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Towards a harmonized treatment of dynamic metrology

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Abstract – **Dynamic metrology is a topic of growing importance at NMIs and in industry, and it is considered in an increasing number of application areas. Guidance documents, such as the *Guide to the Expression of Uncertainty in Measurement* (GUM) [1] and the *International Vocabulary of Metrology* (VIM) [2], are currently not addressing dynamic measurements, and there is an increasing need for a harmonized treatment of dynamic metrology. To this end, we suggest a generic and versatile approach for the mathematical and statistical modeling of dynamic measurements, which also provides a basis for a harmonized vocabulary.**

In its literal definition, a dynamic measurement is one where at least one of the involved quantities is time-dependent. Many different physical quantities and metrological applications fall under or are related to the category *dynamic metrology*, see, for instance, [3, 4] and references therein. When the variable time in the definition is replaced by an unspecified independent quantity this category also contains applications from, e.g., spectrometry and coordinate measurements [6, 7].

Currently, there is a lack of documentary standards for dynamic measurements and there is an increasing need for their harmonized treatment [4]. However, in dynamic metrology currently not even the vocabulary is harmonized between different application areas. In this contribution we suggest a versatile generic mathematical and statistical modeling approach that contains the majority of currently applied approaches for dynamic measurements, see section (i.). In addition, the proposed modeling approach is a natural consistent extension of corresponding static models and it provides a basis for the definition of a generic vocabulary [5]. The proposed models for mathematical and statistical modeling are well established in signal processing system theory and statistics, respectively. Due to their generality, the proposed modeling approaches provide the opportunity to identify generic challenges in dynamic metrology that are present irrespective of the underlying physical nature of the experiment, see section (iii.).

I. MATHEMATICAL MODELING

The GUM applies measurement models and finite dimensional multivariate quantities for the propagation of uncertainties. These models, appropriate for static measurements, are not applicable for dynamic metrology where the value of a quantity varies continuously with time. It has been shown that a natural extension of the GUM framework to metrology on function spaces can be obtained by replacing the probability density functions (PDFs) associated with the incomplete knowledge about the value of a quantity by stochastic processes [3]. Moreover, for specific cases algebraic models in the analysis of dynamic measurements are available that allow the direct application of the GUM framework for the evaluation of uncertainties [7, 9]. In order to derive a viable extension of the GUM framework to dynamic metrology, a generic and harmonized treatment of dynamic measurements is necessary. To this end, system theory of signal processing provides an established framework to cover mathematical modeling of dynamic measurements, whereas time series analysis provides powerful techniques and models for the statistical analysis of dynamic measurements. In the following we briefly outline their application to dynamic metrology.

A. Generic Mathematical Model

A state-space model of a measurement system is defined by its state equation

$$\dot{\mathbf{Z}}(t) = \mathbf{f}(\mathbf{Z}(t), \mathbf{U}(t), t) \tag{1}$$

and its observation or measurement equation

$$\mathbf{Y}(t) = \mathbf{g}(\mathbf{Z}(t), \mathbf{U}(t), t). \tag{2}$$

The quantity $\mathbf{U}$ denotes the, possibly multivariate, input to the measurement system, $\mathbf{Z}$ the system's state variable, and $\mathbf{f}$ and $\mathbf{g}$ the state function and measurement function, respectively. Both, the state equation (1) and the measurement equation (2) can be linear or non-linear, and continuous or discrete in time. These kinds of models also cover so called single-input-single-output (SISO) as well as multiple-input-multiple-output (MIMO) systems. Hence, a dynamic measurement can be modeled using a state-space system model, from a single-sensor measurement up to a complex network of thousands of sensors. Note that this model also covers static measurements by

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dropping the time dependence and completely different areas, such as spectral convolution or dynamic coordinate metrology by replacing the variable time with a corresponding quantity.

