22nd IMEKO TC4 International Symposium & 20th International Workshop on ADC Modelling and Testing
SUPPORTING WORLD DEVELOPMENT THROUGH ELECTRICAL&ELECTRONIC MEASUREMENTS
IASI, ROMANIA, September 14-15, 2017

Effect of Quantization Errors on Power Measurement Error Based on General Cyclostationary Stochastic Process

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Abstract – **In digital energy metering systems, the effect of quantization errors on the power measurement error cannot be ignored. In this paper, to clearly analyze the effect, structural models of a digital energy metering system and a power measurement element (PME) are established. The propagation of errors among the internal components of the digital energy metering system is revealed in detail. A method for analyzing the effect of quantization errors on the power measurement error based on general cyclostationary stochastic process is proposed, and mathematical model for calculating the power measurement error affected by quantization errors is derived. The results by Monte Carlo simulation verify the validity of the proposed method. Compared to conventional methods, the proposed method for analyzing the effect of quantization errors on the power measurement error is universally applicable and solves the persistently unsolved analytical relation of the power measurement error considering the effect of quantization errors. The proposed method can serve as a theoretical basis for power measurement error estimation.**

Keywords – digital energy meter, structural model, quantization error, general cyclostationary stochastic process, power measurement error

I. INTRODUCTION

In digital substations, analog-to-digital (A/D) conversion, IEC 61850-9-2(LE) protocol framing and resolution introduce quantization errors into power measurement process [1]. Quantization errors are a factor that cannot be ignored during power measurement. Errors in power measurement can be classified as algorithmic errors and quantization errors, which are considered separately. Researchers worldwide have long conducted extensive research on algorithmic errors and quantization errors.

Algorithmic errors are theoretical errors of algorithms and therefore system errors. In the past two decades, in the time-domain measurement field, researchers worldwide have studied errors in synchronous, asynchronous and quasi-synchronous sampling measurement (SSM, ASM and QSM) algorithms [2-4] as well as expressions for the error in these algorithms [3-5]. In the frequency-domain measurement field, researchers have derived frequency-domain expressions for the error in different interpolating windowed fast Fourier transform measurement algorithms [6-7]. The aforementioned analytic methods address the problem of evaluating the algorithmic errors in measured fundamental and harmonic signals but do not include the effect of random quantization errors on the power measurement error.

Quantization errors (including A/D and protocol quantization errors) are random errors. Researchers have analyzed the effects of quantization errors by simulations [8-9]. The simulation methods need take all influencing factors into consideration, therefore, they have relatively long cycles and are relatively low efficiency. A random variable-based method for analyzing the effect of quantization errors on the power measurement error has been proposed [10]. It assumes that the probability density function follows an ideal, uniform distribution and that the number of samples is infinite. This method limits the selection of measurement algorithms. The dither technique has been presented to decrease the quantization error via averaging [11-12], but there is no analysis of the factors affecting the quantification error. Therefore, how to analyze the effect of quantization errors on the power measurement error by a universally applicable analytical method still needs further research.

In this paper, we establish structural models of a digital energy metering system and a power measurement element (PME), and analyze the propagation of errors among the internal components of the digital energy metering system. With reference to paper [13], we show that the mathematical characteristics of quantization errors are in accordance with those of a general cyclostationary stochastic process. According to this investigation, we propose a method by deriving a mathematical model for determining how the power measurement error is affected by quantization errors based on a general cyclostationary stochastic process. The advantage of this method is that it provides a clear mathematical expression for the relative power measurement error. Finally, the validity of the proposed method is verified by Monte Carlo simulation.

II. STRUCTURAL MODELS

A. Structural model of a digital energy metering system

Currently, substations generally use a digital method that integrates subsystems, such as a conventional electromagnetic instrument transformer, an analog input-merging unit and a digital energy meter, which is called the Transformers & Merging Unit (TMU) mode. The structural model of a digital energy metering system in the TMU mode established in this paper is shown in Fig. 1.

