20th IMEKO TC4 International Symposium and
18th International Workshop on ADC Modelling and Testing
Research on Electric and Electronic Measurement for the Economic Upturn
Benevento, Italy, September 15-17, 2014

Piecewise affine virtual sensor: a case study - estimation of stepping motor current from long distances

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Abstract –

In this paper we propose the application of a piecewise affine virtual sensor for the estimation of the motor-side current of hybrid stepper motors, which actuate the LHC (Large Hadron Collider) collimators at CERN. The estimation is performed starting from measurements of the current in the driver, which is connected to the motor by a 720 m long cable. Measured current is therefore affected by noise and ringing phenomena. The proposed method does not require a model of the cable, since it is only based on measured data. A circuit architecture suitable for FPGA implementation has been designed and the effects of fixed-point representation of data are analyzed.

I. INTRODUCTION

The LHC (Large Hadron Collider) is a circular particle accelerator which, as all machines of this kind, needs a collimation system to block particles flying off their trajectories. These particles are highly energetic and if they are not properly collected they can seriously damage the accelerator. A collimation system is the module that collects these potentially harmful particles. It comprises some moving parts that need to be actuated with high precision inside the tunnel. Hybrid stepper motors are often used as the actuators in these applications because of their high positioning repeatability and open loop control [1]. These motors, and their electronic drives, are subject to a number of requirements that are relatively unique to accelerators. Since the environment surrounding the motors is highly radioactive and driver electronics are damaged by this radioactivity, the drivers are placed in radiation-safe zones at a distance of up to 1 km from the motors. They must therefore be connected to the drives via long cables. Figure 1 illustrates the connection of a 2 phase hybrid stepper motor to its driver via a long cable.

Pulse Width Modulated (PWM) control voltages are used to increase power efficiency; however, they generate significant electromagnetic interference (EMI) emissions, which can affect neighbouring electronics. High frequency PWM chopping frequencies must therefore be used to shift the emissions to higher frequencies. These high frequency voltage signals, nonetheless, cause the long cables to act as transmission lines, and produce a ringing phenomenon in the currents on the driver-side of the cable, the only side where measurements are possible. Figure 2 shows a comparison of the current in the driver-cable-motor circuit for a single motor phase.

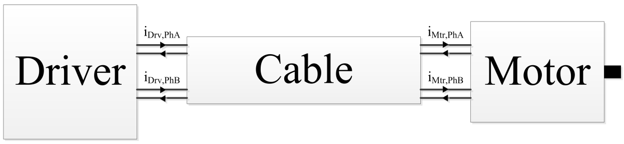

Fig. 1. Illustration of the connection of a 2 phase hybrid stepper motor to its driver via a long cable, with the corresponding phase currents.

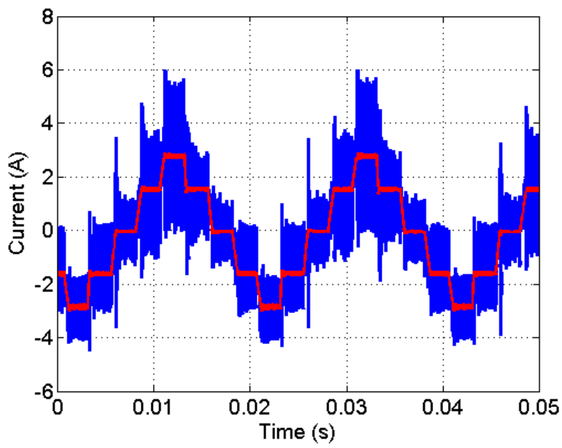

Fig. 2. Comparison of the current in the driver-cable-motor circuit for a single motor phase: Driver-side (blue) and Motor-side (red).

Good motor positioning repeatability is of great importance. It is thus necessary to have real-time knowledge of

ISBN-14: 978-92-990073-2-7

the motor's position, in order that compensatory action can be taken to correct any misalignments (i.e. steps losses). Radiation-hard resolvers are used to measure the motor position and to detect lost steps. It is, nonetheless, desirable to have sensor redundancy. Additionally, even if the stepping motors work at nominal torque, chosen by design to be at least twice the nominal load torque, having an estimate of the real load torque can be useful to warn about mechanical degradation.

