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Design and Calibration of Adaptive Sub-ranging ADCs Resistant to Amplifiers Gain Errors

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***Abstract* – The paper develops a method for designing and calibrating adaptive ADCs robust to internal amplifiers gain errors. As part of the proposed solution, a simple method of postproduction measurement of the actual gain values of amplifiers utilizing the internal ADC, is proposed and analyzed. Efficiency and particularities of application of the proposed approach are discussed on the basis of the results of computer simulations.**

***Keywords* – ADC, adaptive ADC, calibration, gain estimation, performance improvement**

I. INTRODUCTION

The sub-ranging method [1]-[6] has been long and widely used in A/D converters (ADCs), because it enables an excellent compromise between speed, resolution, complexity and power consumption. One of key factors limiting the quality of achievable combinations of performance and exploitation characteristics of a sub-ranging ADC is high sensitivity of its resolution to deviations of amplifiers gains from the nominal values due to manufacturing process variations [1],[2],[7]. This, on one hand, leads to high requirements to the precision of amplifiers, which increases complexity, size and power consumption of a sub-ranging ADC. On the other hand, it necessitates introduction of redundancy in form of using less than all of the bits obtained in subsequent cycles of conversion, as the less significant bits may be invalid due to these errors and need to be corrected with use of dedicated digital algorithms [1],[2] which slows down the speed of conversion. Our researches show that transition to adaptive version of the sub-ranging converter [3]-[9] creates possibilities of a more efficient tackling of the amplifiers imperfections, allowing improvement of the achievable overall (performance and exploitation) quality of a sub-ranging ADC.

A. Principle of functioning of adaptive sub-ranging ADC

The principle of functioning of an adaptive sub-ranging ADC can be described on the basis of the block

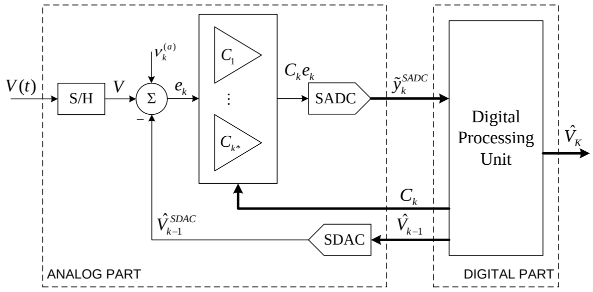

Fig. 1. Block diagram of an adaptive ADC

diagram shown in Fig. 1. Its analog part is the same as in conventional sub-ranging ADCs. The captured sample V of the input voltage signal $V(t)$ is held at the output of the sample-and-hold unit (S/H in Fig. 1) during K cycles (iterations) of conversion. In each k-th cycle, the input voltage V is compensated by means of the feedback signal $\hat{V}_{k-1}^{SDAC}$ delivered to the subtraction unit (Σ) which forms the residue signal e_k :

$$e_k = V - \hat{V}_{k-1}^{SDAC} + v_k^{(a)}. \tag{1}$$

Voltage $\hat{V}_{k-1}^{SDAC}$ is the analog equivalent of the estimate $\hat{V}_{k-1}$ computed in the digital part in the previous cycle of conversion and formed by means of the internal N_{SDAC}-bit D/A sub-converter (SDAC). Variable $v_k^{(a)}$ denotes the analog internal noises. The residue signal e_k diminishes in subsequent cycles and therefore is amplified using the appropriate, depending on the cycle number, internal amplifier (from a bank of amplifiers) with gain C_k correspondingly increasing in subsequent cycles. The amplified residue signal e_k is quantized by an internal coarse flash A/D sub-converter (SADC) with resolution N_{SADC}-bit (2-6 bits), to give the sub-code ("observation") $\tilde{y}_k^{SADC}$. The sub-code $\tilde{y}_k^{SADC}$ is used in the digital part of the converter to determine the code (estimate) of the input sample according to the following relationship:

$$\hat{V}_k = \hat{V}_{k-1} + L_k \tilde{y}_k - L_k C_k (\hat{V}_{k-1} - \hat{V}_{k-1}^{SDAC}) , \tag{2}$$

where L_k is a digital coefficient whose value corresponds to the value of the gain C_k. The last term in (2) compensates the effect of truncation N_{comp} - N_{SDAC} bits of estimate $\hat{V}_{k-1}$ before it is used to form its analog equivalent $\hat{V}_{k-1}^{SDAC}$ by means of SDAC ($N_{comp} > N_{SDAC} >> N_{SADC}$).