In this structural model, the electromagnetic instrument transformers transform measured parameters into a secondary voltage $u(t)$ and current $i(t)$ and then send them to the analog input-merging unit. In this merging unit, the sampled discrete instantaneous values of the voltage and current are obtained through A/D conversion. Sampling messages compliant with the IEC 61850-9-2(LE) protocol are generated by protocol framing and transported via optical fiber. The digital energy meter receives the messages and resolves the corresponding discrete values of the voltage $\hat{u}(n)$ and current $\hat{i}(n)$. In the PME, the discrete values of the instantaneous input power $\hat{p}_i(n)$ are obtained by multiplying $\hat{u}(n)$ by $\hat{i}(n)$. The discrete values of the active power $\hat{p}_o(n)$ are then calculated by removing the high-frequency components of $\hat{p}_i(n)$ using the power measurement low-pass filter (hereinafter referred to as the low-pass filter). In the end, the energy E is calculated and displayed. Because the A/D quantization errors and the protocol quantization errors affect the power measurement error through the same mechanism, in this paper, only the effect of the A/D quantization errors on the power measurement error is analyzed.

B. General cyclostationary stochastic process representation of the quantization errors

Based on the structural model of the digital energy metering system shown in Fig. 1, discrete value of the voltage $\hat{u}(n)$ and discrete value of the current $\hat{i}(n)$ are calculated as follows:

$$\hat{u}(n) = u(n) + \hat{e}_u(n) = \sqrt{2}U_1 \sin(\frac{2\pi T_s n}{T} + \phi_1) + \hat{e}_u(n) \quad (1)$$

$$\hat{i}(n) = i(n) + \hat{e}_i(n) = \sqrt{2}I_1 \sin(\frac{2\pi T_s n}{T}) + \hat{e}_i(n) \quad (2)$$

where $\hat{e}_u(n)$ and $\hat{e}_i(n)$ represent the voltage and current quantization errors resulting from A/D conversion, respectively; $u(n)$ and $i(n)$ represent the ideal instantaneous sampled values of the fundamental voltage and current in the digital substation, respectively; $n \in \mathbb{N}$, $\mathbb{N}$ represents the natural number set; U_1 and I_1 represent the effective values of the fundamental voltage and current, respectively; ϕ_1 is the phase difference between $\hat{u}(n)$ and $\hat{i}(n)$; f_0 is the frequency of the measured signal; T is the period of the measured signal ($T = 1/f_0$); f_s is the sampling frequency; T_s is the sampling period ($T_s = 1/f_s = 1/(Nf_0)$); N is the number of sampling points per cycle and, according to the IEC 61850-9-2(LE) protocol, is set to 256 or 80.

In the analog input-merging unit subsystem, analog input current signals are sampled using an analog-to-digital converter (ADC). Because both the amplitude and the frequency of the input current signals vary with time, at any time ($n = k$) of any data acquisition process ($\lambda = r$), the current quantization error $\hat{e}_{ir}(k)$ is random. The current quantization errors caused by any data acquisition process are a sample function $\hat{e}_{ir}(n)$. At any fixed time, the current quantization errors of different samples are a random variable $\hat{e}_{i\lambda}(k)$ .As time passes, the set of random variables $\{\hat{e}_{i\lambda}(1), \hat{e}_{i\lambda}(2) \cdots \hat{e}_{i\lambda}(k) \cdots\}$ is a stochastic process $\hat{e}_i(n)$.

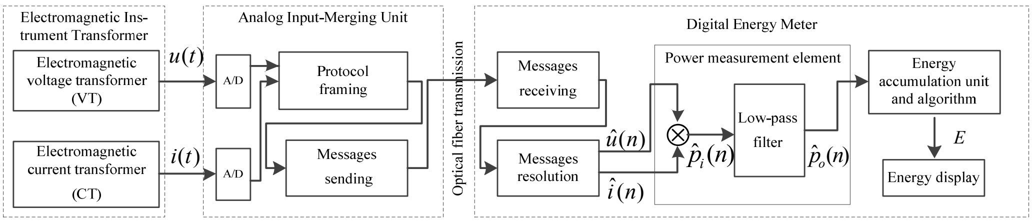

Fig. 1. Structural model of a digital energy metering system

In addition, the simulation (The simulation method is described in Section 6) shows that the mean values and the autocorrelation sequences of the current quantization error both undergo periodic variation, with N as the duration of the period. Therefore, the current quantization error is a general cyclostationary stochastic process. In other words, the above is a general cyclostationary stochastic process representation of the current quantization error $\hat{e}_i(n)$. Similarly, the voltage quantization error $\hat{e}_u(n)$ is also a general cyclostationary stochastic process.

