

# An application to combine and compare measurement results expressed in terms of Random-Fuzzy Variables

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**Abstract** –In the last years, the Authors have proposed the use of the Random-Fuzzy Variables (RFVs) to represent measurements results. An RFV is an extension of a fuzzy variable, which is the natural variable in the mathematical possibility theory (in the same way the random variable is the natural variable in the probability theory).

The aim of this presentation is to show, through a remotely controlled Virtual Instrument (VI) (accessible via the web page <http://131.175.120.11:8000/RFVcalculator.html>), how RFVs are built, starting from the available information about the uncertainty contributions affecting the measurement results; how they are combined, considering also the possible correlations between the uncertainty contributions; and how they are compared.

## I. INTRODUCTION

The standard approach to deal with measurement uncertainty is strictly based on the mathematical probability theory [1, 2]. In the last years, however, the Authors and other researchers have proposed a new method to deal with uncertainty and measurement results, which is not based on the probability theory but on the possibility theory [3-9].

It is important to underline that this new approach represents a generalization of the standard one, both from the mathematical and the methodological point of view. From the mathematical point of view, both probability and possibility represent two particular cases of the more general mathematical theory of evidence [9]. From the methodological point of view, in the standard approach, the measurement results (together with their associated uncertainties) are represented by probability distribution functions (PDFs); while, within this new approach, the measurement results (together with their associated uncertainties) are represented by possibility distributions (PDs), and a PD represents a whole family of PDFs [8]. Moreover, this new approach encompasses all the basic definitions given in [1] about uncertainty, as, for instance, coverage intervals and confidence levels. Last but not least, this new approach allows to represent and

propagate correctly both random and systematic contributions to uncertainty.

In particular, each measurement result is represented by two PDs, which define a Random-Fuzzy Variable (RFV) [6-8]. The advantages of the RFVs are, briefly, the following.

- They represent, separately, the effects on the measurement result of the random contributions to uncertainty and the systematic ones.
- They readily provide all confidence intervals for all levels of confidence.
- They allow one to combine systematic contribution with each other, and random contributions with each other, with different fuzzy operators (different t-norms), according to their different nature and the different way these contributions combine.
- They allow one to compare measurements results, providing three credibility factors which can improve the decision making process.

## II. THE VI PROGRAM

An application has been developed to represent and combine measurement results (together with their associated uncertainties) in terms of RFVs. The program has been developed as a LabVIEW VI, because most measurement practitioners are familiar with its graphical interface, and it features an embedded web server that allows one to make the developed application readily available over the Internet.

The developed VI allows the user to select some common shapes for both the internal and external membership functions of two RFVs, and combine them according to a user-selected algebraic operation. The resulting RFV can be also compared with another user-selected RFV, thus showing the effectiveness of the RFV comparison method.

Fig. 1 shows an example. In the left part, two RFVs are built, choosing a rectangular PD for the internal part and a PD induced by a normal PDF for the random one [6-8]. Then, the two PDs are summed, after the choice of the correlation coefficients for both the systematic and the random contributions. In the right part, two RFVs are compared: two new RFVs can be considered, or one of

them can be the result of the previous operation. Only the external PD is shown in this case, since the whole uncertainty is considered in the comparison. Three coefficients are provided, showing the credibility that one variable is lower/equal/greater than the other one.

### III. DISCUSSION

The presented VI shows some advantages of the approach

based on the RFVs to deal with measurement uncertainty.

- Both random and systematic contributions to uncertainty can be represented and suitably combined.
- The combination can be always performed in an immediate way, since it is based on algebraic operations.

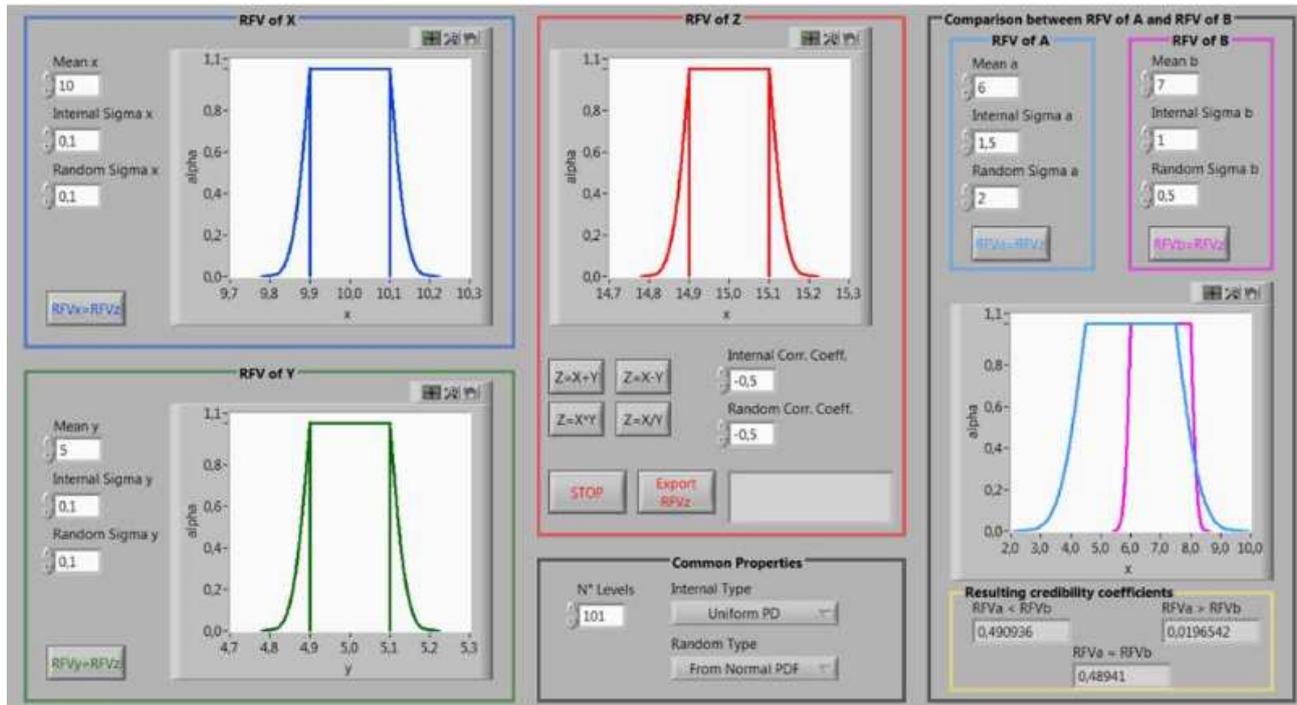


Figure 1. Example of sum and comparison

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