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Challenges in measuring the longitudinal field profile of a magnet by a vibrating wire

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***Abstract* – A vibrating wire is used to measure the longitudinal profile of the magnetic flux density in the aperture of a non-periodical structure such as a quadrupole accelerator magnet. Limitations in terms of measurement repeatability and uncertainty are observed. The possible causes are investigated by experimental characterization of the measurement system. Uncertainty sources are identified and particular cases are reported in which the mathematical assumptions are no longer valid. The challenges are discussed and measures are proposed to improve the metrological performance.**

I. INTRODUCTION

A vibrating-wire system for magnetic measurements uses a conducting wire as a sensing element of magnetic fields. This technique has been proposed by A. Temnykh in [1] and been employed for measuring the position of the magnetic axis of accelerator magnets with resolutions in the micrometer range [2]. In particular, the axes of quadrupole [2, 3] and solenoid [4] magnets are determined by the wire position in the magnet aperture, where the oscillation amplitude takes its minimum. The precision is affected by the relation of the measured axis to the alignment target of the magnet [5]. These limitations have to be overcome to meet the tight requirements of the *Particle Accelerator Components Alignment and Metrology to the Nanometre scale* (PACMAN) project [6][1], where a vibrating wire has to be used as a reference for the alignment of the components of the upcoming *Compact Linear Collider* (CLIC) [7].

Moreover, vibrating wires can be used to measure the longitudinal profile of the transverse field along the wire. This is done by supplying the wire by an alternating current at its resonance frequencies and measuring the displacement caused by the Lorentz force. Knowing the amplitude and the phase of the vibrations at various resonance frequencies, the longitudinal field distribution is reconstructed by an inverse Fourier transform of the wire response [1]. The potential of the vibrating-wire technique in measuring field errors in periodic structures, such as undulators, and in predicting beam trajectories has been studied in [8]. The obtained results have been found to be in good agreement with Hall probe mappings.

In this work the vibrating-wire technique is applied to reconstruct the longitudinal profile of a non-periodic magnetic field structure, consisting of a normal-conducting, reference quadrupole magnet. Technical limitations are highlighted and arising challenges are discussed in order to improve the metrological performance.

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II. THEORY

The mathematical model of the vibrating-wire is derived under the assumptions of plane motion, uniform and constant tension, small wire deflections, constant length, and uniform mass distribution [2, 5]. In such a case, the model yields a second-order linear partial differential equation, whose solution is made up of infinite vibration modes, each contributing to the final response with its own amplitude and phase, as a function of the excitation frequency. Furthermore, each mode can be isolated by setting the excitation frequency to the corresponding resonance value ω_m so that the overtones become negligible[2] and the oscillation amplitude and phase are described by the following equations

$$\delta_m(\omega) \simeq \frac{I_0}{\rho} \frac{\mathcal{C}_m}{\sqrt{(\omega_m^2 - \omega^2)^2 + (\beta\omega)^2}} Y_m(z_0)\,, \quad (1)$$

$$\varphi_m(\omega) = \begin{cases} \arctan\left(\frac{\beta\omega}{\omega_m^2 - \omega^2}\right), & \text{if} \quad \mathcal{C}_m Y_m(z_0) > 0 \\ \pi + \arctan\left(\frac{\beta\omega}{\omega_m^2 - \omega^2}\right), & \text{if} \quad \mathcal{C}_m Y_m(z_0) < 0\,, \end{cases} \quad (2)$$

with

$$Y_m(z) := \sin\left(\frac{m\pi}{L} z\right), \quad (3)$$

$$q_m(t, \omega) := \sin(\omega t - \varphi_m(\omega))\,, \quad (4)$$

where ρ is the linear mass density, $[\rho] = 1\,\mathrm{kg\,m^{-1}}$, β is the damping coefficient normalized with respect to ρ, $[\beta] = 1\ \mathrm{s^{-1}}$, ω is the excitation frequency $[\omega] = 1\ \mathrm{rad\,s^{-1}}$.

