Study of Cylindrical Dielectric Resonators for Measurements of the Surface Resistance of High Conducting Materials

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***Abstract*– We designed a series of dielectric resonators (DR) for the surface resistance measurements of conducting and superconducting samples. The performances of the designed DRs were studied with respect to different geometrical parameters in view of their operation at room temperature. Attention is paid to the measurement of conducting samples with little differences in the surface resistance.**

Keywords –Dielectric resonator, Conducting materials, Surface resistance

I. INTRODUCTION

Microwave techniques are widely diffused as methods to characterise a wide range of materials such as dielectrics, conductors (or superconductors), and magnetic materials. Traditionally, microwave material characterization techniques are divided into two classes: non-resonant or wideband [1, 2] and resonant (at fixed frequency) [3]. Measurements of the surface resistance R_s of metallic materials, including coatings, multilayers and superconductors, require reliable, sensitive and precise methods of the materials characterization. A particular aspect resides in the accurate measurement on conducting samples withsmall differences in surface resistance, a typical issue that arises when one needs to evaluate the effect of some thermal, chemical or mechanical treatment (including thin-film coating of metals).

In this article, we focuson surface impedance (Z_s) measurements obtained throughthe dielectric resonator (DR) technique [4,5, 6, 7]. The DR method was chosen due to its high sensitivity, compactness, and the capability to act as a non-destructive method (contactless), which allows avoiding the sample patterning [8]. A variety of different DRs exists. With the acronym DR we refer here to the class of dielectric loaded resonators, where a conducting shield around a dielectric rod acts to reduce the losses. We also confine ourselves to the cylindrical symmetry. The focus is on room temperature measurements of conductive samples with surface resistance similar to the metal shield surface resistance. Hence, high discrimination capability will be needed, requiring a demanding setup both in terms of sensitivity and in terms of calibration of the resonator and subsequent removal of the background contribution.

II. MEASURED QUANTITIES AND DR CHARACTERISTICS

In the volume or surface perturbation technique one obtains the surface impedance $Z_s = R_s + iX_s$ through only two measured quantities of the resonator, the resonant frequency f_{res} and the quality factor Q, by means of the following relations [6]:

$$\frac{1}{Q} = \frac{R_s}{G_s} + \frac{R_m}{G_m} + \eta \tan\delta, \tag{1}$$

$$-2\frac{\Delta \mathrm{f_{res}}}{f_{res}} = \frac{\Delta X_s}{G_s} + \frac{\Delta X_m}{G_m} + \eta\frac{\Delta\,\varepsilon\prime}{\varepsilon\prime} \tag{2}$$

Here $G_{s,m}$are geometrical factors related to the surface occupied by the sample and the shield (of known surface impedance $Z_m=R_m+iX_m$), respectively. $G_{s,m}$can be estimated through electromagnetic simulations or analytical models, if available. η is a constant value called dielectric filling factor, it can be calculated on the basis of the geometry, and in many cases $\eta \approx 1$. The dielectric permittivity of the dielectric is represented as $\varepsilon = \varepsilon_0\varepsilon'(1+i\tan\delta)$, where tanδ is the loss tangent. "Δ" represents a variation with an external parameter.

It should be noted that in the case of normal conductors or superconductors in the normal state, $R_s=X_s$ and Z_s becomes:

$$Z_S = (1+\mathrm{i})\sqrt{\omega\mu_0/(2\sigma)},$$

where the conductivity σ is real and equal to d.c. conductivity, ω is the angular frequency and μ_0 is the vacuum permeability.

We designed a family of DRs as sensitive devices for the characterization of the conductivity of large metallic

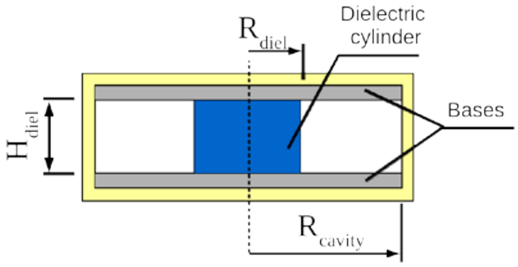

Fig. 1. Sketch of the explored cylindrical DR geometry and identification of the main geometrical dimensions. The dielectric rod, of radius R_{diel} and height H_{diel}, is placed coaxially to a metal cylindrical cavity of radius R_{cavity}, sandwiched between the two conducting bases.

samples, with the aim of characterizing numerically and, in perspective, experimentally the devices. The relation of DR sensitivity to R_s can be found from Eq. (1) [6]:

$$S = \frac{\partial Q}{\partial R_s} = -\frac{Q^2}{G_s} \qquad (3)$$

Thus, to attain high sensitivity, the DR should have high Q-factor and small sample geometrical factorG_s.

