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Design and Implementation of Differential AC Voltage Sampling System based on PJVS

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***Abstract* –Firstly, a differential sampling measurement principle is introduced, then based on this principle, we have developed a precision differential sampling system to measure sine-wave sources with the use of a quantum-accurate ac programmable Josephson voltage standard. And switching measurement technology is adopted in the differential sampling system design. By analyzing the error source of the differential sampling system, a mathematical model is established and the error transfer function is derived.We have performed a variety of measurements to evaluate this differential sampling system. Firstly, the basis of FLUKE 5720A transition process is analyzed, and the selection scheme of sampling window is described. After averaging, the uncertainty obtained in the determination of FLUKE 5720A 1 VRMS amplitudesine wave at 60 Hz is 0.3 μV/V (type A), the uncertainty obtained in the determination of PJVS 1 V RMS amplitude sine wave at 60 Hz is 0.05 μV/V (type A).**

***Keywords* –Differential sampling system, switching measurement, PJVS, quantization**

I. INTRODUCTION

Currently, the main value transfer modes of implementing ACPJVS-synthesized waveform to power frequency sinusoidal signal are lock-in amplifier [1, 2] and sampling technique [3-7], [12]. The measurement mode of lock-in amplifier is to measure the difference between sinusoidal signal and the fundamental of stepwise-approximated sine wave synthesized by ACPJVS. The precise measurement of sinusoidal signals is ensured with ACPJVS as a reference signal. In this mode, the uncertainty of measurement could reach 0.5 μV/V with voltage amplitude less than 0.75V and frequency less than 40Hz [2], However, the harmonic content of the SASW could affect the reading of the LIA. Furthermore, the transients between the successive voltage levels of the SASW [8,9], during which the PJVS is not quantized, causing the root meansquare (RMS) value of the SASW and its fundamental to deviate from their ideal values related to the fundamental physical constants [6].

AC voltage precision measurement program, based on the Full Scale Sampling method, usually used a single sampling voltmeter (such Agilent 3458A) as sampler [3-11], additionally, The sinusoidal signal and AC Programmable Josephson Voltage Standard (ACPJVS) are alternately connected to the sampling voltmeter by means of a signal switch，PTB using the algorithm could reduce the dominant uncertainty contribution from 1 μV/V to 0.38 μV/V (both k = 1) [5]。Differential Sampling is a technique for comparing the stepwise-approximated sine wave synthesized by ACPJVS to the sinusoidal voltages of a secondary source, by means of the Sampling DVM and making sure the input signal of DVM as small as possible by adjusting the phase of two sources. The measured sinusoidal signal could be reconstructed by using the known quantum voltage waveform and integral of the difference signal, thereby obtaining accurate amplitude of the measured sinusoidal signal，NIST have developed a differential sampling method that compares the voltage of an ac source with that of an ac programmable Josephson voltage standard (ACPJVS), the ACPJVS provides a precision quantum-accurate voltage (and phase) reference to accurately measure the amplitude and relative phase of a high-purity 50 Hz sine wave produced by a Fluke 5720A calibrator.

In this paper, we design a differential sampling system that is used to measure the amplitude of a sinusoidal waveform (B) using an ACPJVS reference (A). The differential sampling system is designed by using the commercial ADC chip, and a switching measurement technology is present. Then the transfer function of voltage amplitude error is deduced through the error analysis of the differential sampling system. It is proved that the switching measurement can effectively reduce the influence of channel error by using differential sampling system measure the amplitude of FLUKE 5720A refer to ACPJVS. In this paper, we also investigate the criterion of transition process, the discretization of reconstructed sinusoidal signal within sampling window and the experiment which measuring PJVS with PJVS.

II. SYSTEM DESCRIPTION

A、 *Differential Sampling Measurement Principle*

The paper [13-15] was presented a new reconstruction method of sinusoidal signal, first DFT and then averaging, at the begin of this method, sinusoidal signal has been reconstructed based on ACPJVS and difference signal, as is shown in Fig.1.