It is worth noting that in conventional sub-ranging ADCs the output codes $\hat{V}_k$ are formed using components performing simple low-level bit operations, whereas in the adaptive sub-ranging ADC, the codes $\hat{V}_k$ are actually calculated in a simple computing unit

B. Choice of adaptive ADC parameters C_k , L_k

One of the main problems addressed in hitherto researches (e.g. [3]-[6]) on the adaptive ADC was its optimization, i.e. development of the methods of determining optimal values of the amplifiers gain coefficients C_k and corresponding coefficients L_k in algorithm (2), such that minimize MSE of conversion, or equivalently maximize ENOB (effective number of bits) in each cycle of conversion. The main findings of [3]-[6] and other works, can be summed up as follows. In the pre-threshold interval of cycles of conversion $(k \le k^*)$, in which ENOB grows linearly, the optimal coefficients C_k , L_k can be determined using the following equation:

$$C_k = \frac{D}{\Delta_{SADC} / (2 \cdot C_{k-1}) + \Delta_{SDAC} / 2 + \alpha \sqrt{2} \sigma_a} , \; L_k = C_k^{-1} , \quad (3)$$

where D is a half of the input range $[-D, D]$ of SADC (coinciding with input ranges of SDAC and of adaptive ADC); $\Delta_{SADC} = D \cdot 2^{-(N_{SADC}-1)}$ and $\Delta_{SDAC} = D \cdot 2^{-(N_{SDAC}-1)}$ are the quantization intervals of SADC and SDAC, respectively; σ_a^2 is the variance of internal analog noises; α denotes the saturation factor dependent on the assumed permissible probability μ of saturation of SADC ($\mu \ll 1$). In the first cycle $C_1 = D / (D + \Delta_{SDAC}/2 + \alpha \sigma_a)$.

The threshold number of cycles k^* can be assessed as the minimal value of k for which:

$$\Delta_{SADC} / (2 \cdot C_{k-1}) < \alpha \sqrt{2} \sigma_a + \Delta_{SDAC} / 2 . \quad (4)$$

In the post-threshold interval of cycles of conversion ($k > k^*$), in which ENOB grows more slowly - logarithmically, the gains C_k can be fixed, but the coefficients L_k should be continually adjusted as follows:

$$C_k = C_{k^*} = \frac{D}{\alpha \sqrt{\Delta_{SDAC}^2 / 12 + 2\sigma_a^2}} , \quad L_k = C_k^{-1} \left(1 - \frac{P_k}{P_{k-1}} \right),$$

$$P_k = P_{k-1} - \frac{C_k^2 P_{k-1}}{\sigma_\xi^2 + C_k^2 (\Delta_{SDAC}^2 / 12 + \sigma_a^2 + P_{k-1})} , \quad (5)$$

where σ_ξ^2 denotes the variance of the SADC quantization noise. Initial condition for (5) is $P_{k^*} = \Delta_{SDAC}^2 / 12 + 2\sigma_a^2$.

An adaptive ADC designed according to the algorithm (2)-(5) is optimal in absence of the analog part imperfections but very sensitive to them when they appear.

II. RELATED RESULTS IN THE LITERATURE

This work develops the results of [3]-[6] and other works, concerning examination of the particularities of functioning of the adaptive sub-ranging ADCs and their optimization. It investigates a problem of development of efficient methods of reduction of the influence of analog components imperfections on the quality of conversion that was addressed in many works concerning conventional sub-ranging ADCs, e.g. [1],[2], as well as in recent works concerning adaptive ADCs [8],[9].

Let us notice that the error of the amplifier gain in a given cycle *k* of conversion increases the range of signal e_{k+1} in the following cycle, which in turn increases the probability of saturation of the internal converter SDAC, and the probability of appearance of a large error in the final estimate $\hat{V}_K$. To keep the probability of abnormal errors below the permissible value, one needs to diminish the nominal gains C_k . In known sub-ranging ADCs, for this purpose, the gains of the amplifiers in particular cycles are reduced by half [1],[2], which decrements the speed of conversion by 1 bit per cycle. This decrement by half, although excessive, is unavoidable due to the principle of forming the output codes in conventional sub-ranging ADCs. Determining the output word as a juxtaposition of codes of successive observations $\tilde{y}_k^{SADC}$ imposes a constraint on possible nominal gains values limiting them to exact integer powers of two (as shifting a binary number by several positions is equivalent to a decimal multiplication by an integer power of two).