[2]Numerical simulations have shown that the overtones are at least three order of magnitude smaller than the main tone.

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The resonance frequency ω_m is given by

$$\omega_m = \frac{m\pi}{L}\sqrt{\frac{T}{\rho}}\,, \tag{5}$$

where T is the mechanical tension in the wire, $[T] = 1\,\mathrm{N}$, L the length of the wire, $[L] = 1\,\mathrm{m}$.

The *longitudinal field coefficients* $\mathcal{C}_m$ are derived under the hypothesis of expansibility into Fourier sine series of the magnetic flux density B_n, $[B_\mathrm{n}] = 1$ T and are given by

$$\mathcal{C}_m := \frac{2}{L}\int_0^L B_\mathrm{n}(z)\sin\left(\frac{m\pi}{L}z\right)\mathrm{d}z\,. \tag{6}$$

III. THE MEASUREMENT METHOD

The reconstruction of the longitudinal field profile is accomplished by measuring the local oscillation amplitudes and phases, when the wire is fed by an alternating current at its mechanical resonance frequencies. Differently from what is done in [1], amplitude and phase are fitted separately to the mathematical model.

The measurement method of the longitudinal profile consists of:

- measuring the frequency response of the wire,
- fitting the wire frequency response to Eq. (1) in order to obtain the longitudinal field coefficients $\mathcal{C}_m$,
- calculating the longitudinal field profile from the inverse Fourier transform.

The frequency response of the wire is measured in several points in the neighborhood of the resonance frequency and fitted to the second order mathematical expression

$$\delta_m(\omega) = \frac{a_m}{\sqrt{(b_m^2 - \omega^2)^2 + (c_m\omega)^2}}\,. \tag{7}$$

The unknown parameters a_m, b_m, c_m can be identified by the least-square method. The phase difference ϕ between wire vibration and excitation current function, which can be $-\pi/2$ or $\pi/2$, determines if a_m is positive or negative. The longitudinal field coefficients are finally obtained by comparing Eqs. (7) and (1), which yields

$$\mathcal{C}_m = \frac{\rho}{I_0 Y_m(z_0)}a_m\,. \tag{8}$$

The longitudinal field profile is then established by an inverse Fourier transform.

IV. THE EXPERIMENTAL SETUP

In the presented study, the vibrating-wire technique was employed to measure the longitudinal field profile inside a reference magnet, i.e., a normal-conducting quadrupole magnet which was constructed for the LEP experiment at CERN.

A wire made of Cu-Be alloy, 125 μm in diameter, used for the experiment is kept taut by means of a stepping motor. Two phototransistors Sharp™ GP1S094HCZ0F, orthogonally mounted at the stages, transduce the wire displacements in x and y directions into a voltage signal. This signal is acquired by means of a 18-bits acquisition system NI6289 from National Instruments™ and using a sampling rate of 20 kS/s, 10 times more the maximum wire frequency (2 kHz). Two pairs of high-precision, linear displacement stages from Newport® are used to position the wire extremities. A motor controller Newport® ESP7000 is used to move the stages and acquire the position through a linear encoder with a precision of ± 0.1 μm. The current in the wire is provided by the AC generator Keithley® 6221. The control software is part of the Flexible Framework for Magnetic Measurements [9]. The data are processed and analyzed in Matlab®.

The wire is fed with a sinusoidal current whose amplitude is adjusted to fit properly with the linear region of the phototransistors. The wire excitation current is scanned through 20 resonance frequencies. Moreover, in the neighborhood of each resonance frequency, 11 measurements are done with steps of 0.15 Hz.

The mechanical properties of the setup are given in Tab. 1.

Wire length	L	1568 mm
Mass density	ρ	$1 \cdot 10^{-4}$ kg m^{-1}
Mechanical tension	T	8.264 N
Natural frequency	f_1	87.40 Hz

Table 1: Values of the mechanical properties of the wire used in the experimental setup.

V. EXPERIMENTAL RESULTS

Measurement results are presented and compared with a scan measurement using classical Hall probes.

