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Instantaneous frequency estimation of distorted sinusoidal signal by "quasi" repetitive approach

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***Abstract* – In this paper a control scheme based on a "quasi" repetitive controller is proposed for asymptotic tracking of the instantaneous frequency of distorted sinusoidal signals. The resonant frequency of the repetitive controller is tuned by using an adaptive orthogonal signals generator based on a second-order generalized integrator. Experimental results are performed to highlight the characteristics and performances of the proposed method, in comparison with other existing approaches.**

***Keywords*—** Repetitive control; distorted sinusoidal signals; adaptive control; orthogonal signals generator.

I. INTRODUCTION

Sinusoidal signal is fundamental as stimulus for the test of electronic devices [1] or as reference signal [2, 3]. Naturally, the real sinusoidal signal is distorted, and in several application fields it is necessary to estimate the parameters of this signal. In particular, the frequency estimation of a distorted sinusoidal signal is an important topic for the safety, stability and efficiency of the power system [4, 5, 6, 7, 8] as well as in many applications in signal processing [9, 10, 11, 12].

In the paper, the instantaneous frequency estimation approach of a distorted sinusoidal signal based on a Repetitive Controller (RC) methodology is proposed. In the proposed approach is considered that the signal under examination is a periodic disturbance acting on a positive-real system. This last is an linear approximation of a derivative block. The effect of the input signal on the system output are cancelled by tuning adaptively a modified RC. Adaptive orthogonal signals generator based on second order generalized integrator is used in the adaption scheme (**OSG-SOGI**) for the RC period tuning [13, 14, 15, 16, 17]. OSG-SOGI permits to estimate the instantaneous frequency of the signal with an established level of accuracy [18].

The paper represents a revised version of [18], aiming to provide new contributions in terms of clarification on the state of the art and the advantages of the proposed strategy, description of the fundamental frequency signal tracking method based on RC, and finally presentation of new laboratory results.

The paper is organized as follows. In Section II the state of the art in the field of the research is presented. In Section III the description of the proposed methodology is provided. In Section IV the preliminary experiments results are reported.

II. RELATED RESULTS

Among frequency estimation methods proposed in the literature particular relevance have methods based on: Prony theory, Phase-Locked Loop (PLL) algorithms, variants of the Extended Kalman Filter, enhanced Zero Crossing techniques, Adaptive Notch Filters, and neural networks-based techniques. PLLs have found much attention, mainly due to their simplicity, robustness, and effectiveness [19]. The drawbacks associated with the PLL are: sensitivity to signal variations, difficulty to obtain an accurate tuning of the PLL parameters. PLL are not fast because of their proportional-integral block like the methods based on filters like Adaptive Notch Filters and Extended Kalman Filte. Adaptive Prony methods, zero crossing techniques and neural network have been successfully applied for estimating frequency variations, especially in electric signals. The problem of estimating the amplitude, frequency and phase of a sinusoidal signal by processing a measurement signal corrupted by bias, drift and bounded unstructured disturbances is addressed in [20, 21]. In [22] the accurate estimation of signal frequency and bias is obtained by high-frequency noise rejection method. Other interesting improvements are discussed in [23, 24].

In [25, 26] is proposed and Adaptive Fourier Analysis based on resonator observer. The approach is similar to the proposed one and uses as signal model a band limited periodic signal.

III. DESCRIPTION OF THE PROPOSED METHODOLOGY

RC arises as a practical solution to the tracking or rejection of periodic signals and is based on the well-known internal model principle [27]. According to this principle, in order to achieve zero tracking error in steady state it is necessary and sufficient to include the generators of the reference signal and/or the disturbance signal in the loop, either in the plant itself or in the controller. It is well known that the generator of a sinusoidal signal, i.e., containing only one harmonic component, is a resonant filter. Therefore, following this idea, if a periodic signal has an infinite Fourier series, then an infinite number of harmonic oscillators are required to track or reject such a periodic signal. Fortunately, a simple delay line in positive feedback configuration can be used to produce an infinite number of poles and thereby simulating an infinite number of harmonic oscillators. Thus, similarly to periodic signals, a memory loop can be used which generates an output at frequencies $k\frac{2\pi}{\hat{T}_c}$, with k integer and $\hat{T}_c$ the signal period.

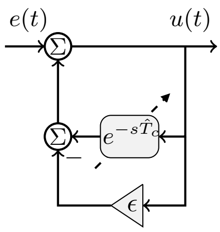

Fig. 1. Quasi repetitive controller.

