

Artificial Bee Colony Algorithm for Peak-to-Peak Factor Minimization in Periodic Signals

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Abstract – Minimization of the peak-to-peak factor in multiharmonic signals is highly desirable in many signal processing applications such as impedance spectroscopy and acoustic signal generation. This minimization allows optimized signal to noise ratio within a given signal amplitude range. Careful selection of the phases of each harmonic of a multiharmonic signal may lead to much better peak-to-peak factors when compared to zero or random phases. This paper presents the use of an Artificial Bee Colony algorithm in the minimization of the peak-to-peak factor by phase optimization of multiharmonic signals. The obtained results are compared with the Schroeder formula and the clipping algorithm. The computation time of each algorithm is also presented for comparison.

Keywords – Peak-to-peak factor minimization, Artificial Bee Colony algorithm, Multiharmonic signals, Schroeder formula, Clipping algorithm

I. INTRODUCTION

In many measurement applications it is important to maximize the signal power of a periodic signal while staying within the available amplitude range, to maximize the signal to noise ratio. Examples include the use of multiharmonic signals in impedance measurements [1] and the generation of acoustic signals [2] among others.

A multiharmonic signal with L frequency components may be defined as

$$s(t) = \sum_{l=1}^L a_{k_l} \cos(2\pi f_{k_l} t + \phi_{k_l}) \quad (1)$$

where a_{k_l} are the amplitudes, $f_{k_l} = k_l f_{fund}$ the frequencies which are integer multiples of the fundamental frequency f_{fund} and ϕ_{k_l} the phases of each frequency component. While the power of the signal is solely defined by the amplitudes of the frequency

components, the amplitude range of the signal is critically affected by the phases of each component. A quantitative measure of the normalized amplitude range is the peak to peak factor [2]

$$p = \frac{\max(s(t)) - \min(s(t))}{2\sqrt{2}s_{rms}} \quad (2)$$

where

$$s_{rms} = \sqrt{\frac{1}{2} \sum_{l=1}^L a_{k_l}^2} \quad (3)$$

is the root mean square value of signal $s(t)$.

To maximize the signal power within an available amplitude range, the phases of each component should be chosen such that (2) is minimized. To this end, different methods have been proposed, such as a recursive formula proposed by Schroeder [3] or a clipping algorithm [4] as proposed by van den Bos.

The Artificial Bee Colony (ABC) algorithm is part of the family of evolutionary algorithms that attempts to mimic the behavior of bee swarms while searching for adequate foods sources [5-9]. The use of three different types of bees allows the ABC algorithm to efficiently explore the global search space while being able to also perform local searches. Because of its low complexity, strong robustness and small parameter set, it has been widely applied in optimization problems.

This paper presents the results obtained by the application of ABC in the phase optimization of multiharmonic signals by minimization of the peak-to-peak factor. The three test cases in [4] are used to compare the performance of the ABC algorithm with the Schroeder formula and the clipping algorithm. Zero phases and random phases are also presented for comparison purposes. The execution time performance of each algorithm is also presented.

II. TRADITIONAL METHODS

The choice of the phases of a multiharmonic signal

critically affects its peak-to-peak factor. Setting all the phases to zero usually results in a signal with a large peak-to-peak factor. Another possibility consists on randomly choosing the phases, which usually results in some improvement over the zero phase case. However, different methods have been proposed to choose the phases so that the peak-to-peak factor is minimized. Two of those methods are Schroeder's formula [3] and van den Bos clipping algorithm [4]. Both methods provide significant improvements over the zero and random phase cases.

Schroeder derived a recursive formula

$$\phi_{k_l} = \phi_{k_{l-1}} + 2\pi(k_l - k_{l-1}) \sum_{m=1}^{l-1} a_{k_m}^2, \quad l = 2, \dots, L \quad (4)$$

for the choice of phases to reduce the signal peak-to-peak factor. However, Schroeder assumed that the signal should have a large number of harmonics and that they should be concentrated in a small band relative to the center frequency of the signal. This condition is not met by many multiharmonic signals, and therefore (4) may not provide the optimized phases.

Another method commonly used is the clipping algorithm developed by van den Bos [4]. This method is iterative, therefore is slower than Schroeder's formula, but does not make any assumptions about the signal spectral content and usually provides better results. The method consists on computing the signal with randomly chosen phases, and then transforming it into a two-level signal $s_2(t) = \text{sgn}[s(t)]$ where $\text{sgn}(\cdot)$ returns the sign of the signal. The spectrum of the two-level signal is computed through a Fast Fourier Transform and the phases of each frequency component are extracted. A new signal $s(t)$ is then created using the phases obtained from the digital signal $s_2(t)$. This process reduces the peak-to-peak factor and is repeated until the peak-to-peak factor stops improving.

