21st IMEKO TC4 International Symposium and
19th International Workshop on ADC Modelling and Testing
Understanding the World through Electrical and Electronic Measurement
Budapest, Hungary, September 7-9, 2016

# Phase Error Determination for the Wideband Resistive Voltage Divider

Jiangtao ZHANG, Xianlin PAN, Ying SONG and Qing HE

*National Institute of Metrology,*
*No.18, Bei San Huan Dong Lu, Beijing 100029 China,*
*zhangjt@nim.ac.cn, +86 10 6452 4514*

***Abstract*** **–A new design of resistive voltage divider with series-parallel connection has been built. A self-calibration method has also been proposed to determine the phase angle errors of the resistive voltage divider. The phase angle displacement has been measured by means of sampling with a high-speed digitizer. The measurement has been done at frequencies from 10 kHz to 100 kHz to calibrate two RVDs with ratio of 100:1 and the measurement results are presented.**

***Keywords*** **–Phase angle errors, Resistive Voltage Divider (RVD), Buffer, Capacitive leakage, Loading errors**

## I. INTRODUCTION

Resistive Voltage Divider (RVD) is widely used at the measurement of ac voltage divider and the establishment of ac power standard especially at high frequencies. For the ac power measurement, it requires not only the amplitude errors, but also the phase angle errors at all the frequency range [1].

Several designs of the RVDs have been proposed by National Metrology Institutes (NMIs). For the RVD, the dominant part of the phase angle errors comes from the capacitive leakage between resistor elements and the housing [2]. Another important part is caused by the time constant influence from the resistor elements themselves. Thermal Power Comparator (TPC) method and sampling method are proposed to determine the phase angle errors of RVD at high frequencies up to 100 kHz [3][4]. Considering the output impedance of RVD is higher resistance and the input capacitance of the digitizer is not zero, a loading error in phase errors will be introduced by the connection cable and connectors directly. The buffer amplifier is necessary to minimize the loading errors.

At National Institute of Metrology (NIM) China, a coaxial RVD structure with series-parallel connection has been developed. A buffer amplifier is used at the outuput of the RVDs. With this design, the influence from the time constant of the resistor can be eliminated. A method has been developed to measure capacitive leakage influence of the RVDs and determine the phase angle errors at frequencies from 10 kHz to 100 kHz.

## II. SERIES-PARALLEL DESIGN RVD

A coaxial RVD with serial-parallel connection has been developed, and the structure is shown in Fig.1. The RVD mainly contains two parts. The upper part consists of $m$ sets of resistor with identical value in series connection and the lower part with $n$ sets of resistor in parallel connection. The series resistors distribute along the axis of the coaxial cylinder housing and the parallel resistors was star-shaped distributed evenly along the circular surface. The ratio K of the RVD can be expressed as equation (1):

Considering the resistive elements of this RVD are selected with identical resistors, the time constant influence from the resistive elements can be ignored. So the dominant part causing the phase angle errors of this RVD is from the capacitive leakage between resistor elements and low terminal housing.

$$\mathrm{K} = mn + 1 \quad (1)$$

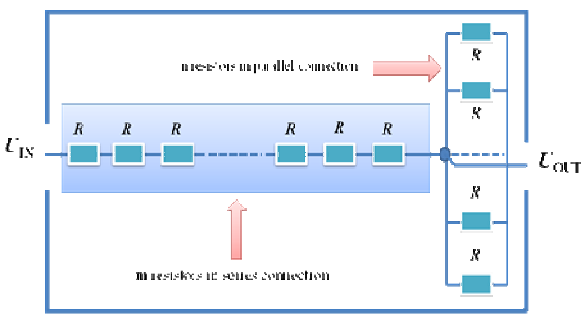


*Fig.1. Structure of the RVD*

For the RVD, the capacitance between the resistor elements and housing is evenly distributed and the influence for the phase angle errors of the RVD can be described as:

$$\theta_C = -(C_R \times \sum_{i=1}^{m} \frac{\left(i \times n \times (m-i) + i\right)}{mn+1} + \frac{mC_0}{mn+1}) \times \omega R \quad (2)$$

where $C_R$ is the capacitance between each resistor element and housing, $C_0$ is the capacitance between input terminal and the housing, $\omega$ is angle frequency, $R$ is the resistance of each resistor element. As seen from equation (2), phase angle errors from the capacitive leakage are in direct proportion to $R$. Based on this special ratio relationship of this RVD, a method has been proposed to accurately measure the phase angle errors..