In sub-ranging adaptive ADCs, thanks to a different method of codes forming – actual computing estimates $\hat{V}_k$ in a computing block, the constraint on possible gains values is removed. The latter creates the possibility to set the gains C_k to arbitrary values, in particular to values that are not lower than just sufficient for elimination of SADC saturations due to amplifiers gains errors. This possibility was first utilized in [8] and the initial results obtained in [8] are further developed in this work.

III. DESCRIPTION OF THE METHOD

The method for making adaptive ADC robust to gain errors is based on the approach proposed in [8] and consists of two stages. The first, pre-manufacturing stage, consists in determination of new, adequately diminished values of nominal gains that take into account the deviations of the amplifiers gains modeled as follows:

$$C_k^{(act)} = C_k (1 + \varepsilon_{C,k}) , \quad (6)$$

where $C_k^{(act)}, C_k$ are the actual and the nominal amplifier gain values in cycle k , respectively. The variable $\varepsilon_{C,k}$ is the relative gain error assumed to be a zero mean random value limited to the interval $[-\delta_C, \delta_C]$.

The second, postproduction stage consists in measuring the actual gains C_k^{act} and setting coefficients L_k accordingly: $\hat{L}_k^{(act)} = (\hat{C}_k^{(act)})^{-1}$. Extending results of work [8], let us additionally consider at the pre-manufacturing stage, the possible inaccuracies of estimates $\hat{C}_k^{act}$, which affect coefficients $\hat{L}_k^{(act)}$, using the following model:

$$\hat{L}_k^{(act)} = \left(C_k^{(act)}\right)^{-1}(1+\varepsilon_{L,k}). \qquad (7)$$

Variable $\varepsilon_{L,k}$ is the relative difference between $\hat{L}_k^{(act)} = (\hat{C}_k^{(act)})^{-1}$ and the adequately set coefficient $L_k^{(act)} = (C_k^{(act)})^{-1}$, resulting from the inaccuracy of estimate $\hat{C}_k^{(act)}$; $\varepsilon_{L,k}$ is assumed to be a zero-mean random value limited to the interval $[-\delta_{L,k}, \delta_{L,k}]$.

Let us now briefly describe the way of determination of adequately diminished nominal gains, taking into account errors (6), (7). First, we determine the values $C_1^{(ref)},...,C_{k*}^{(ref)}$, optimal in absence of analog part imperfections, using the methods developed in [5],[6] and other works. These gains are used as a basis for determination of coefficients $C_k^{(safe)}$ lower than $C_1^{(ref)},...,C_{k*}^{(ref)}$, but not more than necessary to exclude SADC saturations due to gain errors with probability not less than the value 1- μ assumed in determination of $C_1^{(ref)},...,C_{k*}^{(ref)}$. The nominal gains $C_1^{(ref)},...,C_{k*}^{(ref)}$ have been chosen in such a way that in absence of analog part imperfections the signal $C_1^{(ref)}e_k$ is fitted to the input range [-*D*, *D*] of SADC, i.e. the gains values $C_1^{(ref)},...,C_{k*}^{(ref)}$ are maximal among values that fulfill the following condition:

$$\Pr\ \{\ |C_k e_k| > D\ \} \le \mu\ . \qquad (8)$$

The latter allows use of the following approximation:

$$C_k^{(ref)} \cdot \max|e_k| \approx D\ . \qquad (9)$$

Taking into account (9) and the expected identical equality for corrected gain $C_k^{(safe)}$ and signal e_k in presence of gain and gain estimation errors (denoted as $\bar{e}_k$), we obtain the following general formula for the coefficient γ_k reducing the gain $C_k^{(ref)}$ to the maximal safe value $C_k^{(safe)}$:

$$\gamma_k = \frac{C_k^{(safe)}}{C_k^{(ref)}} = \frac{D}{\max|\bar{e}_k|} \cdot \frac{\max|e_k|}{D} = \frac{\max|e_k|}{\max|\bar{e}_k|}\ . \qquad (10)$$