III. ARTIFICIAL BEE COLONY ALGORITHM

The Artificial Bee Colony algorithm makes use of three different bee types: employed bees, onlooker bees and scout bees [8]. Each type of bee has a specific goal. Employed bees evaluate the neighborhood of its assigned food source (*i.e.*, point in the search space) and supply information regarding the suitability of that food source to the onlooker bees. Using that information, the onlooker bees randomly choose an available food source and perform a further search in its neighborhood. The third type is responsible for exploring the remaining search space in search for other available optimum points, therefore are called scout bees.

Initially, N potential solutions $X_i = \{x_{i1}, x_{i2}, \dots, x_{iL}\}$ with $i = 1, 2, \dots, N$ are randomly generated in the search space of dimension L and the fitness fit_i of each potential solution is evaluated. The ABC algorithm

attempts to globally maximize the fitness of the potential solutions. However, in many optimization problems the goal is to minimize a cost function J . Therefore, the fitness of each point X_i is defined as

$$fit_i = \frac{1}{1 + J_i} \quad (5)$$

where J_i is the cost function to be minimized [9].

Each employed bee searches the neighborhood of its assigned point in the search space X_i by randomly generating a new solution V_i and evaluating its fitness. The new solution is generated by mutation of the j^{th} parameter of X_i according to

$$v_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj}) \quad i \neq k \quad (6)$$

where k is a random integer in the range $[1; N]$. Additionally, the perturbation factor ϕ_{ij} is a random number between -1 and 1 [9]. Figure 1 illustrates the process of generating a new solution V_i .

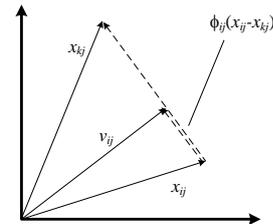


Fig. 1. The new generated solution in ABC algorithm.

The fitness of the new solution V_i is evaluated and it replaces the original solution X_i if the fitness improved. Otherwise, the original solution is kept by the employed bee.

In the next phase, the onlooker bees choose a food source according to the fitness information provided by all employed bees. Better food sources have a higher probability of being chosen by an onlooker bee since a biased roulette-wheel selection scheme is used, where

$$P_i = \frac{fit_i}{\sum_{j=1}^N fit_j} \quad (7)$$

is the probability of each food source being chosen. This means that a better food source will likely be chosen by multiple onlooker bees while the worst food sources may not be chosen by any onlooker. Each onlooker will search the vicinity of its chosen food source by randomly generating a new solution according to (6) in the same manner as the employed bees.

If a solution does not improve after a predetermined number of iterations of the ABC algorithm, it is discarded and the associated employed bee is converted to a scout bee to randomly explore another region of the global search space.

The ABC algorithm iterates over the three phases until the maximum number of iterations is reached. A flowchart of the ABC algorithm is presented in Fig. 2.

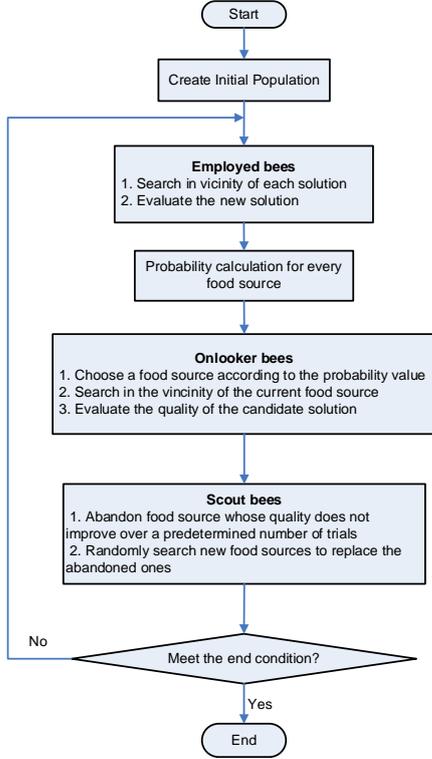


Fig. 2. Flowchart of ABC algorithm.

IV. NUMERICAL RESULTS

This section presents the numerical results for three different test cases. The test cases correspond to the examples presented by van den Bos in [4]. The peak-to-peak factor obtained with the ABC algorithm is presented and compared with the peak-to-peak factor obtained for the cases when: 1) all the phases are zero; 2) the phases are randomly chosen; 3) the phases are obtained by Schroeder formula; 4) the phases are obtained by the van den Bos clipping algorithm. Each method was executed 1000 times and the average and minimum peak-to-peak factors are shown for each method. A comparison of the time per trial of the different methods is also presented.

For all test cases the fundamental frequency of the signal is $f_{fund} = 100$ Hz and the number of samples per period was chosen to be 512 as in [4].

The first test case corresponds to a 16 harmonic signal with amplitudes

$$a_{k_l} = \frac{1}{\sqrt{8}} \sin\left[\frac{\pi(2k-1)}{32}\right], \quad k_l = l = 1, 2, \dots, 16 \quad (8)$$

and frequencies $f_{k_l} = k_l f_{fund}$. Fig. 3 presents one period of this signal for the zero phase and ABC optimized phases. The peak-to-peak factor of the ABC optimized

phases case is much lower than the zero phase case. The comparison between the different methods is presented in Table 1 which shows that the ABC algorithm is able to achieve the lowest peak-to-peak factor (both on average and on its minimum value), despite the fact that is computationally time consuming due to its stochastic nature which leads to a large number of function evaluations.