## III. SELF-CALIBRATION PRINCIPLE

In order to keep the capacitance nearly the same, two RVDs with identical structure and different resistor elements have been built, marked as RVD1 and RVD2. So the phase angle errors of RVD1 and RVD2 from capacitive leakage can be expressed as:

$$\theta_{C1} = -(C_R \times \sum_{i=1}^{m} \frac{\left(i \times n \times (m-i) + i\right)}{mn+1} + \frac{mC_0}{mn+1}) \times \omega R_1 \quad (3)$$

$$\theta_{C2} = -(C_R \times \sum_{i=1}^{m} \frac{\left(i \times n \times (m-i) + i\right)}{mn+1} + \frac{mC_0}{mn+1}) \times \omega R_2 \quad (4)$$

where $R_1$ and $R_2$ is the resistance value of the resistor elements for RVD1 and RVD2 respectively. As seen from equation (3) and (4), the phase angle errors difference between RVD1 and RVD2 can be determined as:

$$\begin{aligned}\Delta\theta &= \theta_{C1} - \theta_{C2} \\ &= -(C_R \times \sum_{i=1}^{m} \frac{\left(i \times n \times (m-i) + i\right)}{mn+1} + \frac{mC_0}{mn+1}) \times \omega \left(R_1 - R_2\right)\end{aligned} \quad (5)$$

According to the equation (5), the phase angle errors of RVD1 and RVD2 can also be determined by:

$$\theta_{C1} = \frac{R_1}{R_1 - R_2} \times \Delta\theta \quad (6)$$

$$\theta_{C2} = \frac{R_2}{R_1 - R_2} \times \Delta\theta \quad (7)$$

As seen from equation (6) and (7), $R_1$ and $R_2$ can be easily known, so the phase angle errors of each RVD can be calibrated by measuring Δθ.

## IV. MEASUREMENT SETUP

A measurement setup to determine the phase angle difference between RVD1 and RVD2 has been developed and shown in Fig.2. The input terminals of the RVD1 and RVD2 are connected directly into the ac voltage source, and the output terminals are connected with an adjusted resistor respectively to keep the output impendence balance. A high-speed digitizer is used to measure the phase angle errors between the voltage drops on the output terminals of RVD1 and RVD2.

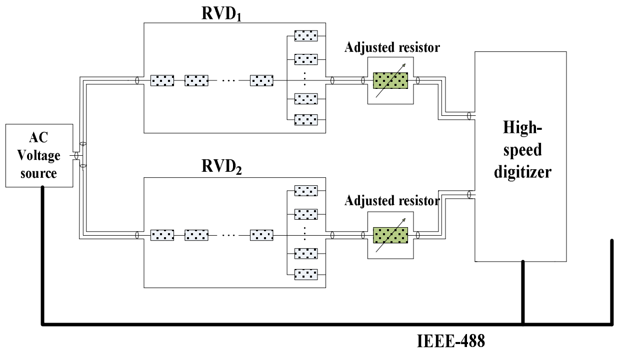


*Fig.2. Measurement setup*

## V. MEASUREMENT RESULTS

Based on the series-parallel connection, two RVDs with identical ratio have been designed and the resistor elements of each RVD are 100 Ω and 1 kΩ respectively. The ratio of the RVD is 100 to 1. The structure was designed by 11 resistors in series connection and 9 resistors in parallel. The phase angle difference between two RVDs can be accurately measured, and then the phase angle errors of each RVD can be determined. The measurement has been done at frequencies from 10 kHz to 100 kHz, and results are given in Table I.