The block diagram of the classical RC is shown in Fig. 1 (for $\epsilon = 0$). The resulting transfer function is

$$C_{RC}(s) = \frac{1}{1 - e^{-\frac{2\pi}{\hat{T}_c}s}}. \quad (1)$$

Thus, due to the delay, this transfer function has infinitely poles on the imaginary axis for frequencies $\omega = k\frac{2\pi}{\hat{T}_c}$, $k = 0, 1, ...$. For these frequencyes the magnitude of the denominator of $C_{RC}(s)$ is zero, making the gain of the transfer function infinite. However instability problems may arise. To overcome this issue, we propose to modify the classical RC by adding the gain ϵ as in Fig. 1. In this case the transfer function from $e(t)$ to $u(t)$ is

$$C_{QRC}(s) = \frac{1}{1 - e^{-s\hat{T}_c} + \epsilon}. \quad (2)$$

From the analysis of the transfer function $C_{QRC}(s)$ is highlighted as for $\omega = k\frac{2\pi}{\hat{T}_c}$ its denominator is ϵ, then, like the classical repetitive controller, the gain of the transfer function is infinite for ϵ tends to zero. In the case $\epsilon \neq 0$ it is possible to impose the gain for the harmonics, from obtaining quasi repetitive controller (**QRC**). An example is show in Fig. 2, where the resonant frequency is set with $\hat{T}_c = 1$ and ϵ is set equal to 0.00, 0.01, 0.10.

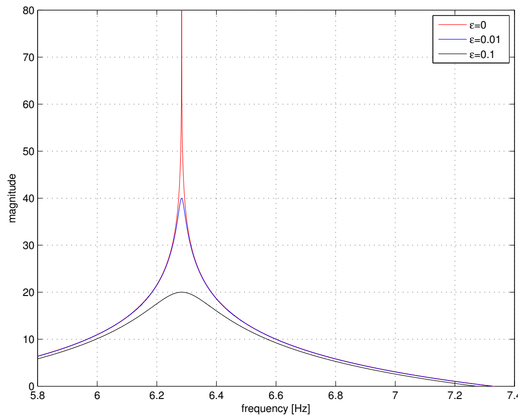

Fig. 2. Trend of the magnitude frequency response of the QRC for $\hat{T}_c = 1$, and ϵ equal to 0.00, 0.01, 0.10.

A. *A periodic disturbance cancellation approach to frequency estimation*

The model $p(t)$ of the distorted sinusoidal signal taken into consideration is :

$$p(t) = a_0 + \sum_{m=1}^{\infty} \left(a_m \cos(m\omega_c t) + b_m \sin(m\omega_c t)\right), \quad (3)$$

where m is the harmonic index, and $a_m, b_m (m = 1, 2, ...)$ are the Fourier coefficients of the mth harmonic; a_0 is the signal offset. The problem is to estimate the frequency ω_c despite of the signal harmonics number. The approach here discussed is based on the periodic disturbance cancellation methodologies based on internal model principle. The idea is to use a QRC to cancel the effect of the input signal $p(t)$, considered as a disturbance, at the output of a system. In order to cancel the bias term present in the signal, the system is chosen as the approximate linearisation of a derivative block. Fig. 3 depicts the control scheme for the instantaneous frequency estimation.

The QRC is tuned to the signal period estimated by the Frequency Estimator System (**FES**). In particular, the FES estimates the frequency by means of OSG-SOGI. Characteristic of the OSG-SOGI is to furnish also to the FES a signal that tends to sinusoid with frequency that asymptotically converges to the unknown fundamental frequency of the $p(t)$. Moreover, it is possible to demonstrate as the estimation uncertainty level depends on the parameter γ. With this aim is considered the transfer function between $p(t)$ and $y(t)$:

$$W_{dy}(s) = \frac{s(1 - e^{-s\hat{T}_c} + \epsilon)}{(\beta s + 1)(1 - e^{-s\hat{T}_c} + \epsilon) + \gamma_c s} \quad (4)$$

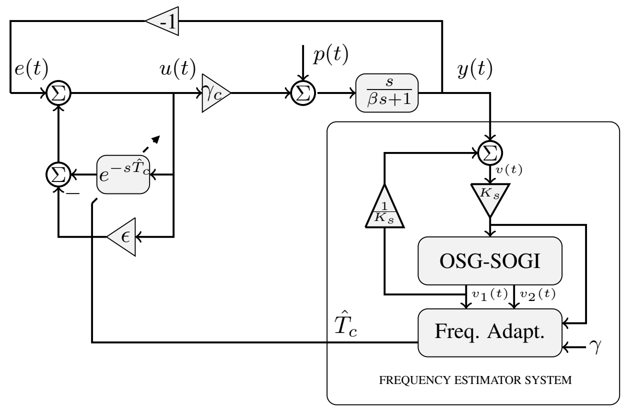

Fig. 3. Quasi repetitive control scheme.