In some cases, special simplifications or transformations of the state-space model can be carried out to simplify notation. For example, in some cases the state-space model can be transformed to a rational transfer function model

$$H(s) = \frac{\sum_{k=0}^{N_b} B_k s^k}{\sum_{k=1}^{N_a} A_k s^k}. \tag{3}$$

This also includes modeling of the measurement system in terms of its frequency response $H(j\omega)$ or a digital filter representation by utilizing the z-transform of $H(s)$.

B. *Examples*

In the following we give some examples of the application of the state space model for different types of dynamic measurements.

Dynamic acceleration

The input-output relation of many accelerometers can be modeled by a second-order ordinary differential equation (ODE)

$$\ddot{x}(t) + 2\delta\omega_0\dot{x}(t) + \omega_0^2 x(t) = \rho a(t). \tag{4}$$

In the state-space model approach this becomes $\mathbf{f}(t) = \big(Z_2(t), -2\delta\omega_0 Z_2(t) - \omega_0^2 Z_1(t) + \rho A(t)\big)$ and hence, $U(t)$ is the input acceleration $A(t)$. Note that the second-order ODE has been transformed to a system of two first-order ODEs. In a calibration, the ODE parameters δ, ω_0 and ρ have to be determined.

Sampling oscilloscope

Dynamic calibration of high-speed sampling oscilloscopes is typically realized by means of their impulse response [10]. In the state-space model approach this is related to a digital filter representation of the state-space system (1)-(2). The impulse response can then be viewed as a FIR-type filter model of the measurement system.

Single sensor in complex system

Utilization of a dynamically calibrated sensor in a complex system, such as, e.g., a force sensor in a fatigue testing machine, can be modeled by the state-space approach as follows. In a first step, a state-space model of the sensor itself is derived in the calibration step. This model is then incorporated into a larger state-space model of the fatigue testing machine.

Pressure sensor sensitive to vibrations

Typically, pressure sensors are built such that vibrations of the housing and other parts of the measurement system do not impact the sensor output signal. Then, the sensor model is of SISO type, similar as for the accelerometer above. However, when this sealing is not sufficient in a specific measurement setup (e.g., shock tubes), a MISO type system model might be more appropriate [13]. In the state-space approach the system input $\mathbf{U}(t)$ then consists of the input pressure and acceleration of the sensor housing.

C. *Relation to GUM*

Assuming a measurement is modeled by the state-space model (1)-(2), the *indication value* in the measurement is $\mathbf{Y}(t)$ and the *measurand* is the system input $\mathbf{U}(t)$. A dynamic calibration is thus an identification of the functions $\mathbf{f}$ and $\mathbf{g}$, typically in terms of parameter identification. Estimation of the measurand requires to solve the state-space equation model for the function $\mathbf{U}(t)$. This is an inverse problem, which is generally ill-posed. That is, small changes in the indicated values $\mathbf{Y}(t)$ result in large changes in an estimated measurand. Hence, some kind of regularization is required in order to render the estimation stable. The general ill-posedness of input estimation in dynamic measurements is typically not found in static measurements and poses a significant challenge. For linear time-invariant models this has been accomplished, for instance, by means of a digital filter representation of the inverse model [9] or the utilization of the (discrete) frequency response [10]. In both cases, regularization of the inverse problem has been realized by means of low-pass filtering. Note that, in a Bayesian approach to input estimation an inversion of the system model is not necessary explicitly and regularization can be achieved by using appropriate proper prior distributions [7]. For an approach in line with GUM, we assume that the inverted system is also modeled as a state-space model

$$\dot{\mathbf{Z}}(t) = \widetilde{\mathbf{f}}(\mathbf{Z}(t), \mathbf{Y}(t), t) \tag{5}$$

$$\mathbf{U}(t) = \widetilde{\mathbf{g}}(\mathbf{Z}(t), \mathbf{Y}(t), t). \tag{6}$$