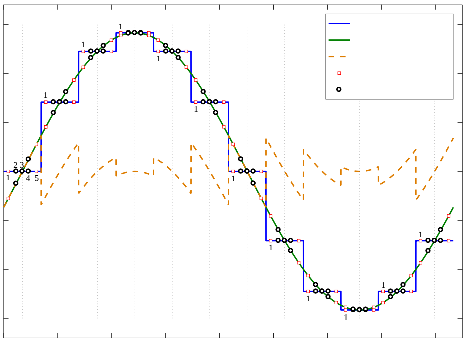

Fig. 1 Ideal simulated ACPJVS staircase-approximated sine wave with 12 samples and ac source sine wave (both with 1 V rms amplitude and zero relative phase). The resulting differential waveform is represented by the dotted line. Free of transients, the open circle samples are used for waveform amplitude reconstruction. The open circle samples that contain the transients are discarded

For simplicity, we have chosen in this example an ACPJVS waveform with 12 samples. Both waveforms have 1 V RMS. The stepwise signal divides the sinusoidal into equal parts of the number of step N, the points of the same position for each plateau have the same identification number (such as number 1 in Fig.1). Then all number 1 in each step will reconstruct a sine waveform, and number 2 to number 5 have the same situation like number 1. Because the ACPJVS waveform is only accurate on each constant-voltage step, the contributions from the transients must be removed, red square points in Fig 1 (number 1 and number 5). Therefore, the retaining points that number 2 to number 4 could reconstruct 3 sinusoidal signal, and obtaining three RMS amplitudes using FFT algorithm, then calculating the RMS amplitude of reconstruction sinusoidal signal by averaging all those three RMS amplitudes. It is proposed that the relative Fourier coefficients of reconstruction sinusoidal signal concerned be calculated as :

$$a_1(m)=\frac{2}{N}\sum_{n=0}^{N-1}x_{n,m}\sin(n,m),\quad a_1=\frac{1}{t-s+1}\sum_{m=s}^{t}a_1(m)\quad (1)$$

$$b_1(m)=\frac{2}{N}\sum_{n=0}^{N-1}x_{n,m}\sin(n,m),\quad b_1=\frac{1}{t-s+1}\sum_{m=s}^{t}b_1(m)\quad (2)$$

s is the beginning of sampling window, t is terminal point, m is the number of selected points in sampling window. $m=t-s+1$.

B. General measurement scheme of differential sampling system

Fig. 2 shows the measurement configuration for the differential sampling method, which combines three main components such as a sinusoidal-wave voltage source of high spectral purity and stability, an AC-PJVS system that provides a reference voltage waveform, and. a sampling system to measure differential voltages.

The PJVS system is developed by National Institute of Standards and Technology (NIST), The 12-bit AC-PJVS chip enable a maximum output of 2 V for 18 GHz microwave input, For some of the measurements reported in this paper, a Fluke 5720A was used as the sinusoidal-wave source. The reference signal for the phase and frequency locking of the Fluke 5720A is provided by an arbitrary waveform function generator. The differential sampling system is designed by ourselves, in Fig 2, we only describe one differential sampler, and acutely there are two sampler in the sampling system for the purpose of power measurement. Detailed differential sampler design will be described bellow.

III. SWITCHING MEASUREMENT AND ERROR TRANSFER FUNCTION

A. Switching measurement

We use LTC2378-20 as analog-to-digital conversion chip, and develop a differential sampling system. The operating principle as shown in the block diagram 3.

Differential sampling system mainly includes reversing switch, voltage follower, pascaline, differential drive circuit, low pass filter, ADC, FPGA timing control circuit and host computer. The reversing switch is photoMOS, with the purpose of A-B and B-A, in this way, we will measure twice, sine wave minus stepwise and stepwise minus sine wave, then calculate the mean of two amplitude. Voltage follower can increase the input impedance of differential sampling system. Signal conditioning fulfill the requirements for input common voltage of differential driver amplifier. The LTC2378-20, used in differential sampling system, is a 20-bit successive approximation analog-to-digital conversion chip with integral non-linearity error of ±0.5 ppm, quantization error of 2 ppm, maximum sampling rate of 1 MHz and input differential voltage range of ±5 V .