which can be rewritten as

$$W_{dy}(s) = \frac{\frac{1}{\gamma_c}(1 - e^{-s\hat{T}_c} + \epsilon)}{1 + G(s)}, \tag{5}$$

where

$$G(s) = \frac{1}{\gamma_c}\frac{\beta s + 1}{s}(1 - e^{-s\hat{T}_c} + \epsilon). \tag{6}$$

In [18] is demonstrated as no intersections are possible between the function $\gamma_c L(\jmath\omega)$ and the negative real axis and that the closed loop system with transfer function $W_{dy}(s)$ is asymptotically stable. Moreover, by the analysis of the transfer function it is obvious the importance of the term $\epsilon \neq 0$ to guarantee the closed-loop stability.

B. Frequency estimation by means of OSG-SOGI

In order to estimate the frequency of the input signal is proposed the use of a system by an OSG-SOGI (represented in Fig. 3) and a frequency adaptive block. The OSG-SOGI is a second-order filter characterized by a gain K_s and a resonant frequency ω_s tuned by the frequency adaptive block. The output signals of the OSG-SOGI $v_1(t)$ and $v_2(t)$ are orthogonal and defined as:

$$v_1(t) = K_s\omega_s x_2(t) \tag{7}$$
$$v_2(t) = K_s\omega_s^2 x_1(t) \tag{8}$$

where the $x_1(t)$ and $x_2(t)$ are obtained according to the following differential equations:

$$\dot{x}_1(t) = x_2(t), \tag{9}$$
$$\dot{x}_2(t) = -\omega_s^2 x_1(t) - K_s\omega_s x_2(t) + K_s v(t) \tag{10}$$

As shown in [18] for ω_s constant, the signals $v_1(t)$ and $v_2(t)$ are the outputs of linear time-invariant systems $F_1(s)$ and $F_2(s)$ that represent second order filters, respectively band-pass filter and low-pass filter.

$F_1(s)$ has a bandwidth depending on the gain K_s and a resonant frequency equal to ω_s and has no attenuation and no phase shift at the resonant frequency. $F_2(s)$ is characterized by static gain K_s.

Thus, for a input sinusoidal signal $v_1(t)$ and $v_2(t)$ are orthogonal and converge exponentially to isofrequential sinusoidal signals with amplitude function of ω_s and ω_c [18].

In the case ω_s ($\omega_c \equiv \omega_s$), the OSG-SOGI generates sin/cos waves that have the same magnitude as $v(t)$ and with $v_1(t)$ in phase with the input signal. In order to determine the unknown frequency ω_c, the resonant frequency ω_s can be adapted according to the following differential equation

$$\dot{\omega}_s = -\gamma K_s\omega_s\left(K_s v(t) - v_1(t)\right) v_2(t) \tag{11}$$

where $\gamma > 0$ is the adaptation gain.

IV. PRELIMINARY EXPERIMENTAL RESULTS

Experimental results are provided in order to investigate about the effectiveness of the proposed method (**QRC-SOGI**). The measurement stand configuration is show in Fig. 4, and includes an Arbitrary Waveform Generator Tektronix AWG420 (**AWG**), a digitizer National Instruments NI USB-6211 (**ACQ**) and a personal computer (**PC**).

The marcher output of the AWG feeds the digital input of the ACQ in order to synchronize [28, 29, 30, 31, 32] the two devices. PC interfaces AWG by a GPIB port and ACQ by USB ports. CQ is set to digitize the input signal with sampling frequency of 25kHz, full-scale 2V. The acquisition starts with external trigger mode [33]. Moreover it runs a proper program developed in NI LabView environment. The main steps of the program are:

1. AWG is set to generate a specific periodic standard function. The generation and the output channel are enabled;
2. ACQ is set to digitize the input signal with sampling frequency of 25kHz, full-scale 2V. The acquisition starts with external trigger mode;
3. once the signal is acquired, the samples are transferred into the PC memory implementing QRC-SOGI.