Using the last term in (10) as well as formulas (1), (2), the second equation in (3), (6) and (7), one can obtain formulas for the corrected gains $C_k^{(safe)}$ presented in Tab. 1 (to which we further refer to as correction (Tab. 1)) that take into account maximal relative errors δ_C and $\delta_{L,k}$. For $\delta_{L,k} = 0$, taking into account that in most cycles $\Delta_{SDAC}/2 + \alpha\sqrt{2}\sigma_a << \Delta_{SADC}/(2C_{k-1}^{(ref)})$, we obtain the following approximate recursion:

$$C_{1<k<k*}^{(safe)} \approx C_k^{(ref)} \cdot \left(\frac{C_{k-1}^{safe}}{C_{k-1}^{(ref)}} \cdot \frac{1-\delta_C}{1+\delta_C} \right), \qquad (11)$$

which shows the general "mechanism" behind the gain reduction realized by formulas (Tab.1) solely due to gain errors δ_C. It responds to such factors influencing the range of signal e_k in given cycle k as: diminishing of previous nominal gains to the maximal safe values (factor $C_{k-1}^{(safe)}/C_{k-1}^{(ref)}$), gain error in the previous cycle increasing error of estimates $\hat{V}_{k-1}^{SDAC}$ in (1) (factor $1-\delta_C$) and possible positive gain error in the current cycle resulting in overamplification of signal e_k (factor $1/(1+\delta_C)$).

Assuming $\delta_C = 0$, $\Delta_{SDAC}/2 + \alpha\sqrt{2}\sigma_a << \Delta_{SADC}/(2C_{k-1})$, we obtain the approximate recursive formula:

$$C_{2<k<k*}^{(safe)} \approx C_k^{(ref)} \cdot \left(\frac{C_{k-1}^{(ref)}}{C_{k-1}^{(safe)}} \cdot \left(1+\delta_{L,k-1}\right) + \frac{C_{k-1}^{(ref)}}{C_{k-2}^{(safe)}} \cdot \delta_{L,k-1} \right)^{-1}, \qquad (12)$$

allowing analysis of the reduction solely due factors connected with non-ideality of measurement of $C_k^{(act)}$ resulting in $\delta_{L,k}$. The diminishing factor in (12), similarly like its counterpart in (11) for a given δ_C, depends on the extent of diminishing of the previous gains (factor $C_{k-1}^{(ref)}/C_{k-1}^{safe}$) and on the error $\delta_{L,k-1}$ in the previous cycle (term $1+\delta_{L,k-1}$). Moreover, in (12), there appears an additional term $(C_{k-1}^{(ref)}/C_{k-2}^{safe}) \cdot \delta_{L,k-1}$ resulting from appearance of a relatively large error term $\delta_{L,k-1} \cdot \xi_{k-1}/C_{k-1}^{(ref)}$ in estimates $\hat{V}_{k-1}^{SDAC}$ in (1), which makes the influence of inadequacy of coefficient $\hat{L}_k^{(act)}$ much stronger than the influence of gain errors to a large extent reducible by setting coefficients L_k in tune with C_k^{act}: $L_k = (C_k^{(act)})^{-1}$.

Tab. 1. Corrections of nominal gains $C_1^{(ref)},...,C_{k*}^{(ref)}$ that adequately diminish them to maximal safe values

$$C_1^{(safe)} = C_1^{(ref)} \frac{1}{1+\delta_C}\ , \qquad C_2^{(safe)} = C_2^{(ref)} \cdot \frac{1}{1+\delta_C} \cdot \frac{\Delta_{SADC}/\left(2C_1^{(ref)}\right) + \Delta_{SDAC}/2 + \alpha\sqrt{2}\sigma_a}{\delta_L D + \dfrac{\Delta_{SADC}\left(1+\delta_{L,1}\right)}{2C_1^{(safe)}\left(1-\delta_C\right)} + \left(1+\delta_{L,1}\right) \cdot \dfrac{\Delta_{SDAC}}{2} + \alpha\sqrt{2+2\delta_{L,1}+\delta_{L,1}^2}\,\sigma_a}\ ,$$

$$C_k^{(safe)} = C_k^{(ref)} \cdot \frac{1}{1+\delta_C} \cdot \frac{\Delta_{SADC}/\left(2C_{k-1}^{(ref)}\right) + \Delta_{SDAC}/2 + \alpha\sqrt{2}\sigma_a}{\dfrac{\delta_{L,k-1}\Delta_{SADC}}{2C_{k-2}^{(safe)}\left(1-\delta_C\right)} + \dfrac{\Delta_{SADC}\left(1+\delta_{L,k-1}\right)}{2C_{k-1}^{(safe)}\left(1-\delta_C\right)} + \left(1+\delta_{L,k-1}\right) \cdot \dfrac{\Delta_{SDAC}}{2} + \alpha\sqrt{2\left(1+\delta_{L,k-1}+\delta_{L,k-1}^2\right)}\,\sigma_a}\ , \quad k > 2\ .$$

