrt{\varepsilon_r \mu_r} (d+r) \right]$  [10].

### III. RESULTS

We present here the results of the measurements performed on two geo-materials: the first is a magnetite granular sample extracted from the Malmberget mine in Sweden (courtesy of MINELCO BV), the second is a non-magnetic clayey soils collected in Rome, Italy, mainly containing Montmorillonite  $\left( (Na, Ca)_{0.3} (Al, Mg)_2 Si_4 O_{10} (OH)_2 \cdot n(H_2O) \right)$ , a mineral of the Smectite group. The coaxial-cage line is inserted in a cryostat cooled down to about 200K by using  $CO_2$  ice. The measurements were performed first dropping the temperature from 298 K to 200 K and then allowing the system to slowly reaches again the temperature of 298 K. Each cycle lasted about 50 h, 10 h for cooling and 40 h for warming up. A PT-100 probe has been inserted in the coaxial-cage line to measure sample temperature during the cooling and heating processes.

The results are represented in terms of real and imaginary parts of the electromagnetic parameters. The uncertainties on  $\varepsilon_r$  and  $\mu_r$ , which depend on  $S_{11}$ ,  $S_{21}$ ,  $l$ , and  $F_g$ , are estimated using the statistical error propagation formula [11], taking into account the uncertainties of the S-parameters given in the VNA accuracy specifications (E5071C ENA Network Analyzer Data sheet document).

In Figure 1 the electromagnetic parameters of magnetite are shown as a function of frequency at five temperatures. The sample shows a dispersive behavior both in the electrical and magnetic spectra. In particular, the electric spectrum is characterized by a conductive behavior in the imaginary part and is altered by the resonance of the probe at about 150MHz (see Eq.(5)). The same “noise” is also present, in the magnetic spectrum, where the resonance apparently produces two broad peaks at about 50 MHz and 170 MHz. However, both in the real and imaginary part of permeability, it is still recognisable a single magnetic

polarization process centred at about 80 MHz and describable by a Debye model. Furthermore, Figure 1 also shows the dependence of the constitutive parameters on temperature which is more pronounced in the electric permittivity than in magnetic permeability. The temperature dependence of the real part of permittivity [12] could indicate that the values of  $\epsilon_r$  are due to a relaxation process at frequencies higher than those investigated in this paper. Moreover, the imaginary part increases as the temperature decreases as it occurs in metallic materials. On the other hand, the relaxation process observed in the magnetic permeability is probably due to magnetic domain walls [13] whose polarization apparently does not depend on temperature [14].

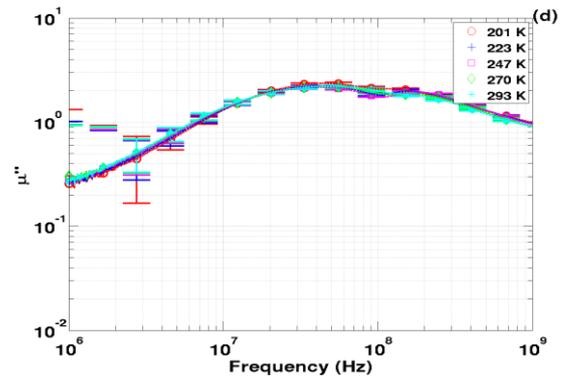
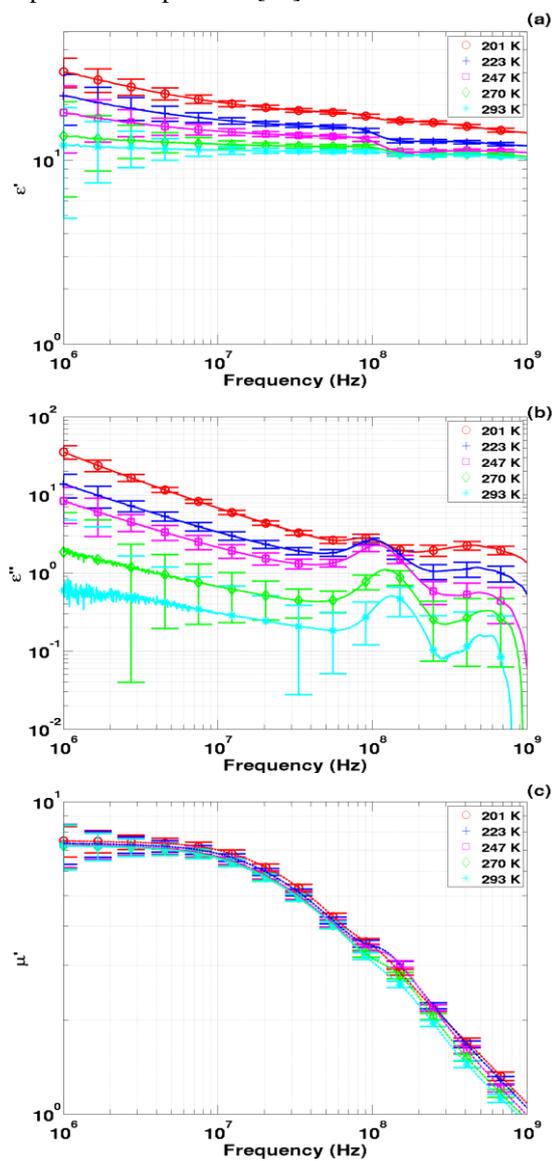
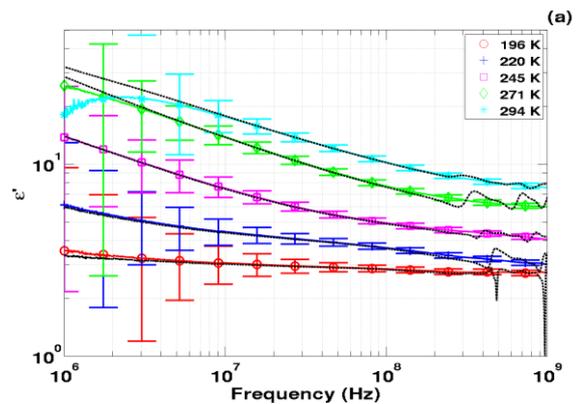