The results demonstrate the frequency dependence of the damping coefficient, which points to a nonlinearity of the drag forces, despite the assumption of a constant value in the mathematical model.

The approximation error due to the field reconstruction with only 20 harmonics is 3%; compare the blue and red curves in Fig. 1). This approximation error was computed as a RMS difference of the measured and reconstructed profiles. Moreover, the reconstructed field coefficients and profile of the horizontal field component, averaged over three measurement repetitions, have a standard deviation of 2% with respect to the field peak. Results are shown in Fig. 1.

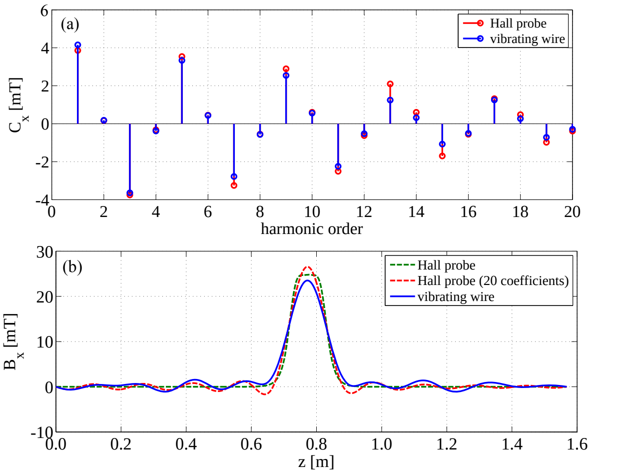

Figure 1: Reconstructed longitudinal field profile of the LEP-LPI-QS quadrupole magnet by the vibrating-wire compared to Hall probe measurement. (a) Longitudinal field coefficients. (b) Field profile.

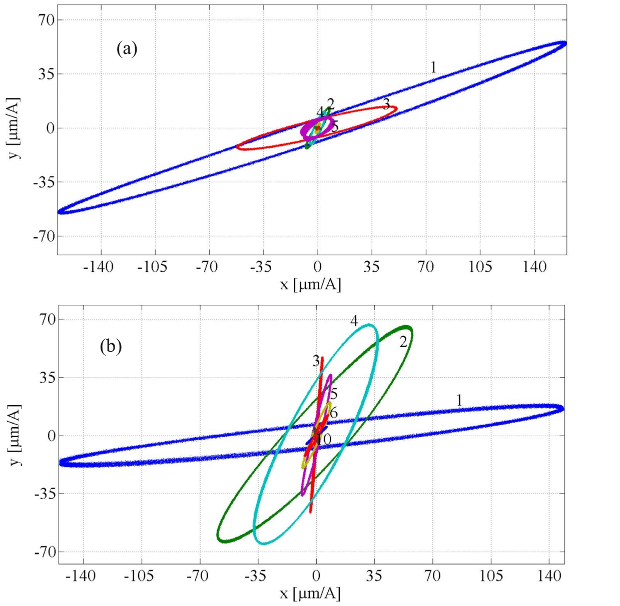

Figure 2: Trajectory of the stretched wire in the transverse plane when the first ten natural modes are excited and the tensioning motor is mounted. (a) Without tensioning motor. (b) Tensioning motor mounted.

VI. EXPERIMENTAL CHARACTERIZATION

The measurement of the longitudinal field profile is intrinsically limited by the bandwidth of the measurement system, which results in a maximum number of measurable vibration modes that are distinguishable from noise. In addition, the above reported experiment shows limitations on the measurement repeatability and accuracy, even though attention is restricted solely to the measured modes. In this section we describe the hunting of uncertainty sources which degrade the system performance.

A. *Plane motion*

The hypothesis of plane motion is no longer valid under certain conditions, for example, a 45 degree inclination of the field with respect to the ground plane. This is observed when the wire is forced to vibrate close to a magnetic pole in the setup described in section IV. In this case, the wire describes an elliptically polarized trajectory in the transverse plane rather than a straight line. This behavior of vibrating strings was already studied in [10]. In such a situation, the projection into x and y axes is no longer justified and the measurement procedure should be extended. On the other hand, the ellipses collapse to a straight line when the wire is forced to vibrate horizontally or vertically in the region between two magnetic poles.