By using this measurement stand, it has been possible to examine a wide range of signal conditions. Preliminary experimental tests investigate about the influences of the noise affecting the input signal. In this case, the AWG capability to pollute the signal with a white Gaussian noise is considered. The noise level within the range $[-113, -95]$ dBm/Hz is used. The experiment consists of generating

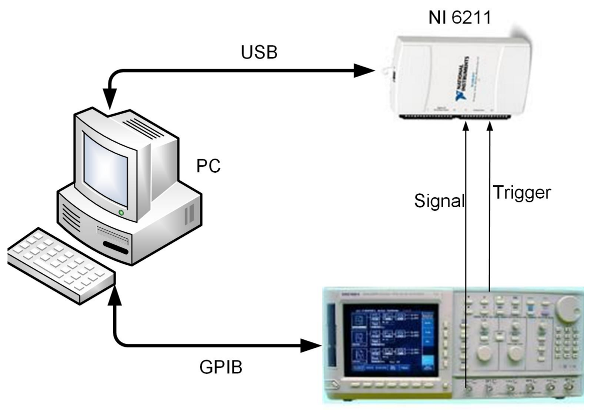

Fig. 4. Block scheme of the experimental measurement stand.

noisy sinusoidal signal with known frequency of 50Hz and with different noise levels. For each noise level, 50 experiments were conducted on different generated signals. Figs. 5 and 6 report the mean absolute error in percent and the standard deviation respectively.

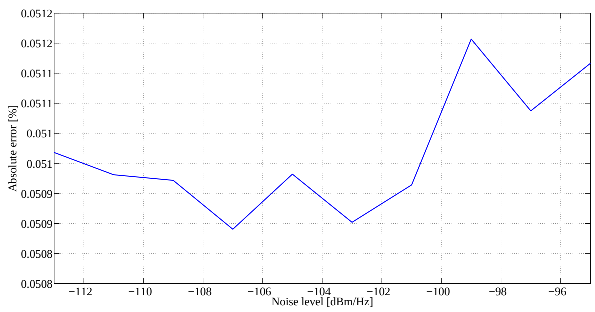

Fig. 5. Absolute error percentage in the frequency estimation in noisy signals with frequency of 50Hz. Logarithmic scale.

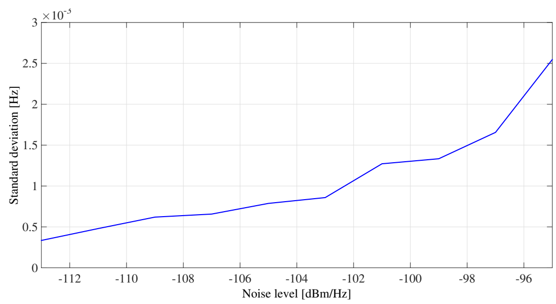

Fig. 6. Standard deviation in the frequency estimation in noisy signals with fundamental frequency of 50Hz. Logarithmic scale.

In order to evaluate both advantages and drawbacks of the presented approach, a comparison with the PLL topology proposed in [34], namely SOGI-PLL, with the enhanced phase locked loop (EPLL) proposed in [35] and with the adaptive filters bank based on SOGI (mSOGI) proposed in [36] is provided. Experiments were conducted producing by the AWG a voltage signal which contains third, fifth, and seventh harmonics, each of amplitude equal to 5% of the fundamental. Values of free parameters of each method are reported in Table 1.

Table 1. Algorithms parameters

Method	Parameters
EPLL	$K_p = 30$, $K_i = 300$, $K = 100$
SOGI-PLL	$K_p = 10$, $K_i = 100$, $K = 100$
mSOGI (m=2)	$K_{s_1} = K_{s_2} = 1, \gamma = 10$
QRC-SOGI	$K_s = 1, \epsilon = 10^{-6}$, $\gamma = 10^{-3}$, $\gamma_c = 10^{-2}$, $P(s) = \frac{s}{0.01s+1}$

The first comparison considers a distorted sinusoidal signal characterized by a frequency step from 47Hz to 52Hz. Fig. 7 shows the frequency estimation results obtained by all the considered methods. The figure highlights as QRC-SOGI and mSOGI methods track the reference frequency value with similar transient response. The other two methods present higher oscillations in the frequency estimate. Moreover, in the inset of Fig. 7 is shown an enlarged plot of the difference between the estimated frequencies and the reference one in the last part of the acquired signal. This difference shows as the QRC-SOGI provides better precision in the frequency estimation and low fluctuation in comparison with the other methods.