B. Error transfer function

Main source of error in the differential sampling system include the gain error and offset error respective in positive input and negative input, the gain error and offset error in differential driver circuit, the reference error and sampling error in analog-to-digital conversion. As a result of FFT analysis of sinusoidal signal, we only consider the fundamental RMS, thus only the effect of gain error, reference error and sampling error on measurement of voltage amplitude is analyzed when modeling is established.

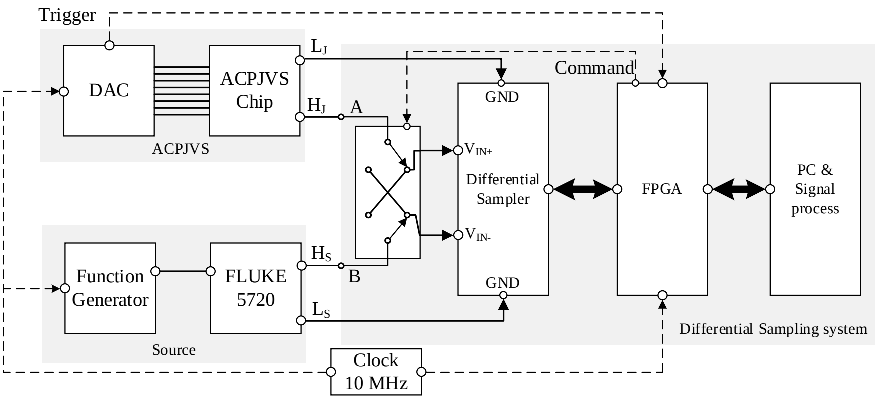

Fig. 2. AC signal measurement system block diagram based on PJVS

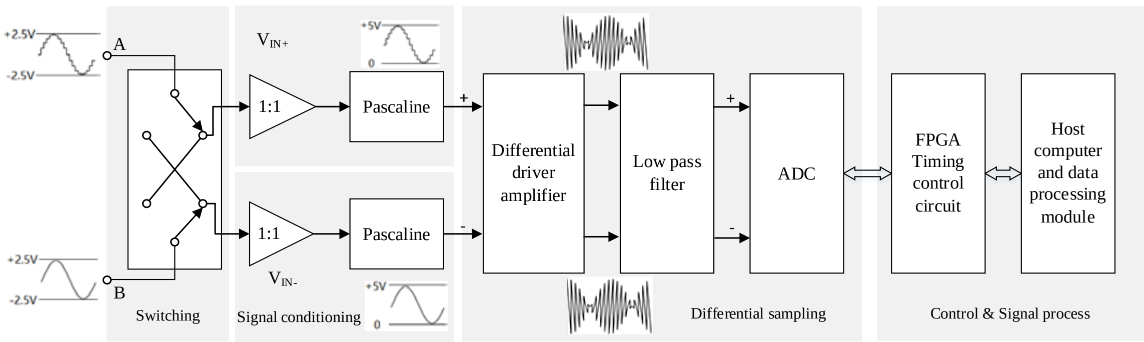

Fig. 3. The block diagram of differential sampling system

We assume that the sinusoidal signal is $V_s(n)$, the stepwise signal is $V_j(n)$, the differential signal is $V_d(n)$, the positive gain error of signal conditioning circuit is $E_p(n)$ and the negative gain error of signal conditioning circuit is $E_n(n)$; Differential drive circuit error, reference error and sampling error as a whole to consider, collectively referred to as differential circuit gain error, denoted by $E_d(n)$. Thus the differential signal before and after the switching measurement can be expressed by the Equation (3).

$$\begin{cases} V_{d1}(n) = \left[V_s(n)\cdot(1+E_n(n)) - V_j(n)\cdot(1+E_p(n))\right]\cdot(1+E_d(n)) \\ \qquad\qquad\qquad\qquad\qquad\qquad n = 1,2\cdots L \\ V_{d2}(n) = \left[V_j(n)\cdot(1+E_p(n)) - V_s(n)\cdot(1+E_n(n))\right]\cdot(1+E_d(n)) \end{cases} \tag{3}$$