B. *Background fields*

Experience has shown that the wire can be made to vibrate by background magnetic fields consisting of the Earth magnetic field, fringe fields arising from the tensioning motor, and possible environmental field distortions. The effect of the fringe field from the tensioning motor, which is mounted in proximity of the wire, can be seen by comparing Figs. 2 (a) and 2 (b): Fig. 2 (a) shows the trajectories in the xy plane when the motor is replaced by a simple weight fixed to one of the wire extremities. Fig. 2 (b) shows the same trajectories when the tensioning motor is put in place. The presence of the motor produces an amplification of the even modes and a rotation of the odd, but also the first mode is slightly changed. The oscillation amplitude of the high order modes is amplified by 2%-3%.

Even if the tensioning motor is not used, there are still some parasitic fringe fields (Fig. 2(a)). The direction of the main component corresponds well to the earth-magnetic field map in central Europe. The amplitude of the first-order mode oscillation is about 2% of the amplitude measured with the reference quadrupole mounted on the system. For the other resonances, the effect is below 1%.

C. *Nonlinearities and overtones*

The assumption of linearity implies that when the resonance condition is excited, the respective mode is dominant with respect to the overtones, which can therefore be neglected. In practice, however, also higher order modes are measured, highlighting the intrinsic nonlinearity of the system. Experiments have shown that if, for instance, the first mode is excited, then also the second mode is measured and its amplitude varies from 2% to 7% of the fundamental, depending on the system configuration. The other overtones are also present but with fast decreasing amplitude. The analysis of the current excitation signal on a broad frequency range (100 - 5000 Hz) excluded that such

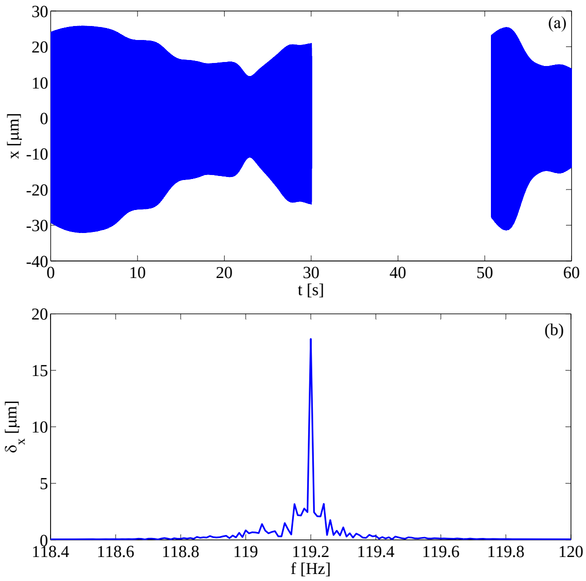

Figure 3: Steady-state modulation of vibrating-wire motion. (a) Time domain vibration. (b) Frequency domain oscillation amplitude.

overtones are coming from the signal generator.

D. Steady-state modulation

The assumptions under which the mathematical model is derived lead to a constant oscillation amplitude of the vibrating string in the steady state. This behavior is betrayed in practice as it is shown in Fig. 3 (a), where the wire vibration describes an amplitude modulated signal, with a variable modulation frequency. Indeed, the frequency spectrum of this signal, shown in Fig. 3 (b) contains two side bands of 0.5 Hz in width, in which higher order modes are present. This intermodulation depends on the excitation frequency; in particular it was observed that the effect is minimal when the excitation frequency corresponds to the resonance frequency (a residual fluctuation of 5% is measured). This hints that if it was possible to measure precisely the resonance frequency[3], the measurement of the wire vibration only at the resonance could be a better technique than fitting the steady state model. Nevertheless, scanning the frequency through the resonance allows small perturbations on the system to be detected, making the procedure more robust.