Once manufactured, the adaptive ADC in design of which the nominal gains were corrected using (Tab.1), has to be calibrated. Its calibration consists in measurement of the actual gains values $C_k^{(act)}$, and setting the coefficients L_k to the values calculated on the basis of the estimates $\hat{C}_k^{(act)}$ using (3),(5). To facilitate the post-production adjustments, we propose to implement a special calibration mode in the adaptive ADC. In this mode, the feedback is switched off (output of SDAC is zero) and the adaptive ADC works as a flash ADC with gain $C_k^{(act)}$, depending on which amplifier is chosen. Instead of determining the gain $C_k^{(act)}$ of a given k-th amplifier $(1 \le k \le k^*)$ directly, we determine the gain of the block "amplifier + SADC" which has the same gain $C_k^{(act)}$ and the transfer curve shown in Fig. 2. For this purpose, we can use one of the methods for evaluation of ADC gain and offset described in IEEE Std. 1241 [10], either the independently based or the terminal based method described in Sects. 7.4.1 and 7.4.2 of [10], respectively. Our researches show that the use of latter, much simpler one requiring measurement of only two extreme thresholds $T_k[1]$ and $T_k[2^{N_{SADC}}-1]$, (marked in Fig. 2 with blue), is in many cases sufficient and hence, will be considered in this work. The extreme thresholds can be measured by identifying transitions of the codes at the output of SADC in response to known voltages delivered to the input of the adaptive ADC. On the basis of $T_k[1]$ and $T_k[2^{N_{SADC}}-1]$, one can calculate the gain estimates $\hat{C}_k^{(act)}$ using the formula (37) from [10]:

$$\hat{C}_k^{(act)} = \frac{\Delta_{SADC}(2^{N_{SADC}}-2)}{T_k[2^{N_{SADC}}-1]-T_k[1]} . \qquad (13)$$

An algorithm for estimation of the gains $C_k^{(act)}$ according to (13) and succeeding calculation of the coefficients $\hat{L}_k$ can be implemented in the digital part of the adaptive ADC as a semi-autonomous calibration procedure. In this case, values of the testing voltages have to be saved in the memory of the adaptive ADC.

If value δ_C was taken with a relevant excess, possible gain error of SADC will not worsen the efficiency of the calibration procedure. The main practical issue needing attention is the choice of the step $\Delta V_{cal,k}$ of the sequence of testing voltages. Too large a step will result in worse estimation of the thresholds, and, in effect, of $C_k^{(act)}$, which will cause drop of ENOB, too little will unnecessarily complicate and increase duration of the calibration procedure. Analysis of this issue allowed us to obtain on the basis of (13), the dependence between the step $\Delta V_{cal,k}$ and maximal relative error $\delta_{L,k}$ of $\hat{L}_k^{(act)}$ (7) calculated using $\hat{C}_k^{(act)}$ whose error is determined by $\Delta V_{cal,k}$:

$$\delta_{L,k} = \frac{\Delta V_{cal,k}}{F_k},\; F_k = \frac{\Delta_{SADC}(2^{N_{SADC}}\text{-}2)}{\hat{C}_k^{(act)}} \approx \frac{\Delta_{SADC}(2^{N_{SADC}}\text{-}2)}{C_k^{(safe)}} . \qquad (14)$$

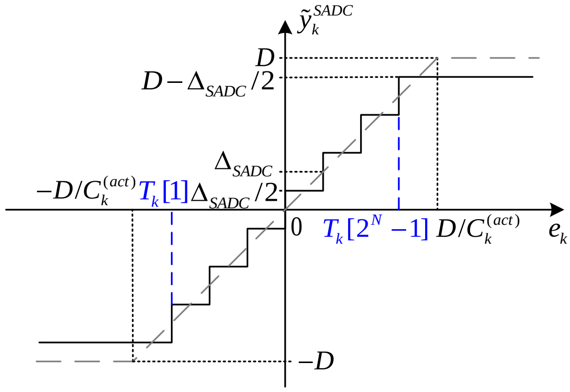

Fig. 2. Transfer function of the block "amplifier + SADC"