For accurate and sensitive measurements, special attention should be paid to the choice of the material from which the DR is made. In the following we describe some of the main characteristics useful for the design.As DR, we have chosen the Hakki-Coleman geometry [9] due to its simple structure, well-known electromagnetic model and (usually) high Q [10]. Itstypical structure is presented in Fig. 1. Here the dielectric rod with radius $R_{diel}$and height $H_{diel}$is sandwiched between two conducting plates and placed in a cylindrical cavity of the radius R_{cavity}. TE_{01n} modes are usually exploited for the study of planar materials,to generate in-plane microwave currents on the sample surface. In this geometry, the base is replaced by the sample under study with surface resistance R_s. We concentrate here on the TE_{011} mode as the operating resonant mode, by virtue of its large separation (in frequency) from other modes.

The geometry of the assembly, and in particular the geometry of the dielectric rod, determine the resonant frequency, the Q factor, the mode chart, and the geometrical factors, whence the sensitivity. However, such parameters are not free from practical considerations: by reducing the operating frequency one can benefit from lower-cost Vector Network Analyzers (VNA), at the expense of increasing the dimensions. In turn, large dimensions require large samples (whence a reduced flexibility of the device), otherwise the sensitivity worsens. We selected as an acceptable frequency range a resonant frequency not higher than 20 GHz. We will see that this choice allows to safely measure samples of linear dimensions of the order of 10 mm. The operating frequency range is, mainly, defined by the permittivity and size of the dielectric cylinder.

An additional constraint is the sensitivity of the DR at room temperature, which requires high Q. It can be deduced from Eq. (1) that low loss (low tan δ) dielectrics and high conductivity metals for the resonating assembly are required.As a first consequence, highly conductive, large diameter cavities are required in order to reduce the conductive losses in the lateral wall. The typical choice is oxygen free copper, with $R_s < 30\text{m}\Omega$ at 10 GHz. Moreover, low-losses dielectrics are unavoidable: from Eq. (1), it is seen that tan δ is a limiting factor in Q.Single crystal sapphire (Al_2O_3) cylinders are the typical choice for the right combination of properties, although care should be taken for possible miscut of the crystals [11]:low dielectric losses, tan $\delta \sim 10^{-6}$, and sufficiently large permittivity, $\varepsilon' \approx 9$, which allows to reduce the cavity diameter and then to measure small-size samples.

III. GEOMETRY DESIGN

The choice of the DR dimensions can be done using analytical models [12] as well as finite elements simulations[13]. Here we combine both methods to obtain more reliable results. We look for an optimization of the geometry of the resonator with respect to the sensitivity and the practical operation (absence of spurious nearby modes, robustness with respect to tolerances in the dimensions of the sapphire rod). Thus, two features (at least) must be taken into account at the same time: the mode separation and the sensitivity of the selected measuring mode (TE_{011}), as a function of the aspect ratio R_{diel}/H_{diel}(R_{diel} and$H_{diel}$are the radius and height of the dielectric rod, see Fig.1).

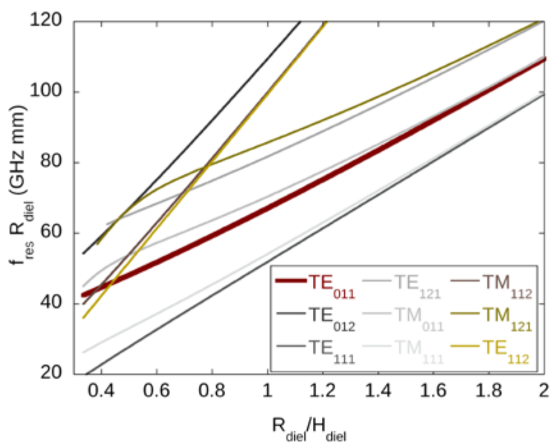

Fig. 2. Partial mode chart around the TE_{011} mode as a function of the dielectric cylinder aspect ratio.