TABLE I
Self-Calibration Results in Phase Angle Errors of the RVDs with Ratio of 100:1 at Frequencies from 10 kHz to 100 kHz

| | Phase angle errors in μrad at frequency in kHz | | | | |
|---|---|---|---|---|---|
| Frequency | 10 | 15.92 | 20 | 50 | 100 |
| Δθ | -169 | -275 | -349 | -904 | -1827 |
| $\theta_{c1}$ ($R$=1 kΩ) | -188 | -305 | -388 | -1005 | -2031 |
| $\theta_{c2}$ ($R$=100 Ω) | -19 | -31 | -39 | -100 | -203 |

## VI. LOADING EFFECT

In the power measurement, normally the input voltages to RVD are in hundreds of volt, and the output resistance of RVD is in several kΩ. When the output voltage is measured with a digitizer, the input capacitance of the digitizer is not zero. In addition, the cable and connectors also have the capacitance, and all the capacitance can be thought to be parallel with the output resistance of the RVD. It will result in a significant phase angle errors which is so-called loading errors. In order to minimize the loading errors, normally a wideband buffer amplifier is used to transfer the high output resistance of the RVD down to lower resistance. A kind of precision buffer amplifier with three-stage voltage follower was developed at National Measurement Institute, Australia (NMIA). Several buffers were used for the resistance transition of the RVDs. The phase shift of the buffer can be measured by a two-channel digitizer or a Lock-in amplifier. The input and output of the buffer are directly connected to the two channel of a high speed digitizer respectively and the phase shift can be measured by measuring the phase angle difference of the two channels. Twice measurements are done by exchanging the channels of the digitizer to eliminate the error from the inconsistency of the two channels. The phase shift between the output and input voltage of the buffer are measured to be around 2 μrad per kHz. The combination of the RVD and the buffer is shown in Fig.3.

The loading error is still exist even with the buffer, because the buffer still has the non-ignorable input capacitance. The loading error between the RVD and the buffer can be determined by inserting another identical buffer to measure phase shift. The improved measurement setup to measurement the phase error of RVDs is shown in Fig.4. The buffers are used for the RVDs, and the phase errors of the RVDs and buffer are taken into account integrally and measured. RVD1 and RVD2 connect to the ac voltage resource directly. Two buffers connect to the output of RVD1 and RVD2. The digitizer NI 5922 is used to measure the output voltages of the buffers. Another buffer is inserted in paralel with the buffer connected with RVD1, and the phase shift is measured in the two condition of connecting and disconnecting the additional buffer.

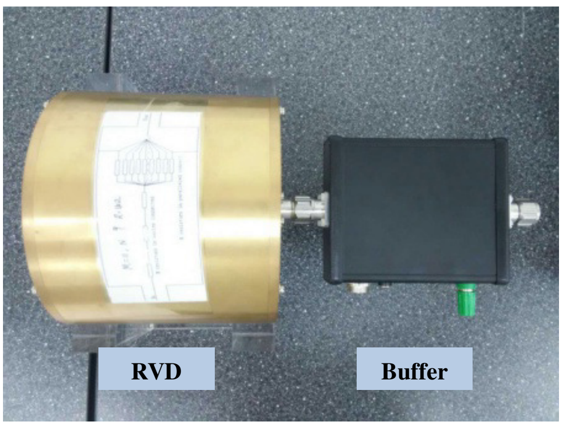

*Fig.3. Picture of the RVD connected with the buffer*

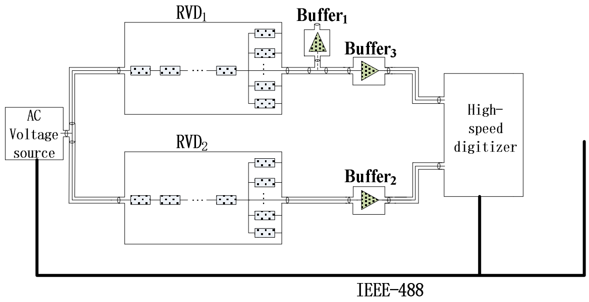

*Fig.4. Measurement of loading error*

The loading error $\varphi_{load}$ is obtained in equation (8).

$$\varphi_{load} = \varphi_1 - \varphi_0 \tag{8}$$

Where $\varphi_1$ and $\varphi_0$ are the phase when the additional buffer is connected and disconnected.