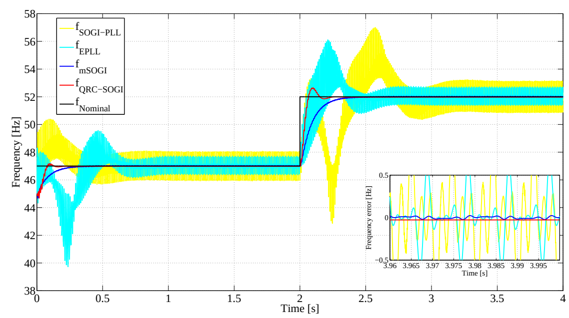

Fig. 7. Trends of the estimated frequency in the case of a frequency step from 47Hz to 52Hz obtained by means of different methods. In the inset the difference between the nominal frequency trend and the estimated frequency trends.

The second comparison takes into account a distorted sinusoidal signal characterized by a frequency sweep from 47Hz to 52Hz. Fig. 8 shows the results of the frequency estimation obtained by all the considered methods. The trend of the frequency estimated by QRC-SOGI and mSOGI methods follow the nominal trend of the frequency with better precision and low fluctuation respect to the other methods. Moreover, in the inset of the figure is shown the difference between the estimated frequencies and the nominal one. The analysis of the inset underlines as QRC-SOGI is more robust than the other methods also in the case of time-varying frequency.

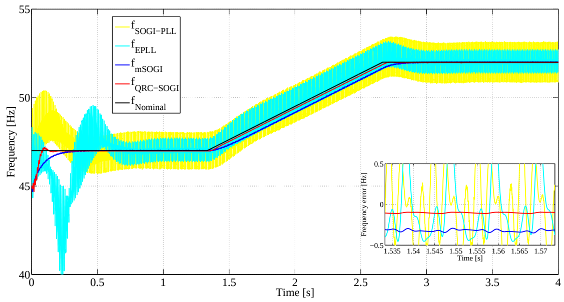

Fig. 8. Trends of the estimated frequency in the case of a frequency sweep from 47Hz to 52Hz obtained by means of different methods. In the inset the difference between the nominal frequency trend and the estimated frequency trends during the sweep.

Table 2. experimental test parameters

Parameter	Value
Amplitude	1 V
Frequency range	[50; 69] Hz
Frequency step	1 Hz
Sampling Frequency	25 kHz
Observation time	4 s

Other experimental test are devoted to analyse the results of QRC-SOGI with canonical periodic signal. With this aim AWG is forced to generate a sine, a triangular and a square waveform with known amplitude and frequency. The parameters of the generated signals are shown in Tab. 2. QRC-SOGI is tuned as shown in Tab. 1. The frequency of the test signal is increased within the frequency range of a frequency step in order to verify the quality of the QRC-SOGI in the case of constant frequency signals. For each frequency value the test is repeated 50 times. In order to verify the quality of the frequency estimates and quantify the fluctuation of the results, 50 experiments were conducted for each frequency value and the mean value and the standard deviation of the estimated frequency, after the QRC-SOGI transient, are considered.

In particular, Fig.9 shows the trend of the mean absolute error percentage in the frequency estimation for the considered canonical waveforms. The figure highlights as the error is lower than 0.072% for all test and, in the case of

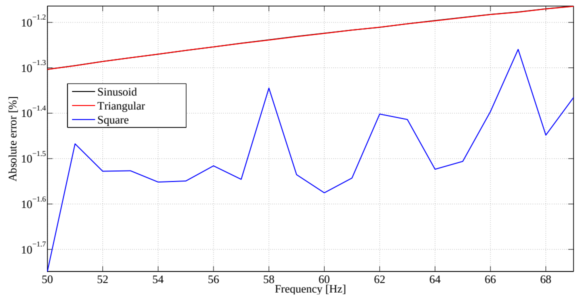

Fig. 9. Absolute error percentage in the frequency estimation. Logarithmic scale.

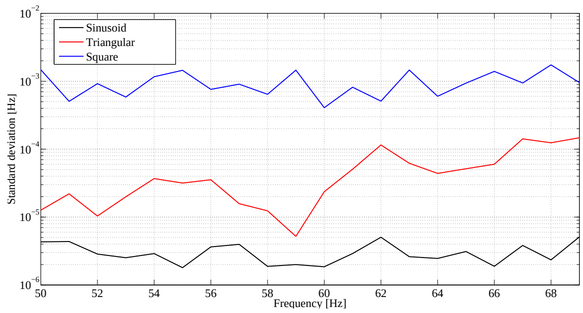

Fig. 10. Standard deviation in the frequency estimation. Logarithmic scale.

square signal, is lower than 0.056%. The standard deviation of the estimated frequency is reported in Fig. 10.

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