Through the above analysis, the effect of quantization errors on the power measurement errors can be transformed into a problem that the quantization error, expressed as a general cyclostationary stochastic process, passes through a power measurement low-pass filter system. And then, the average power output by the system is estimated based on a general cyclostationary stochastic process. The problem is solved by establishing a structural model of the PME based on a general cyclostationary stochastic process.

C. Structural model of the PME based on a general cyclostationary stochastic process

Based on the PME function described in part A and considering the effect of the general cyclostationary stochastic process quantization error on the active power calculation, a structural model of the PME is established based on a general cyclostationary stochastic process, as shown in Fig. 2.

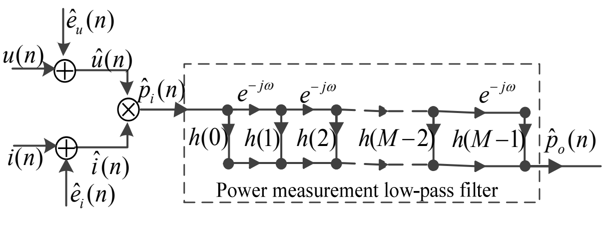

Fig. 2. Structural model of the PME

Fig. 2 shows the relationships between the seven elements ($u(n)$, $i(n)$, $\hat{e}_u(n)$, $\hat{e}_i(n)$, $\hat{u}(n)$, $\hat{i}(n)$ and the low-pass filter) and $\hat{p}_o(n)$. $\hat{p}_i(n)$ is expressed as follows

$$\begin{aligned}\hat{p}_i(n) &= \hat{u}(n)\hat{i}(n) = \left[u(n)+\hat{e}_u(n)\right]\times\left[i(n)+\hat{e}_i(n)\right] \\ &= p_i(n)+u(n)\hat{e}_i(n)+i(n)\hat{e}_u(n)+\hat{e}_u(n)\hat{e}_i(n) \\ &\approx p_i(n)+u(n)\hat{e}_i(n)+i(n)\hat{e}_u(n) \qquad n\in\mathbb{Z}\end{aligned} \tag{3}$$

where $\hat{e}_u(n)\hat{e}_i(n)$ is an infinitesimal higher-order term and can therefore be ignored. In Fig. 2, $h(n)$ represents the unit impulse response of the finite impulse response filter. It is calculated using the moving average of the sampling window and numerical integration. M is the filter length, which is related to the number of sampling points N per cycle.

This paper uses the complex rectangle algorithm as an example, and then $M = N$, $h(n)$ equals $1/N$, with $n\in[0, N-1]$ and $n\in\mathbb{Z}$. Based on Fig. 2, an input–output relationship is established for the low-pass filter as follows.

$$\begin{aligned}&\hat{p}_o(n) = \sum_{k=0}^{N-1} h(k)\hat{p}_i(n-k) = \sum_{k=0}^{N-1} h(k)p_i(n-k) \\ &+\sum_{k=0}^{N-1} h(k)\left[u(n-k)\hat{e}_i(n-k)\right]+\sum_{k=0}^{N-1} h(k)\left[i(n-k)\hat{e}_u(n-k)\right] \\ &= p_o(n)+\hat{e}_i^u(n)+\hat{e}_u^i(n)\end{aligned} \tag{4}$$

To analyze the effect of quantization errors on the power measurement error more clearly, the structural model of the PME is expressed in parallel as shown in Fig. 3.