The phase angle error $\varphi$ of the RVD1 with buffer can be expressed as equation (9).

$$\varphi = \varphi_{RVD} + \varphi_{buffer} + \varphi_{load} \tag{9}$$

where $\varphi_{RVD}$ is the phase angle error of the RVD1 and $\varphi_{buffer}$ is the phase angle error of the buffer.

## VII. PHASE ERRORS OF RVD WITH BUFFER

Based on the series-parallel connection structure, the coaxial RVD constructed with 100 Ω resistors with ratio of 100:1 has been designed and the phase angle error has been measured at frequencies from 10 kHz to 100 kHz. The phase angle error of the buffer and the loading error are also measured. As mentioned above, the phase error of the RVD with buffer can then be calibrated and the measurement results are given in Table II.

TABLE II

Measurement Results in Phase Angle Errors of the RVD with Buffer in μrad at Frequencies from 10 kHz to 100 kHz

| $f$ | 10 | 15.92 | 20 | 50 | 100 |
|---|---|---|---|---|---|
| $\varphi_{RVD}$ | -19.8 | -30.4 | -38.6 | -100.5 | -203.0 |
| $\varphi_{buffer}$ | 19.4 | 30.6 | 38.4 | 95.6 | 190.1 |
| $\varphi load$ | -15.3 | -25.8 | -33.2 | -91.1 | -184.1 |
| $\varphi$ | -14.7 | -25.6 | -33.4 | -96.1 | -197.0 |

## VIII. FUTURE DESIGN OF AN ACTIVE RVD

The series-parallel connection RVD combined with buffer mentioned above solves the phase angle tracebility of the voltage divider in high frequency power measurement. Anyway this kind of RVD still have some disadvatages. Firstly the star-shaped parallel part used printed circuit board causes unavoidable large capacitance which results in relative larger phase errors.

Secondly the connection between RVD and buffer with N-type connector also has several pF capacitance. For the RVD with high resistance output, the loading effect is still larger and need to be compensated. An active RVD is proposed to be developed in the future to overcome the disadvanges. In the design, single resistive element is used as the lower part to minimize the output capacitance of the RVD. In addition, the RVD is directly designed and embeded in the print circuit board ofthe buffer. The demention of the RVD will be designed as small as possible, and several pads are reserved in the upper part for the adjustment of the phase errors calibrated against the phase errors known series-parallel RVD. The phase errors of the new design is expected to be less than 100 μRad at 100 kHz.

## IX. CONCLUSION

A series-parallel RVD with buffer for wideband power measurement has been proposed. The method to determine the phase angle error of the RVD with buffer is developed. The measurement results for the loading effect and the phase errors are presented. This kind of RVD is used as the phase error traceability standard for the development of the high frequency power standard at frequency up to 100 kHz at NIM China.

## *REFERENCES*


[1] Hagen, T., and I. Budovsky, "Development of a precision resistive voltage divider for frequencies up to 100 kHz", CPEM 2010 Conf. Dig., pp.195 – 196, 2010.

[2] Karl. E. Rydler, S. Svensson, and V.Tarasso, "Voltage dividers with low phase angle errors for windband power measuring system," CPEM 2002 Conf. Dig., pp.382-383, 2002.

[3] I. Budovsky, "Measurement of phase angle errors of precision current shunts in the frequency range from 40 Hz to 200 kHz," IEEE Trans. Instrum. Meas., vol. 56, no. 2, pp. 284–288, Apr. 2007.

[4] Bergsten,T., V.Tarasso, and K.E.Rydler, "Precision measurement system for character of phase displacement of voltage dividers up to 1 MHz", CPEM 2010 Conf. Dig., pp. 259-260, 2010.