Fig. 1 Panels (a) and (b) show the real  $\epsilon'$  and imaginary part  $\epsilon''$  of electric permittivity; panel (c) and (d) show the real  $\mu'$  and imaginary part  $\mu''$  of magnetic permeability. The data, retrieved using Eqs. (3) and (4), are plotted as a function of frequency at the five temperatures reported in the legend.

Figure 2 shows the real and imaginary parts of electrical permittivity of clayey soil sample. The permittivity has been estimated using Eq. (6) (colored lines) and Eq. (3) (black dashed lines). The results, obtained applying the two algorithms, are in good agreement in the frequency range far from the resonant frequencies of the probe (see Eq. (5)) and confirm that the sample does not exhibit magnetic properties. The behavior of the electrical permittivity spectrum is dominated by the presence of water in the sample which gives place to an ionic conduction and an increase of real part of permittivity. In the low frequency range the Maxwell-Wagner polarization is recognisable [12] and is more pronounced at high temperature as the electrical conductivity is higher (see panel b). At low temperature, when the water is completely frozen, the electric spectrum becomes less dependent on frequency both in real and imaginary part.



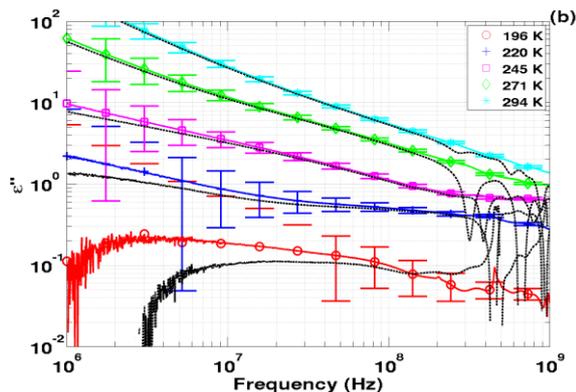


Fig. 2 Panels (a) and (b) show the real  $\epsilon'$  and imaginary part  $\epsilon''$  of electric permittivity. The colored lines and symbols represent the data obtained by using Eq.(6). The dashed black lines represent the data retrieved applying Eq.(3). All the data are plotted as a function of frequency at the five temperatures reported in the legend.

#### IV. CONCLUSIONS

The results presented in this paper showed the possibility to accurately measure the dielectric and magnetic properties of granular materials using a coaxial-cage transmission line connected to a VNA. This measurement setup, together with a modified version of the NRW and Boughriet algorithms, allowed the evaluation of the real and imaginary parts of the complex permittivity and permeability in a wide range of frequencies and temperature. The disadvantages of this experimental device lie on the presence of probe resonances and higher modes above the cut-off frequency, which limit the use of this probe approximately between 1MHz-1GHz. A possible way to improve the investigable frequency range is to reduce the transversal dimension and the length of the probe moving the resonances and the cutoff to higher frequencies. However, a drastic reduction of the cage-coaxial cell determines the investigation of a small volume which could not be fully representative of the bulk electromagnetic properties of the geo-materials.

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