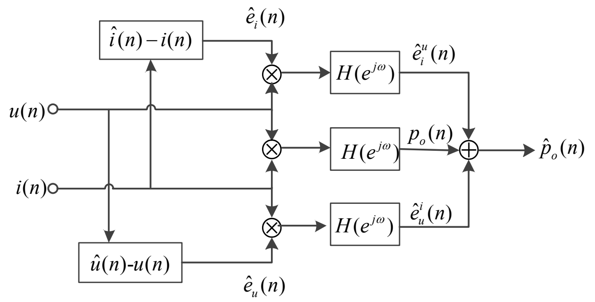

Fig. 3. Parallel structural model of the PME

In this model, $p_o(n)$ is the discrete value of the active output power in the absence of the effect of quantization errors. $\hat{e}_i^u(n)$ and $\hat{e}_u^i(n)$ represent the output error of the PME resulting from $\hat{e}_i(n)$ and $\hat{e}_u(n)$, respectively. $\hat{e}_i^u(n)$ is obtained by passing the general cyclostationary stochastic process $\hat{e}_i(n)$ through the linear low-pass filter (a linear system) after modulating it with the known signal $u(n)$. Similarly, $\hat{e}_u^i(n)$ is obtained. Thus, $\hat{e}_i^u(n)$ and $\hat{e}_u^i(n)$ are both general cyclostationary stochastic processes. Therefore, the key to determining the power measurement error lies in solving for the power of $\hat{e}_i^u(n)$ and $\hat{e}_u^i(n)$. This paper proposes a method for calculating the power of $\hat{e}_i^u(n)$ and $\hat{e}_u^i(n)$ based on the idea of passing a general cyclostationary stochastic process through a linear system. This method is to derive a mathematical model for calculating the effect of quantization errors on the power measurement error.

III. MATHEMATICAL MODEL

The average power of a general cyclostationary stochastic process can be represented by the average of the autocorrelation sequence of the process with a time shift of 0. The calculation of the average power of $\hat{e}_i^u(n)$ ($\hat{P}_i^u$) is shown as an example to demonstrate the process.

$$\begin{aligned}
\hat{P}_i^u &= \bar{\hat{R}}_i^u(n,0) = \frac{1}{N}\sum_{n=0}^{N-1}\hat{R}_i^u(n,0) \\
&= \frac{1}{N}\sum_{n=0}^{N-1}E[\hat{e}_i^u(n)\hat{e}_i^u(n)] \\
&= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1}h(k)h(l)u(n-k)u(n-l)E\left[\hat{e}_i(n-k)\hat{e}_i(n-l)\right] \\
&= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1}h(k)h(l)u(n-k)u(n-l)\sigma_i^2\delta(k-l) \\
&= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}h^2(k)U_1^2\sigma_i^2\left[1-\cos(2k\beta-2n\beta-2\phi_1)\right]
\end{aligned} \quad (5)$$

where $\hat{R}_i^u(n,0)$ represents the autocorrelation sequence of $\hat{e}_i^u(n)$ starting at time n with a time shift of 0; σ_i^2 represents the average power of $\hat{e}_i(n)$; and $\tilde{E}_R(N)$ is the quadratic sum of $h(n)$. In the complex rectangle algorithm, $\tilde{E}_R(N)$ equals $1/N$. Eq. (5) can be expressed as follows:

$$\hat{P}_i^u = \tilde{E}_R(N)U_1^2\sigma_i^2 - \frac{U_1^2\sigma_i^2}{N^2}\mathrm{Re}\left[\frac{1}{N}\sum_{n=0}^{N-1}e^{-2(n\beta+\phi_1)j}\sum_{k=0}^{N-1}e^{2k\beta j}\right] \quad (6)$$

In Eq. (6),

$$\begin{aligned}
\sum_{n=0}^{N-1}e^{-2(n\beta+\phi_1)j} &= \frac{1-e^{-2N\beta j}}{1-e^{-2\beta j}}\cdot e^{-2\phi_1 j} \\
&= \frac{e^{-N\beta j}}{e^{-\beta j}}\cdot\frac{e^{N\beta j}-e^{-N\beta j}}{e^{\beta j}-e^{-\beta j}}\cdot e^{-2\phi_1 j} \\
&= \frac{\sin(N\beta)}{\sin(\beta)}e^{-[(N-1)\beta-2\phi_1]j}
\end{aligned} \quad (7)$$

Similarly,

$$\sum_{k=0}^{N-1}e^{2k\beta j} = e^{(N-1)\beta j}\frac{\sin(N\beta)}{\sin(\beta)} \quad (8)$$

Substituting Eqs. (7) and (8) into Eq. (6), we obtain

$$\hat{P}_i^u = \tilde{E}_R(N)U_1^2\sigma_i^2 - \frac{U_1^2\sigma_i^2}{N^2}\cdot\frac{1}{N}\frac{\sin^2(N\beta)}{\sin^2(\beta)}\cos(2\phi_1) \quad (9)$$

Next, $\hat{P}_i^u$ is calculated in the SSM and ASM algorithm modes.