Simulationsyieldingthe resonant frequencies of modes close tothe TE_{011} are reported in Fig. 2in the customary plot $f_{res}\cdot R_{diel}$vs.R_{diel}/H_{diel}. It should be emphasized that, in the simulation,a $R_{cavity} >> R_{diel}$ was chosen to minimize the effect of the cavity shield on $f_{res}$and Q. We show that in the interval of R_{diel}/H_{diel} = 0.65 – 1.1 the TE_{011}mode is characterized by the best mode separation, and then this is the region where the measuring device should be designed to avoid mode contamination.

Unfortunately the sensitivity is inversely affected by R_{diel}/H_{diel}: since Qdecreases with R_{diel}/H_{diel}, as we have verified with the simulations, it contributes to the decrease of sensitivity since$S \sim Q^2$. This is only partially compensated by the change of the geometrical factor G_s

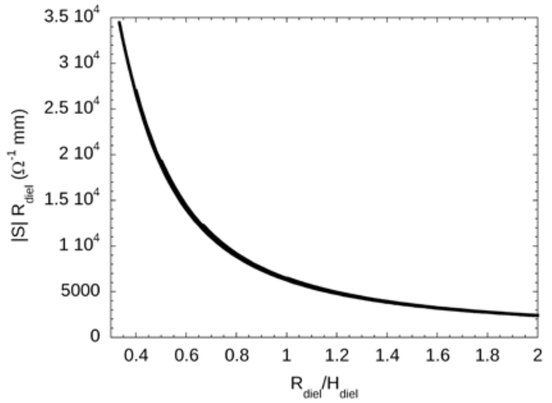

Fig.3. Universal plot of the product $|S|\cdot R_{diel}$[S is the sensitivity of the resonator, from Eq.(3)], as a function of the aspect ratio of the sapphire rod R_{diel}/H_{diel}, for the TE_{011} mode. Although the sensitivity rises at small R_{diel}, the region R_{diel}/H_{diel}< 0.6 is affected by spurious nearby modes (see Fig.2). Moreover, an appropriate design benefits from a relative insensitivity of $|S|\cdot R_{diel}$ with respect to mechanical tolerances.

with R_{diel}/H_{diel}.The full simulation is reported in Fig. 3, where we report the almost universal curve $|S|\cdot R_{diel}$vs. R_{diel}/H_{diel}.

Finally, we separately studied the possible variationof the resonant frequencies of the spurious modes because ofpossible resonator inhomogeneities, micro gaps or slightly inaccurate dielectric mounting. An aspect ratio R_{diel}/H_{diel} = 0.7 – 0.95 gave best isolation of the TE_{011}mode from other modes, and then it was selected for the final design. The final optimization with sensitivity and mode separation gave a value R_{diel}/H_{diel} ~ 0.8.

Based on the operational frequency f_{res}around 15 GHz we have selected a dielectric withR_{diel} = 3.65 mm, H_{diel} = 4.50 mm and$TE_{011}$$f_{res}$ = 16.36 GHz, obtaining minimum frequency distance to the nearest mode 1.2 GHz. With this particular choice, we have considered the geometrical most favourable configuration, where the sample constitutes one of the bases. The same base can be substituted by a Cu block. All the remaining metal parts are made of Cu, with R_m = 32 mΩ (at f_{res}).

In order to evaluate the discrimination capabilities of the designed resonator, we took a cavity diameter R_{cavity} = 20 mm, and the sapphire dielectric losses and permittivity as $\tan\delta = 10^{-6}$ and $\varepsilon' = 9.58$. We then calculated the Q variation with the change of R_S of a single base of the resonator (the sample, in practical measurements). The results are reported in Table 1. To comment on the results, one should bear in mind that each real measurements will require a partial disassembly of the resonator, so that Q variations$\Delta Q \sim 50$ can arise due to the procedure: we then consider as an appreciable difference only$\Delta Q \geq 100$. We then find from Table 1 that even a sample surface resistance only 10% higher than that of Cu gives rise to an appreciable change in Q[14], and then it should be detected with sufficient accuracy.