The L is the number of sample points in a period of sinusoidal signal. According to differential sampling principle, the error transfer function($E_{r1}(n)$,$E_{r2}(n)$) of reconstructed sinusoidal signal ($V_{s1}(n)$,$V_{s2}(n)$) could be derived.

$$\begin{cases} E_{r1}(n) = E_n(n) - \dfrac{V_j(n)}{V_s(n)}\cdot E_p(n) + \eta\cdot E_d(n) \\ E_{r2}(n) = E_p(n) - \dfrac{V_j(n)}{V_s(n)}\cdot E_n(n) + \eta\cdot E_d(n) \qquad n = 1,2\cdots L \\ \eta = \dfrac{V_s(n) - V_j(n)}{V_s(n)} \end{cases} \tag{4}$$

In this paper, η is defined as difference coefficient. According to Equation (4), η is a magnitude that varies with the sampling point, and the magnitude of η is much less than 1. Therefore when after the differential circuit error multiply by η, the influence of the differential circuit error on the reconstruction sinusoidal signal voltage accuracy is reduced. $V_j(n)/V_s(n)$ is the amount of approximately 1, thus the positive gain error and the negative gain error could transfer into the error of reconstruction sinusoidal signal with an equal radio, but the positive gain error and the negative gain error are opposite to the effect of reconstruction sinusoidal signal error. Therefore if the positive circuit and the negative circuit are exactly same, the influence of signal conditioning circuit error on the voltage accuracy of reconstruction sinusoidal signal will become very small, but the actual circuit design process could not guarantee the consistency of the positive circuit and the negative circuit. Consequently, we propose to apply switching measurement to the differential sampling system. Exactly, we calculate the average of V_{s1}(n) and V_{s2}(n), and then the voltage error of reconstruction sinusoidal signal could be obtained.

$$E_r(n) = \eta \cdot \left[\frac{E_p(n) + E_n(n)}{2} + E_d(n) \right] \qquad n = 1, 2 \cdots L \quad (5)$$

As can be seen from Equation (5) , gain error of signal

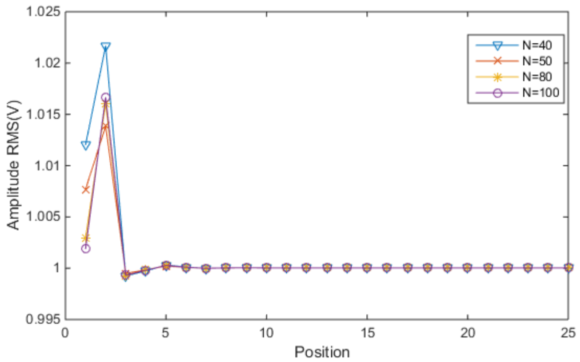

Fig. 4. RMS amplitude of reconstruction waveform in different position (the position is from number 1 to number 25)

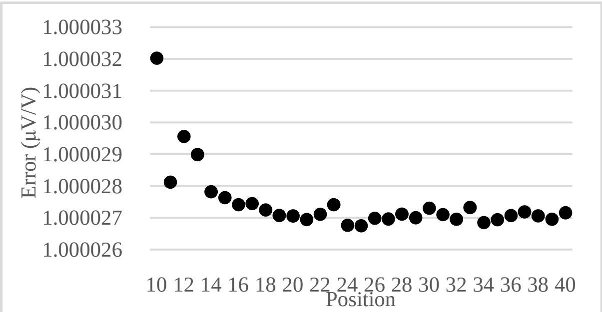

Fig. 5. RMS amplitude of reconstruction sine waveform in different number point

conditioning circuit multiplied by the factor η, the signal conditioning circuit achieve the effect of differential sampling after switching measurement. Therefore, the effect of gain error on the accuracy of reconstructed sinusoidal signal measurement is greatly reduced by switching measurement.