The input quantities in the sense of the GUM are then the indicated values $\mathbf{Y}$, the (parameters of the) state equation function $\widetilde{\mathbf{f}}$ and the (parameters of the) measurement equation $\widetilde{\mathbf{g}}$. In general, this model is non-linear and requires linearization of (5) and (6), resulting in the linear state-space system

$$\dot{\mathbf{Z}}^{(l)}(t) = \mathbf{A}(t)\mathbf{Z}^{(l)}(t) + \mathbf{B}(t)\mathbf{Y}(t) \tag{7}$$

$$\mathbf{U}(t) = \mathbf{C}(t)\mathbf{Z}^{(l)}(t) + \mathbf{D}(t)\mathbf{Y}(t), \tag{8}$$

where $[\mathbf{A}]_{i,j} = \partial f_i/\partial Z_j$, $[\mathbf{B}]_{i,j} = \partial f_i/\partial Y_j$, $[\mathbf{C}]_{i,j} = \partial g_i/\partial Z_j$ and $[\mathbf{D}]_{i,j} = \partial g_i/\partial Y_j$. Finally, the simplest, but yet often applied, model in dynamic metrology is a linear time-invariant system model, which in state-space form becomes

$$\dot{\mathbf{Z}}(t) = \mathbf{AZ}(t) + \mathbf{BY}(t) \tag{9}$$

$$\mathbf{U}(t) = \mathbf{CZ}(t) + \mathbf{DY}(t). \tag{10}$$

Note that the GUM considers only quantities whose values can be represented by finite dimensional vectors. In dynamic measurements the value of the dynamic quantity is typically a continuous function, and thus requires a discretization or finite parametrization in order to be treated in line with GUM. In addition, (uncertain) knowledge about the state and measurement equation functions f and g has to be represented by a finite dimensional parametrization.

II. UNCERTAINTY IN DYNAMIC METROLOGY

The values of the dynamic quantities $U(t)$ and $Y(t)$ are continuous functions of time. A natural way to model uncertain knowledge about the values of a continuous function is a stochastic process with continuous trajectories. That is, for a continuous function $x(t) \in C([a,b],\mathbb{R})$ the state of knowledge can be modeled by a continuous-time stochastic process $X_t : [a,b] \times \Omega \to \mathbb{R}$ with probability space $(\Omega, \mathcal{M}, \mathbb{P})$, such that for *any* set of time samples $(t_1, \ldots, t_N)$ the corresponding probability measure induced by the process models the state of knowledge about the values of $x(t)$ at these time samples. The same holds true for multivariate functions with continuous time dependence. It has been shown that an uncertainty framework based on such a model of the state of knowledge provides a consistent extension of the GUM framework [3]. Thus, future guidelines on dynamic metrology may be based upon such a general framework from which discrete-time scenarios can be derived easily.

For a treatment of uncertainty completely in line with the existing GUM, discrete-time signals have to be considered. As the GUM only considers finite dimensional vectors as representation of the value(s) of a quantity, the dynamic quantities have to be considered either sequential in time with measurement model considered at each time instant individually or as a discretized signal on a finite interval.

A. *Sequential estimation*

The state-space model for the estimation of the measurand provides a sequential estimation. That is, at each time instant an estimate of the measurand at that particular time is obtained by using estimates from previous time instants. Thus, at each time step the measurand is a finite dimensional vector to which uncertainty can be assigned in line with the GUM. However, the values of a dynamic quantity typically show correlation between different time instants which has to be taken into account in the evaluation of uncertainty.

An often applied stochastic process model class in discrete time series analysis is the class of auto-correlated-moving-average (ARMA) processes [8]

$$\eta(t_n) = \sum_{k=0}^{N_b} b_k \varepsilon(t_n - k) + \sum_{k=1}^{N_a} a_k \eta(t_n - k) + \varepsilon(t_n) \quad (11)$$

with $\varepsilon(t)$ i.i.d. $\mathcal{N}(0,\sigma^2)$ with variance σ^2. As the value of the process at one time instant is calculated only from previous values it is sequential in time. Thus, ARMA models provide a means of representing correlation in a stochastic process by a sequential model. For the identification and utilization of ARMA processes a large amount of literature and many established versatile methods are available. It is worth noting that, similar to the state-space model for the mathematical description of a dynamic measurement, the techniques of time series analysis can be employed not only for time-dependent quantities by replacing time with any other independent quantity. In the multivariate case, the model (11) can be written as a linear time-invariant state-space model with input being the noise process ε.