Moreover, the steady-state modulation is compatible with the scenario of a varying resonance frequency which in turn generates the amplitude change. This could be ascribed to non constant tension and/or length. Several author claim that tension does vary, even for very small deflections [11] which are 5-20 micro meters on about

[3]This can be done by sending to the wire a current unit impulse in order to excite the free vibration. The fundamental frequency is then measured from the spectrum of the recorded signal.

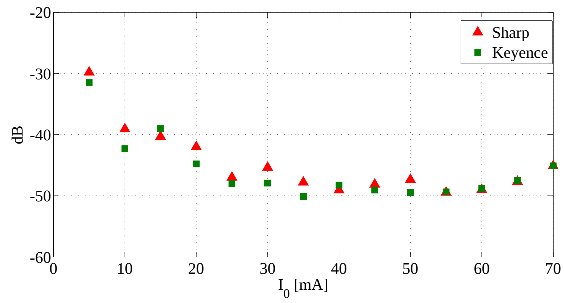

Figure 4: Total harmonic distortion of the Sharp® photodetectors and the Keyence® CCD sensor, as a function of the wire current amplitude.

1.5 meter wire length. In order to exclude other possible sources, the coupling of the wire bench with the ground vibration was investigated [12]. The signals acquired by the phototransistors were compared to measurement of ground motion by means of two geophones placed in the vicinity of the measurement bench. It was observed that no correlation exists between the signals from the two devices, therefore the source of the modulation must be considered intrinsic to the system.

E. Linearity of optical sensors

The advantage of the Sharp® phototransistors is their low price and their high sensitivity. However, the domain of linear voltage/displacement response is relatively small (40 μm, about 1 V) [4]. In order to exclude that measured overtones result as artifacts from a non-linear sensor response, the phototransistors were tested against a CCD sensor from Keyence®, which has a linear measurement domain of about 6 mm, but a lower Signal to Noise Ratio (SNR).

These tests showed that no harmonic distortion is introduced by the Sharp® photodetectors, evident from Fig. 4, where the total harmonic distortions for the Sharp and Keyence sensors are plotted against the current amplitude in the wire. Consequently, the overtones measured are indeed in the wire motion and are not introduced by the wire displacement sensors.

F. Damping

The damping coefficient describes a set of several loss mechanisms: viscous drag damping (air), internal damping, transfer of energy to other vibrating systems, electromagnetic energy loss (emission damping), effect of induced currents.

Experience has shown that damping due to these mechanisms varies with frequency. Also, by using the model of the viscous drag damping, one can see that over a frequency span for example of 200-10000 Hz, the damping coefficient should vary in a range wide 5 times the value at the fundamental frequency.

More work is needed to understand how far the linear damping model can be hold and where a more realistic, frequency depending model should be adopted.

VII. CONCLUSIONS

The vibrating-wire technique was employed to measure the longitudinal field profile in a reference quadrupole magnet. Limitations on the metrological performance were highlighted and critical aspects discussed. The reconstruction accuracy is limited by the mechanical bandwidth of the system, that limits the number of measurable coefficients. This approximation error is limited to 3%. It was shown that some of the assumptions underlying the linear mathematical model are not valid in practice. In particular, the plane motion does not hold when the wire vibrates in proximity of a magnetic pole, describing an elliptical polarized trajectory in the transverse plane to the wire direction. Intrinsic mechanical nonlinearities were detected by observing overtone excitation in resonance condition; the possibility of overtone excitation by external sources were excluded by performing test on the signal generator and on the optical measurement devices. A modulation of the steady-state wire oscillation was observed and characterized with respect to the excitation frequency. It was observed that this modulation is minimized when the excitation frequency matches the resonance.

The influence of background magnetic fields was also discussed. In particular, the excitation of the main component of the wire oscillation corresponds to the earth-magnetic field in central Europe. An influence of the fringe field coming from the electrical tensioning motor was also observed.

Future efforts have to be directed toward the compensation of deterministic error sources, the extension to a more sophisticated mathematical model, and the technical improvement of the measurement system.

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