To ensure neglectible influence of the relative error $\delta_{L,k}$ on ENOB of the adaptive ADC, i.e. to provide that:

$$\max\left|\Delta\hat{V}_k\right|_{\delta_{L,k}>0} - \max\left|\Delta\hat{V}_k\right|_{\delta_{L,k}=0} \ll \ll \max\left|\Delta\hat{V}_k\right|_{\delta_{L,k}=0} , \qquad (15)$$

where $\Delta\hat{V}_k = \hat{V}_k - V$, we established, using (2),(7), the following requirements for $\delta_{L,k}$:

$$\delta_{L,k} \ll \begin{cases} \dfrac{\Delta_{SADC}/(2C_1^{(safe)})+\alpha\sigma_a}{\dfrac{D}{1+\delta_C}+\dfrac{\Delta_{SADC}}{2C_1^{(safe)}}+\alpha\sigma_a}, & \text{for } k=1 , \\ \dfrac{\Delta_{SADC}/(2C_k^{safe})+\alpha\sigma_a}{\dfrac{\Delta_{SADC}}{2C_{k-1}^{(safe)}}+\dfrac{\Delta_{SADC}}{2C_k^{(safe)}}+\dfrac{\Delta_{SDAC}}{2}+\alpha\sqrt{2}\sigma_a}, & \text{for } k>1 . \end{cases} \qquad (16)$$

Combining (14) and (16), allows obtainment of the direct requirements for the value of the step $\Delta V_{cal,k}$:

$$\Delta V_{cal,k} = \delta_{L,k} \cdot F_k = \left(\lambda \cdot err_{L,k}\right) \cdot F_k ,\; \lambda << 1 , \qquad (17)$$

under given coefficient λ regulating the extent of fulfillment of (16); $err_{L,k}$ is equal to value of the expression on the right side of inequality (16). Closer analysis of (17), (14), (16) shows that the required step $\Delta V_{cal,k}$ decreases approximately C_k times per cycle.

The ranges Z_L and Z_U of the calibrating discrete ramp signal guaranteeing successful determination of the lower and the upper extreme threshold in (13), respectively, can be determined using the following dependence:

$$Z_{L/U} = \left[\mp\frac{\Delta_{SADC}(2^{N_{SADC}-1}\text{-}1)}{C_k^{(safe)}(1\pm\beta\cdot\delta_C)}, \mp\frac{\Delta_{SADC}(2^{N_{SADC}-1}\text{-}1)}{C_k^{(safe)}(1\mp\beta\cdot\delta_C)}\right] . \qquad (18)$$

For $\beta = 1$, formula (18) gives the ranges of possible values of the actual thresholds resulting from δ_C, however, the assumed method of their estimation and preferably larger step $\Delta V_{cal,k}$ necessitate taking $\beta = 2$.

A different approach to postproduction calibration of adaptive ADCs with gains errors was investigated in [9].

IV. RESULTS OF SIMULATIONS

The developed method for design and calibration of adaptive ADCs robust to amplifiers gains errors was used in development of relevant models of the adaptive ADC in MATLAB. The potential advantages from application of the proposed approach were assessed on the basis of analysis of changes of ENOB calculated in accordance with the IEEE Std. 1241 [10]:

$$ENOB_k = \log_2\left(\frac{FSR}{NAD_k\sqrt{12}}\right),\ NAD_k = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(\hat{V}_k^{(m)} \text{-} V^{(m)}\right)^2}, \quad (19)$$

as a function of two variables - number of cycles $k = 1,...,10$ and maximal relative gains errors δ_C changed from 0 to 10%. $FSR = 2D$ in (19) is the full-scale range of the adaptive ADC and NAD_k is the rms value of noise and distortion [10]. ENOB (19) was calculated on the basis of the results of conversion of the same (in all experiments) sequences of signal samples $V^{(m)}$ $(m = 1,..., M)$, $M = 100,000$, uniformly distributed in the entire input range of the adaptive ADC. We assumed step-wise models of SADC and SDAC with resolutions $N_{SADC} = 4$ and $N_{SDAC} = 12$, respectively; other parameters had the values: $FSR = 2$ V, $\alpha = 4$, $\hat{V}_0 = 0$, analog noise $v_k^{(a)}$ was modeled as a white Gaussian noise, $\sigma_a = 10\ \mu$V. For these values of parameters $k^* = 4$. The actual gains errors were modeled according to (6); to enable comparison of the results for different values δ_C, in all experiments instead of random values, we used the following pseudorandom sequence of errors $\varepsilon_{C,k}: -0.9\cdot\delta_C$, $0.8\cdot\delta_C$, $-0.6\cdot\delta_C$, $0.7\cdot\delta_C$ for $k = 1,...,4$, respectively.