A sensorless driver, based on an extended Kalman Filter (EKF), which can work with arbitrarily long power cables between the driver and motor has been developed in [2]. Using exclusively the motor model in the EKF means that the algorithm's inputs and measurements are the motor-side voltages and currents, respectively. However, as previously stated, these signals are not directly available in ordinary operation since measurements can only be made on the drive side of the cable. The approach proposed in [2] is to use cable-length-adaptive estimators on the drive-side data. In particular a cable model is used to obtain a transfer function relating the motor-side current to the drive-side current. The current estimation is therefore performed by means of a digital filter whose coefficients can be easily adapted to the used cable.

When the model is not known or it is not sufficiently accurate, it is possible to estimate a quantity starting from measurements of other measurable variables related to that quantity. We refer to a function of past inputs and measured outputs of a system as *virtual sensor*. The virtual sensor can be used to estimate an unmeasurable system output, without needing the knowledge of a model. A procedure for the design of a virtual sensor is described in [3], which relies on choosing a suitable set of basis functions, so that the resulting virtual sensor satisfies the assumptions required to apply the theoretical results in [4]. In [5], [6] and [7], PWAS functions (i.e. PieceWise Affine functions defined over a Simplicial partition) have been proposed as basis functions for the design of virtual sensors. The main advantage of using PWAS functions is that they can be implemented very efficiently in digital circuits such as FPGAs, providing low power consumption, fast response times, and, at least for high-volume applications, low cost. Moreover, convergence and optimality properties of the general approach [3] are maintained.

In [5] a single PWAS function is used to obtain the estimate (*standard direct virtual sensor*). If a relatively large number of inputs or measurable outputs is available, or if a large number of past data are used, the exponential increase of the complexity (*curse of dimensionality*) makes the approach impractical. An alternative solution has been proposed instead in [6] and [7], that leads to a complexity reduction. The resulting *reduced-complexity direct virtual sensor* is expressed as the sum of lower-dimensional PWAS functions instead of using a single higher-dimensional PWAS function.

The method used for estimation of the motor-side currents in [2] is based on an approximation of a first principles model of the dynamic system relating the drive-side current to the motor-side current. It therefore requires multiple, time intensive steps of modeling, parameter estimation and approximation to produce a real-time implementable, reduced-order estimator.

In this paper we propose the application of a PWAS reduced-complexity virtual sensor for the estimation of the motor-side current starting from measurements of the drive-side current. A circuit architecture suitable for FPGA implementation has been also designed and simulation results considering fixed-point data representation effects are shown.

II. PWAS VIRTUAL SENSORS

Consider the following nonlinear discrete-time dynamical model:

$$\begin{aligned} x_{k+1} &= g(x_k, u_k) \\ y_k &= h_y(x_k) \\ z_k &= h_z(x_k) \end{aligned}$$

where $x \in \mathbb{R}^{n_x}$ is the state vector, $u \in \mathbb{R}^{n_u}$ is the exogenous input vector of manipulated variables, $y \in \mathbb{R}^{n_y}$ is the vector of measurable outputs, and k denotes the discrete-time instant. Vector $z \in \mathbb{R}^{n_z}$ collects a set of variables to be estimated. We assume that the vector z_k can be measured by a *real sensor* for $k = 0, \ldots, K-1$. A portion of these measurements, from $k = 0$ to $k = K_t - 1$, $K_t < K$, is considered as a *training set* and is used to design the virtual sensor, which will operate without measuring z_k; the remaining data, from $k = K_t$ to $k = K - 1$ is used as a *validation set* to verify the estimation capabilities of the virtual sensor. We aim to construct a virtual sensor that estimates z_k for $k \geq K$, when the real sensor is no more available. The functions $g(\cdot,\cdot) : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_x}$, $h_y(\cdot) : \mathbb{R}^{n_x} \to \mathbb{R}^{n_y}$, and $h_z(\cdot) : \mathbb{R}^{n_x} \to \mathbb{R}^{n_z}$ are assumed to be unknown. For the sake of simplicity, we assume $n_z = 1$; non-scalar z can be easily estimated component-wise.