In the SSM algorithm mode,

$$\beta = 2\pi\frac{T_s}{T} = \frac{2\pi}{T/T_s} = \frac{2\pi}{N} \quad (10)$$

Substituting Eq. (10) into Eq. (9), we obtain

$$\hat{P}_i^u = \tilde{E}_R(N)U_1^2\sigma_i^2 \quad (11)$$

In the ASM algorithm mode,

$$N\beta = 2\pi\frac{T_sN}{T} = 2\pi\frac{T+\Delta T}{T} = 2\pi+\Delta(f) \quad (12)$$

where ΔT is referred to as the synchronization deviation or periodic deviation and $\Delta(f)$ is the angular frequency of the sampling truncation ($\Delta(f)=2\pi\Delta T/T$). Since $\Delta(f)<<1$ and $\beta<<1$, substituting Eq. (12) into Eq. (9), we obtain

$$\begin{aligned}
\hat{P}_i^u &= \tilde{E}_R(N)U_1^2\sigma_i^2 - \frac{U_1^2\sigma_i^2}{N^3}\frac{\sin^2(2\pi+\Delta(f))}{\sin^2(\beta)}\cos(2\phi_1) \\
&= \tilde{E}_R(N)U_1^2\sigma_i^2 - \frac{U_1^2\sigma_i^2}{N^3}\left[\frac{\Delta(f)}{\beta}\right]^2\cos(2\phi_1) \\
&= \tilde{E}_R(N)U_1^2\sigma_i^2\left[1-\frac{1}{N^2}\left(\frac{2\pi\Delta T/T}{2\pi T_s/T}\right)^2\cos(2\phi_1)\right] \\
&= \tilde{E}_R(N)U_1^2\sigma_i^2\left[1-\left(\frac{\Delta T}{NT_s}\right)^2\cos(2\phi_1)\right]
\end{aligned} \quad (13)$$

Therefore, the average power of $\hat{e}_i^u(n)$ in the SSM and ASM algorithm modes can be expressed as

$$\hat{P}_i^u = \tilde{E}_R(N)U_1^2\sigma_i^2\left[1-\left(\frac{\Delta T}{NT_s}\right)^2\cos(2\phi_1)\right] \quad (14)$$

Obviously, $\Delta T = 0$ in the SSM algorithm mode, and $\Delta T \neq 0$ in the ASM algorithm mode. Similarly, the average power of $\hat{e}_u^i(n)$ ($\hat{p}_u^i$) can be calculated. Therefore, based on Eqs. (4) and (14) a mathematical model for calculating the power measurement error that considers the effect of quantization errors can be derived as Eq. (15).

$$\hat{P}=\sqrt{\hat{P}_i^u+\hat{P}_u^i+2\operatorname{cov}\left(\hat{e}_i^u(n)\,,\,\hat{e}_u^i(n)\right)}$$
$$=\sqrt{\hat{P}_i^u+\hat{P}_u^i+2\operatorname{cov}\left(\frac{1}{N}\sum_{k=0}^{N-1}\left[u(n-k)\hat{e}_i(n-k)\right],\frac{1}{N}\sum_{k=0}^{N-1}\left[i(n-k)\hat{e}_u(n-k)\right]\right)}$$
$$=\sqrt{\hat{P}_i^u+\hat{P}_u^i+\frac{2}{N^2}\sum_{k=0}^{(N/2)-1}\operatorname{cov}\begin{bmatrix}\left[u(n-k)\hat{e}_i(n-k)\right]+\left[i(n-k)\hat{e}_u(n-k)\right],\\ \left[u\left(n+\frac{N}{2}-k\right)\hat{e}_i(n+\frac{N}{2}-k)\right]+\\ \left[i\left(n+\frac{N}{2}-k\right)\hat{e}_u(n+\frac{N}{2}-k)\right]\end{bmatrix}} \quad (15)$$
$$=\sqrt{2\left(\hat{P}_i^u+\hat{P}_u^i\right)}$$
$$=\sqrt{2\tilde{E}_R(N)\left(U_1^2\sigma_i^2+I_1^2\sigma_u^2\right)\left[1-\left(\frac{\Delta T}{NT_s}\right)^2\cos(2\phi_1)\right]}$$