Trajectories of ENOB in Fig. 3a were obtained for adaptive ADC employing the standard algorithm (2)-(5). The reason of the observed serious decrease of ENOB values with growing gain deviation δ_C is fast increasing frequency of saturations. There can be seen no interval of tolerance – errors of relative magnitude of less than one percent cause a significant decrease of ENOB, which confirms that the adaptive ADC realized according to (2)-(5) is very sensitive to gains errors.

Plots in Fig. 3b-d correspond to the adaptive ADC with nominal gains reduced according to (Tab. 1), after calibration performed according to the description in Sect. III, for different values of parameter λ regulating the extent of fulfillment of (16). In order to determine the values of the thresholds to be used in (13), we used input voltages in the ranges (18), with $\beta = 2$, which allowed significant reduction of sizes of the calibrating sequences. The corrected nominal gains $C_k^{(safe)}$ and steps $\Delta V_{cal,k}$ were determined according to the following algorithm:

1. Assume initially $\delta_{L,k} = 0$.
2. Find values $C_k^{(safe)}$ using $\delta_{L,k}$ and (Tab.1).
3. Determine values $\delta_{L,k}$ for the given λ, using $C_k^{(safe)}$ and (16): $\delta_{L,k} = \lambda \cdot err_{L,k}$.
4. Repeat steps 2-3 until values $C_k^{(safe)}$ stabilize.
5. Determine $\Delta V_{cal,k}$ using $\delta_{L,k}$, $C_k^{(safe)}$ and (17), (14).

The efficiency of the developed method for design and calibration of adaptive ADCs robust to amplifiers gains errors can be assessed by comparing plots in Fig. 3b-d to the reference trajectory of ENOB in Fig. 3a for $\delta_C = 0$, which is the upper limit of adaptive ADC performance achievable in absence of gains errors. For $\lambda = 1\%$ (Fig. 3b), we observe almost full restoration of growth of ENOB for all δ_C. The gentle decrease of ENOB with growth of gains deviation δ_C is caused by the necessary, increasing with δ_C and accumulating in subsequent cycles, adequate reduction of the nominal gains according to (Tab. 1). For $\lambda = 10\%$, which allows 10 times larger step $\Delta V_{cal,k}$, the loss of resolution compared to $\lambda = 1\%$ is only by a fraction of a bit. For $\lambda = 50\%$, there can be observed a more significant loss of ENOB, by about 1.5 bits in the last cycle. The results show that the choice of $\lambda = 10\%$, provides a good compromise between complexity of calibration procedure and achievable performance of the calibrated adaptive ADC.

Comparing the trajectories of ENOB in Fig. 3a-d for $\delta_C = 0$, one can observe the strong influence on accuracy of estimates $\hat{V}_k$ of sole error $\delta_{L,k}$ resulting from the assumed value γ. For a given λ, according to (17), (16), δ_L needs to be 5-20 times smaller: for $\lambda = 1\%$, $\delta_{L,k} \in (0.06\%, 0.2\%)$; for $\lambda = 10\%$, $\delta_{L,k} \in (0.6\%, 2\%)$; for $\lambda = 50\%$, $\delta_{L,k} \in (3\%,\ 9.5\%)$. For given λ and δ_C, value $\delta_{L,k}$ increases with the cycle number k, which indicates its strongest influence on $\hat{V}_k$ errors in initial cycles.

The results presented in Fig. 3 confirm the capability of the proposed approach to reduce the negative influence of the gains errors in a noticeably wide range of their magnitude. In the simulations corresponding to Fig. 3b-d, there were registered no cases of signal $C_k^{(act)}e_k$ exceeding the input range of SADC, which additionally confirms validity of gains corrections (Tab. 1) and formula (17) for required (maximal) step of the calibrating signal.

V. NOVELTIES IN THE PAPER

The novelty of this work is development, as well as simulational verification, of an efficient method of designing and calibration of adaptive sub-ranging ADCs robustified against amplifiers gains errors. The method facilitates compromise between achievable performance (ENOB) and simplicity of the calibration procedure, by taking into account errors of actual gains measurement.