Since the actual variables y, u and z at past time instants are not available, the noisy measurements of them are assumed to be

$$\begin{aligned} \tilde{u}_k &= u_k + \eta_{u,k} \quad k \geq 0 \\ \tilde{y}_k &= y_k + \eta_{y,k} \quad k \geq 0 \\ \tilde{z}_k &= z_k + \eta_{z,k} \quad 0 \leq k < K \end{aligned}$$

where η_u, η_y, and η_z are unmeasured stochastic variables. For given values of M_u, M_y and M_z, the inputs of the virtual sensor will be noisy sequences of measurements of

y and u, and a vector of past values of $\hat{z}$, namely

$$\begin{aligned}\tilde{U}_k &\triangleq \left[\begin{array}{cccc}\tilde{u}^T_{k-M_u+1} & \tilde{u}^T_{k-M_u+2} & \cdots & \tilde{u}^T_k\end{array}\right]^T \\ \tilde{Y}_k &\triangleq \left[\begin{array}{cccc}\tilde{y}^T_{k-M_y+1} & \tilde{y}^T_{k-M_y+2} & \cdots & \tilde{y}^T_k\end{array}\right]^T \\ \hat{Z}_k &\triangleq \left[\begin{array}{cccc}\hat{z}^T_{k-M_z} & \hat{z}^T_{k-M_z+1} & \cdots & \hat{z}^T_{k-1}\end{array}\right]^T\end{aligned}$$

The values of M_u, M_y, M_z are generally considered as tuning parameters.

A. *Standard virtual sensor*

The standard virtual sensor is obtained by estimating z_k in the following way:

$$\hat{z}_k = f_{PWAS}(\tilde{U}_k, \tilde{Y}_k, \hat{Z}_k) \tag{1}$$

f_{PWAS} being a PWA function defined over a simplicial partition made up of N_s vertices and represented through the α-basis, i.e.,

$$f_{PWAS}(x) = \sum_{i=1}^{N_s} w_i \alpha_i(x) \tag{2}$$

By denoting as v_j, $j = 1, \ldots, N_s$, the vertices of the simplicial partition, functions $\alpha_i(x)$ are PWAS functions such that

$$\begin{aligned}\alpha_i(v_j) &= 1, \quad \text{if } i = j \\ \alpha_i(v_j) &= 0, \quad \text{if } i \neq j\end{aligned}$$

Once the basis functions and the simplicial partition are selected, weights w_i uniquely define f_{PWAS} and, in case the α-basis is used, they correspond to the value of the function in the partition vertices, i.e., $w_i = f_{PWAS}(v_i)$. A detailed discussion about PWAS functions can be found in [8].

Weights vector $w = [w_1 \ldots w_{N_s}]^T$ is obtained by solving the following optimization problem:

$$\min_w \left\{ \sum_{k=M}^{K_t-1} \left[\tilde{z}_k - f_{PWAS}(\tilde{U}_k, \tilde{Y}_k, \hat{Z}_k)\right]^2 + \sigma w^T \Gamma w \right\} \tag{3}$$

K_t being the number of elements in the training set and σ the Tikhonov regularization parameter and $M = \max(M_u, M_y, M_z)$.

The first term can be reformulated as a quadratic function of w and takes into account the square error between the value of the PWAS function (i.e., the estimated data) and the actual data $\tilde{z}_k$. The second term performs a Tikhonov regularization that depends on the structure of Γ. In the simplest case, $\Gamma = I$ provides the zero-order Tikhonov regularization. First-order (or higher order) Tikhonov regularizations can be obtained alternatively by considering the gradient of the PWAS function f_{PWAS} (or higher order derivatives) in constructing Γ. The choice of the regularization parameter σ is quite critical since it can highly influence the performances of the virtual sensor. A low value may lead to an ill-conditioned problem, i.e. the solution is sensitive to small changes in data. A high value may lead to an inaccurate estimate of the unmeasurable output.