The theoretical average output power of the PME is

$$P=U_1I_1\cos(\phi_1) \quad (16)$$

The mathematical expression for the relative power measurement error γ can be derived from Eqs. (15) and (16).

$$\gamma=\frac{\hat{P}}{P}=\frac{1}{\cos(\phi_1)}\sqrt{2\tilde{E}_R(N)\left(\frac{\sigma_u^2}{U_1^2}+\frac{\sigma_i^2}{I_1^2}\right)\left[1-\left(\frac{\Delta T}{NT_s}\right)^2\cos(2\phi_1)\right]} \quad (17)$$

Eq. (17) shows that regardless of the sampling mode, γ decreases as the effective values of the voltage, current, power factor and filter length increase and increases with the average power of the quantization error. In the ASM algorithm mode, γ not only satisfies the above relation with the aforementioned influencing factors, but also increases with the periodic deviation.

IV. SIMULATION AND VERIFICATION

With reference to the OIML R46-1/-2 international recommendation, the extreme value of the measurement error is used to describe the maximum permissible error. Therefore, the theoretical and simulated extreme values of the relative power measurement error are compared to verify the validity of the proposed method.

In this study, 30,000 repeated simulations were performed using the Monte Carlo method [9]. First, in the scope of the one thousandth positive and negative values of the voltage and current, we generated 30,000 values as the effective values of 30,000 measured signal samples with uniform distributions such that the voltage and current analog signal sample values were different in each power measurement test. Then, they were quantized using a 16-bit ADC to obtain the voltage and current samples involved in the quantization error. Finally, the power measurement algorithm was used to calculate the power, and the relative error was obtained.

The parameters for the simulation were as follows. In the SSM algorithm mode, $f_0=50\text{Hz}$, $U_1=1\text{V}$, $I_1=1\text{A}$ or $I_1=0.1\text{A}$, $N=256$ or $N=80$, $\cos(\phi_1)=1$, $\cos(\phi_1)=0.8\text{C}$ or $\cos(\phi_1)=0.5\text{L}$ and the measured voltage and current were sampled for three complete periods. In the ASM algorithm mode, $f_0=49\text{Hz}$, $f_0=50\text{Hz}$ or $f_0=51\text{Hz}$, $U_1=1\text{V}$, $I_1=1\text{A}$, $N=256$, $\cos(\phi_1)=1$, $f_s=12800\text{Hz}$ and the measured voltage and current were sampled for three periods.

We use γ_t to represent the theoretical extreme values of the relative power measurement error within the 95% confidence interval. Based on the aforementioned parameters settings, the maximum values of the average power of the voltage and current quantization errors ($\sigma_{u\max}^2$ and $\sigma_{i\max}^2$) under different conditions were obtained. Consequently, we calculated γ_t by substituting aforementioned parameters and $\sigma_{u\max}^2$, $\sigma_{i\max}^2$ into Eq. (17) with 1.96 γ as the relative error bandwidth (It is equivalent to 95% confidence interval). We use γ_s to represent the simulated extreme values of the relative power measurement error within the 95% confidence interval. To obtain γ_s, we research the distribution characteristics of the relative error. The Kolmogorov-Smirnov test method was adopted in this paper to test whether error obeys normal distribution. We record the maximum relative error value within the 95% confidence interval if the error obeys the normal distribution or draw a frequency distribution histogram to calculate the maximum relative error value with a 95% probability.

The Kolmogorov-Smirnov test results indicate that the relative power measurement error follows the normal distribution when the current is 1 A, and it doesn't follow the normal distribution when the current is 0.1 A. Fig. 4 shows the frequency distribution histogram of the power measurement relative error using the complex rectangle algorithm when $U_1=1\text{V}$, $I_1=0.1\text{A}$, $f_0=50\text{Hz}$, $\cos(\phi_1)=1$ and the filter length is 256.