VI. CONCLUSIONS

In the paper, there was proposed a method for manufacturing adaptive sub-ranging ADCs robust to amplifiers gain errors, with the ENOB obtained in each cycle not or

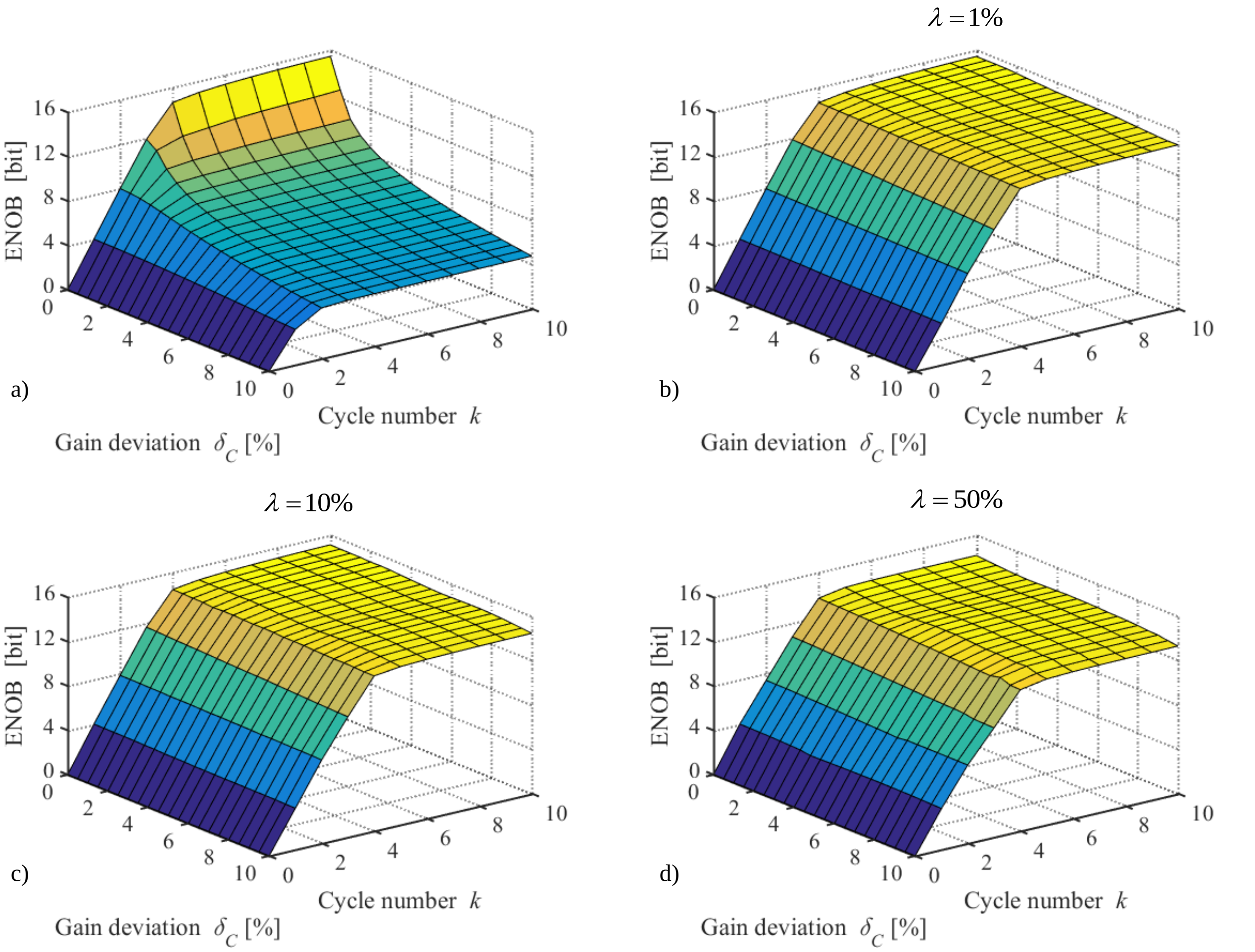

Fig. 3. ENOB of adaptive ADC: employing the standard algorithm (2)-(5) *(a); with nominal gains corrected according to (Tab. 1), after calibration, with coefficient* λ *regulating the extent of fulfilment of (17) equal to: 1% (b), 10% (c), 50% (d)*

only little worse than achievable in the case of ideally realized amplifiers. Efficiency and feasibility of implementation of the proposed method was verified and analyzed using simulations. A more general conclusion of the paper is that transition to adaptive version of the sub-ranging ADC creates the possibilities of a more efficient solution of the problem of strong influence of gain errors on performance of the sub-ranging ADCs than the nowadays used. In effect, there can be reduced the requirements concerning precision of the amplifiers.

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