The regularized least squares problem (3) can be recast as an unconstrained quadratic programming (QP) problem in the form:

$$\min_w \; w^T H w + \frac{1}{2} f^T w \tag{4}$$

A rigorous convergence analysis of the standard virtual sensor is reported in [5].

B. *Reduced complexity virtual sensor*

The reduced-complexity virtual sensor approach expresses the estimate $\hat{z}$ as a sum of lower-dimensional PWAS functions, in order to mitigate the effects of the curse of dimensionality, which prevents the applicability of the standard virtual sensor in many applications. For the sake of compactness, henceforth the input of the virtual sensor is referred to as

$$\Xi_k \triangleq \left[\begin{array}{ccc}\tilde{U}^T_k & \tilde{Y}^T_k & \hat{Z}^T_k\end{array}\right]^T \in \mathbb{R}^{n_\xi} \tag{5}$$

where $n_\xi \triangleq M_u n_u + M_y n_y + M_z$. Assume to split vector Ξ into $\nu \in \mathbb{N}$ subsets Ξ^1, Ξ^2, ..., Ξ^ν, such that all elements of Ξ are included in one and only one of these subsets. Each subset Ξ^j ($j = 1, ..., \nu$) has dimension equal to n_j, such that $1 \leq n_j \leq n_\xi$, and $n_1 + n_2 + ... + n_\nu = n_\xi$. The n_j elements of each Ξ^j are denoted as $\xi_{j,1}, \xi_{j,2}, ..., \xi_{j,n_j}$. The proposed reduced-complexity virtual sensor is defined through a sum of PWAS functions f^j_{PWAS}, $j = 1, \ldots, \nu$, each being the weighted sum of N_j PWAS basis functions:

$$\hat{z}_k = f_{PWAS}(\Xi_k) = \sum_{j=1}^{\nu} f^j_{PWAS}(\Xi^j_k) = \sum_{j=1}^{\nu} \sum_{i=1}^{N_j} w_{j,i} \alpha_{j,i}(\Xi^j_k) \tag{6}$$

where $f_{PWAS} : \mathbb{R}^{n_\xi} \to \mathbb{R}$ (for fixed w), $\alpha_{j,i}$ denote the i-th α-basis of the j-th PWAS function and N_j is the number of basis functions in each domain Ξ^j. Also,

$$w \triangleq \left[\, w_{1,1} \cdots w_{1,N_1} \; w_{2,1} \cdots w_{2,N_2} \cdots w_{\nu,1} \cdots w_{\nu,N_\nu} \right]^T \tag{7}$$

The vector of parameters w (which determines the shape of f_{PWAS}) is obtained, as for the standard virtual sensor, by solving the least-squares problem

$$\min_w \left\{ \sum_{k=M}^{K_t-1} [\tilde{z}_k - f_{PWAS}(\Xi_k)]^2 + \sigma w^T \Gamma w \right\} \tag{8}$$

The regularized least squares problem (8) can be recast again as an unconstrained QP problem in the form (4). A rigorous convergence analysis of the reduced-complexity virtual sensor is reported in [7].

III. RESULTS

We exploited the capabilities of MOBY-DIC Toolbox for MATLAB [9] to train a reduced-complexity PWAS virtual sensor for the estimation of the motor-side current, starting from measurements of the drive-side current. The same tool also allowed the automatic generation of VHDL files describing a circuit architecture for the FPGA implementation of the sensor. The architecture, described in detail in [7], has a latency of 8 clock cycles (i.e., $160ns$ at $50MHz$).

In our case we have $n_u = 0$, $n_y = 1$ and $n_z = 1$, in particular y represents the drive-side current and z the motor-side current. We designed the sensor by heuristically setting $M_y = 1$ and $M_z = 4$. The domain of the PWAS functions constituting the virtual sensor has been partitioned by using 3 subdivisions per dimension.

We employed a dataset made up of 500000 samples of the drive-side current and the corresponding motor-side current, sampled at 500 kS/s. The first 350000 samples (70% of the data) have been used as a training set, to derive the weights w. The remaining 150000 samples have been used to validate the sensor.