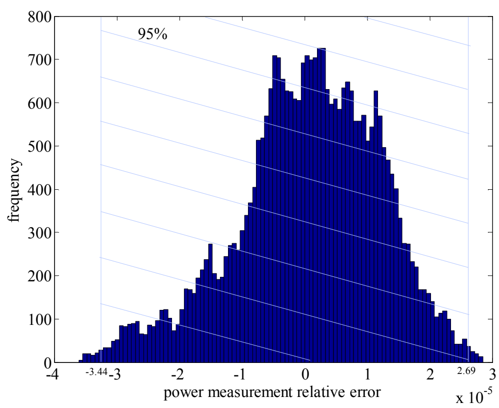

Fig. 4. Frequency distribution histogram of the power measurement relative error when using the complex rectangle algorithm (I_1=0.1 A)

Based on the aforementioned methods, γ_t (the theoretical extreme values of the relative error) and γ_s (the simulated extreme values of the relative error) within 95% confidence interval were obtained. Table 1 and Table 2 list the results including $\sigma_{u\max}^2$, $\sigma_{i\max}^2$, γ_t and γ_s in the SSM and ASM algorithm modes, respectively.

By analyzing the data shown in Table 1 and Table 2, it is found that γ_t and γ_s have the same order of magnitude and that γ_t is greater than γ_s in different sampling modes under various influencing factors. These results indicate that γ_t can cover γ_s. In other words, γ_t covers more comprehensive random condition effects of impact factors. Consequently, the validity of the proposed method is verified.

V. CONCLUSIONS

The structural models of the digital energy metering system and the power measurement element have been established, which could reveal the propagation of errors among the internal components of the digital energy metering system in detail. The mathematical model for calculating the power measurement error that considers the effect of quantization errors in the SSM and ASM algorithm modes has been derived based on general cyclostationary stochastic processes. The factors that influence the relative power measurement errors have been revealed by the mathematical model. The validity of the proposed method has been verified using Monte Carlo simulation. The proposed method is universally applicable and can serve as a theoretical basis for power measurement error estimation.

VI. ACKNOWLEDGMENTS

This work is supported by the National High Technology Research and Development Program of China (863 Program, No. 2015AA050404) and the National Natural Science Foundation of China (No. NSFC-51577006).

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Table 1. $\sigma_{u\max}^2$, $\sigma_{i\max}^2$, γ_t and γ_s in the SSM algorithm mode.

error term	*N*=256						*N*=80					
	I_1=1A			I_1=0.1A			I_1=1A			I_1=0.1A		
	$\cos(\phi_1)=1$	$\cos(\phi_1)=0.8c$	$\cos(\phi_1)=0.5l$	$\cos(\phi_1)=1$	$\cos(\phi_1)=0.8c$	$\cos(\phi_1)=0.5l$	$\cos(\phi_1)=1$	$\cos(\phi_1)=0.8c$	$\cos(\phi_1)=0.5l$	$\cos(\phi_1)=1$	$\cos(\phi_1)=0.8c$	$\cos(\phi_1)=0.5l$
$\sigma_{u\max}^2\left(10^{-10}\right)$	4.63	4.63	4.63	4.63	4.63	4.63	4.55	4.55	4.55	4.55	4.55	4.55
$\sigma_{i\max}^2\left(10^{-10}\right)$	4.63	4.63	4.63	4.58	4.58	4.58	4.55	4.55	4.55	4.48	4.48	4.48
$\gamma_t\left(10^{-6}\right)$	5.27	6.59	10.54	37.26	46.58	74.52	9.35	11.69	18.70	65.92	82.40	131.84
$\gamma_s\left(10^{-6}\right)$	3.42	3.49	5.54	34.44	34.48	35.50	7.16	7.89	9.74	63.90	68.36	84.34

Table 2. $\sigma_{u\max}^2$, $\sigma_{i\max}^2$, γ_t and γ_s in the ASM algorithm mode.

error term	$f_0=49\text{Hz}$	$f_0=50\text{Hz}$	$f_0=51\text{Hz}$
$\sigma_{u\max}^2\left(10^{-10}\right)$	4.65	4.63	4.65
$\sigma_{i\max}^2\left(10^{-10}\right)$	4.65	4.63	4.65
$\gamma_t\left(10^{-6}\right)$	5.28	5.27	5.29
$\gamma_s\left(10^{-6}\right)$	4.48	3.42	4.57

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