The ARMA model provides a means of representing uncertain knowledge about discrete-time signals and is a viable tool for modeling stochastic measurement noise. Difficulties arise when the uncertain knowledge contains non-stationary components. To this end, under certain conditions so called ARIMA models can be employed. However, in practice the uncertainty associated with the dynamic quantities contains a mixture of (non-stationary) stochastic, deterministic and unknown components, and a general stochastic model is thus still to be found.

Uncertainty associated with the knowledge about the measurement system, i.e. $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$, is available in terms of an associated probability distribution [1]. Hence, the uncertainty associated with the discrete-time sequence U has sequential and non-sequential contributions. In some special cases at least the (co-)variances can be evaluated sequentially by techniques related to the well-known Kalman filter [11]. However, a general framework for the evaluation and representation of infinite sequences U is a topic of future research.

B. *Estimation on finite intervals*

When the discretized signals are considered on a finite interval then their values can be represented by finite dimensional vectors. Hence, uncertainty can be associated by means of multivariate probability distributions. Then the GUM framework for evaluation and interpretation of uncertainties is readily applicable [3]. A practical problem of this approach is the dimensionality of the dynamic quantities. That is, although the mathematical formulation of the corresponding propagation of uncertainties is straight-forward, its practical implementation often results in numerical and computational difficulties [12].

In addition, a practical problem regarding the evaluation of the uncertainty in the measurement occurs when uncertainty is to be determined from repeated measurements. That is, if the covariance matrix associated with a random vector is to be determined from repeated measurements then the number of these measurements has to be larger than the dimension of the vector. However, in practice the dimensionality of the discretized dynamic quantities is much higher than the number of repeated measurements that can be carried out in a reasonable amount of time. To this end, parametric approaches such as ARMA models may be employed or some kind of approximate representation of the covariances has to be determined.

III. GENERIC CHALLENGES IN DYNAMIC METROLOGY

The generic description of the mathematical and statistical model of dynamic measurements provides a basis for identifying generic challenges in metrology. The mathematical abstraction of these challenges in turn allows the development of generic solutions to these challenges irrespective of the underlying physical experiment. Challenges for dynamic metrology regarding uncertainty evaluation are, for instance, to combine expert knowledge of imperfections of measured signals with classic time series analysis, the efficient and reliable generic propagation of uncertainties and the reliable quantification of the uncertainty contribution of regularizing the inverse problem.

In applications in dynamic metrology signal processing tasks, such as, e.g., FFT, windowing, sub-sampling, filtering, are applied routinely and all major software environments provide corresponding tools. However, the uncertainty associated with these tasks is often neglected due to the lack of established standards and methodologies. Identification of correlations between different time instants of the dynamic quantities is in general challenging due to the high dimension of these measurements compared to their static counterparts. For instance, the measurement of a time-varying force signal may contain several thousand time samples whereas the force at a specific time is one-dimensional. Related to the high-dimensionality of dynamic measurements is the challenge of transferring measurement results in terms of estimate and uncertainties, because the associated covariance matrix is of high dimension as well.

IV. CONCLUSIONS

Dynamic metrology is a still-growing area in metrology and many investigations and developments are carried out at NMIs and industry. At present, there is a lack of documentary standards and guidelines. To this end, we proposed a generic mathematical and statistical framework which covers most currently applied models. We also pointed to a number of generic challenges in dynamic metrology. In some areas, such as waveform metrology for high-speed electronics and dynamic measurement of mechanical quantities, first attempts have been made to tackle these challenges. The generic framework proposed here puts these and other developments in this area under a common mathematical and statistical description. This allows the identification of synergies and may serve as a basis for future standardizations towards a harmonized dynamic metrology. In addition, a generic framework provides a basis for the development of a common vocabulary, which is a prerequisite for a generic documentary standards.

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