Figures 3 and 4 show the measured motor-side current of the validation dataset (blue), the estimated current in MATLAB double precision (red), and the estimation obtained by VHDL simulation in fixed-point precision (black).

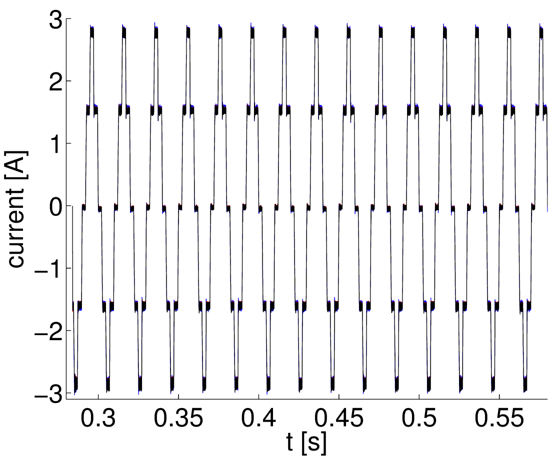

Fig. 3. Motor-side current: measured (blue), estimated in double precision (red), estimated in fixed precision (black).

The root mean squared (RMS) estimation error obtained in MATLAB is 0.0546 A and the one obtained in fixed-point precision is 0.0564 A, whilst the overall current waveform has an RMS value of 2 A.

The motor-side estimated current obtained with the estimator in [2] in MATLAB double precision is shown in Figure 5 for comparison. The RMS estimation error is 0.0433 A. Despite being slightly lower than that obtained

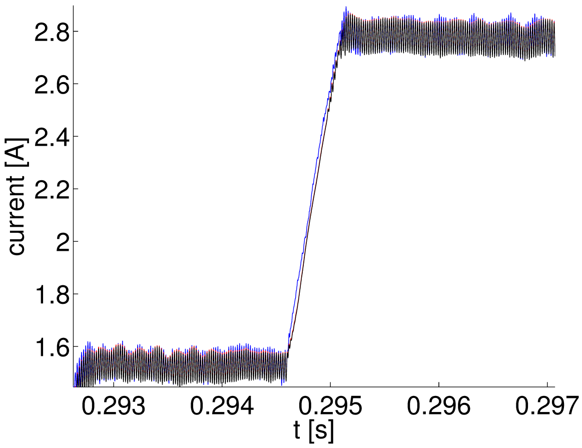

Fig. 4. Zoom of Fig. 3

with proposed method, it requires significantly more effort to obtain the estimator. Depending on the application, this extra effort may not be worth the small increase in estimation precision.

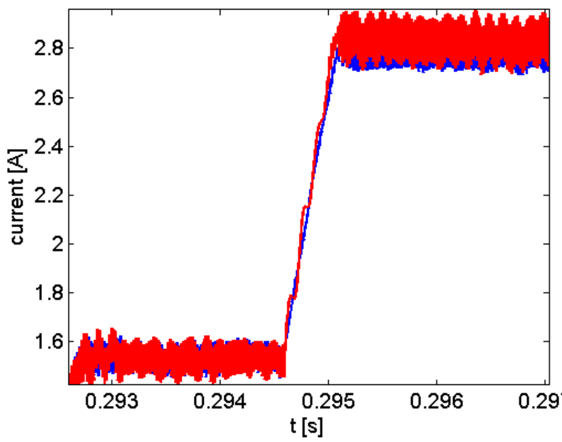

Fig. 5. Motor-side measured current (blue) and current estimated with the estimator in [2] (red).

IV. CONCLUSIONS

We designed a piecewise-affine virtual sensor for the estimation of motor-side current of hybrid stepper motors, starting from measurements of drive-side currents through a 720 meters long cable.

A circuit architecture has also been designed for the implementation of the virtual sensor. The proposed solution exhibits performances comparable with the ones obtained by standard model-based estimators, but results in a much faster and simpler design, since the identification of a model is not required.

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