Table1. Changes in Q due to one resonator base having different R_s.

R_s (mΩ)	32	35	40	50	65	85
Q	5200	5100	4940	4620	4140	3500

IV. EXPERIMENTAL VALIDATION

To experimentally validate the simulated structure we performed a series of measurements. For the experimental measurements, a pre-existing copper cavity cell has been used. The details of the cavitystructure could be found in ref.[6]. Briefly, the cell has been designed for Hakki-Coleman type DRs and it is capable to work in transmission. The magnetic coupling is realized through loops made from the central conductor

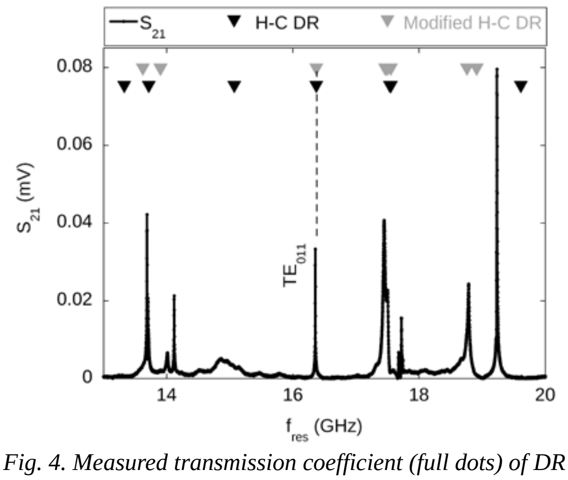

Fig. 4. Measured transmission coefficient (full dots) of DR with sapphire rod having R_{diel} =3.65 mm andH_{diel} = 4.50 mm. Black triangles indicate the resonant frequencies obtained through the analytical model with Hakki-Coleman geometry; gray triangles indicate resonant frequencies obtained by the full electromagnetic simulationreproducing the experimental measuring cell geometry. Vertical position of triangles is arbitrary and chosen for the sake of representation. The TE_{011} mode is highlighted.

of two tiny coaxial cables, which enter from the upper base through small holes. This structure is capable to accommodate large-size dielectric rods.

Sapphire rods with different aspect ratios R_{diel}/H_{diel} have beenstudied. Wehave selected sapphire rods with R_{diel}/H_{diel} ratio0.43, 0.81 and 1.11 to cover the large range of the dielectrics dimensions where TE_{011} are well separated from the spurious modes. We obtain good matchings between the simulated values of f_{res} and the experimentally measured values.

Taking into account the geometry of the experimental measuring cell,we performed an additional full electromagnetic (e. m.) simulation of the DR structure close to the real one. As a typical example, we present the part of the results regarding the sapphire rod with the dimensions chosen in the previous section (see Sec. III), i.e. R_{diel} = 3.65 mm, H_{diel} = 4.50 mm, hence R_{diel}/H_{diel} = 0.81. In Fig. 4 we report the comparison between the f_{res} as measured by the transmission coefficient frequency sweep,as obtained through the analytical model and as obtained by the full electromagnetic simulation. In the latter case, the real experimental cell was modelled, including the coupling holes through one of the bases but excluding the coupling coaxial cables.The full e.m. simulation, reproduces very well the experimental TE_{011} mode frequency, whereas the analytical model, despite its oversimplification, attains a quite good agreement with the data. We observe a larger discrepancy in the estimate of the resonant frequencies f_{res} of the spurious modes,with both the analytical and the full e. m. simulation. This is probably due to the various degrees of approximation of the cell geometry used for the computations.

In any case, the discrepancieslead only to a slight under-/over-estimate of the TE_{011} mode separation from the spurious modes. In overall, the agreement between the experimental and simulated f_{res} (Fig. 4)is a validation of the design process.

V. CONCLUSIONS

In this communication we designed and optimized a dielectric resonator for non-destructive microwave measurements at room temperature of conducting samples. We focused on measurements on materials with surface resistance close to the R_s of Cu.A careful geometrical design and suitable choice of the resonator materials allows to perform the desired measurements within common experimental capabilities. We have validated the design through experimental measurements, showing a very good agreement between design and measured resonant frequencies of the proposed DR.

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