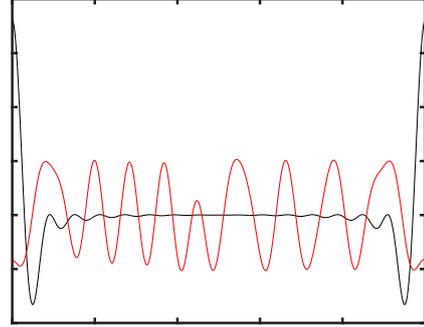


Fig. 3. One period of the signal in test case 1 for: 1) zero phase (black); 2) ABC optimized phases (red).

Table 1. Obtained peak-to-peak factors for test case 1.

	Average	Minimum	Time per Trial
Zero phase	2.63	-	-
Random	1.69	1.29	0.272ms
Schroeder	1.18	1.13	0.355ms
Clipping	1.18	1.05	28.9ms
ABC	1.08	1.03	33.5 s

The second test case used has 31 harmonics, all with the same amplitude

$$a_{k_l} = \frac{1}{\sqrt{31}}, \quad k_l = l = 1, 2, \dots, 31 \quad (9)$$

with frequencies $f_{k_l} = k_l f_{fund}$. Fig. 4 shows one period of the signal for the cases with zero phase and with the ABC optimized phases. It can be seen that the peak-to-peak factor obtained with the ABC optimized phases is much lower than the case in which the phases are all taken as zero. The comparison between the methods is presented in Table 2, where the average and minimum peak-to-peak factors are shown along with the time per trial of each method. As in test case 1, the ABC method, on average, performs better than any of the other methods. However, the clipping algorithm was able to achieve a minimum value which was lower than the value obtained with the ABC optimized phases. However, if more generations were used in the ABC method, it is likely that the minimum value would fall below the value obtained by the clipping algorithm, because the ABC algorithm may not have had enough iterations to reach a global minimum.

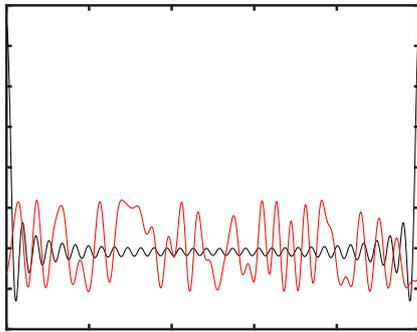


Fig. 4. One period of the signal in test case 2 for: 1) zero phase (black); 2) ABC optimized phases (red).

Table 2. Obtained peak-to-peak factors for test case 2.

	Average	Minimum	Time per Trial
Zero phase	3.44	-	-
Random	1.87	1.44	0.500ms
Schroeder	1.27	1.19	0.843ms
Clipping	1.27	1.11	51.7ms
ABC	1.19	1.13	59.2 s

The final test case consists on a special case that does not meet Schroeder's assumption. It has a spectrum with amplitudes

$$a_{k_l} = \frac{1}{\sqrt{6}}, \quad k_l = 2^{l-1}, \quad l = 1, 2, \dots, 6 \quad (10)$$

with frequencies $f_{k_l} = k_l f_{fund}$. Fig. 5 shows one period of the signal for both the zero phase case and the ABC obtained phases. Table 3 presents the comparison between all the methods considered and it can be seen that the ABC algorithm performs better than all the other methods, both on average and on the minimum peak-to-peak factor achieved. However, the ABC method has the disadvantage of being much slower than any of the other methods.

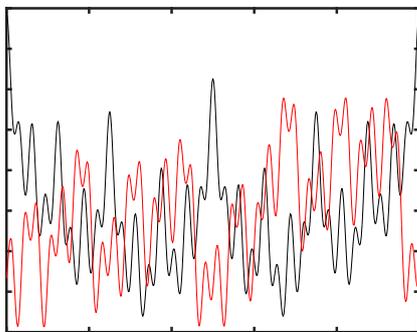


Fig. 5. One period of the signal in test case 3 for: 1) zero phase (black); 2) ABC optimized phases (red).

Table 3. Obtained peak-to-peak factors for test case 3.

	Average	Minimum	Time per Trial
Zero phase	1.88	-	-
Random	1.79	1.49	0.133ms
Schroeder	1.78	1.48	0.178ms
Clipping	1.60	1.45	13.8ms
ABC	1.42	1.41	19.38 s

V. CONCLUSIONS

This paper analyzes the use of an Artificial Bee Colony algorithm to minimize the peak-to-peak factor of multiharmonic signals. The ABC algorithm was benchmarked against the traditionally used Schroeder formula and clipping algorithm, both in terms of average and minimum peak-to-peak factors achieved and also in terms of time per trial. Although the ABC algorithm is much slower, due to its stochastic nature, it is able to achieve better peak-to-peak factors in almost all tests.

VI. ACKNOWLEDGMENT

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