IV. EVALUATION OF THE DIFFERENTIAL SAMPLING SYSTEM

A. FLUKE 5720A measurements

Precision comparisons were performed between the ACPJVS and the Fluke 5720A sine wave with various numbers of steps and relative phase configurations. Both of the 60 Hz waveforms had 1.2 V RMS amplitude. With the use of a coaxial cable between the various instruments, this configuration also provides better shielding. The sampling rate of differential sampling system is 480 kHz. The reconstruction waveform is averaged by 300 cycles (5 s), In order to avoid the impact of the transition, the reconstruction waveform need to be intercepted by sampling window. For this reason, it is first necessary to determine the size of the transition. According to the principle of differential sampling, we calculate each sinusoidal signal RMS amplitude from number 1 to number M (M is the number of sampling point on one step). Because the transition only appears on the rising and falling edges of the stepwise, therefore, Fig. 4 describe the sinusoidal signal RMS amplitude from number 1 to number 25 on different N (N is the number of steps of stepwise).

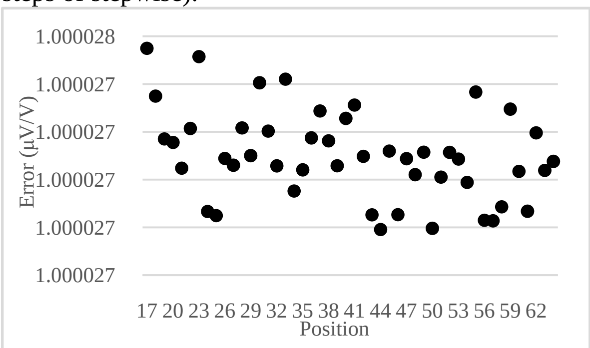

Fig. 6. RMS amplitude of reconstruction sine waveform in sampling window

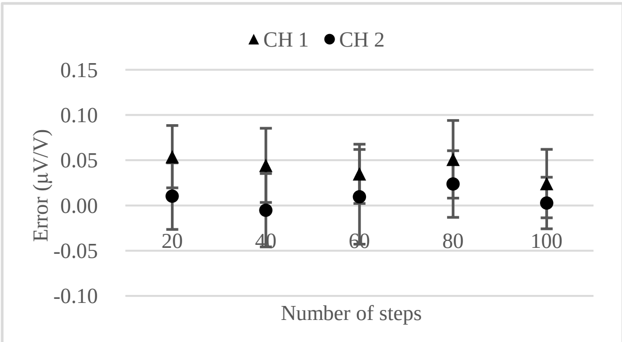

Fig. 7. RMS amplitude Error of PJVS measurements

As can be seen from Table 1, the RMS amplitude, which calculated by the point include transition, is bigger than 1.01 V, thus it is necessary to eliminate the influence of the transition on the RMS amplitude calculate through effective judgment basis. In this paper, we adopt 3σ principles to judge the transition. First calculate the standard deviation (σ) of RMS amplitude in the middle flat position. And then calculate the difference between the two adjacent RMS amplitudes. When the difference is less than 3σ, we believe that the effect of the transition on the accuracy of the RMS amplitude measurement is already included in the self-discretization of the reconstructed sinusoidal signal.

The following example is the number of steps N = 100, the judgment method of the transition will be described in detail. The RMS amplitude after switching measurement is obtained by averaging the voltage value of the previous two times, and the transition of the previous two measurements is the same. Therefore, the transition process can be judged by the signal before switching. The σ equal to 0.2 V, then 3σ equal to 0.6 V. Calculate the difference between the two adjacent RMS amplitude. Since the first eight dots are far greater than 0.6 V, only the number 11 to 38 point RMS amplitude are shown in Fig. 5.

The difference between number 14 and number 15 is 0.2 μV, smaller than 3σ, in Fig. 5. Then from number 14 the effect of the transition on the accuracy of the RMS amplitude measurement is already included in the self-discretization of the reconstructed sinusoidal signal. For insurance purposes, in the selection of sampling window, we will discard several points more than transients. Here, we discard 16 point.Therefore, the retaining points are from number 17 to number 64, then calculate amplitude RMS of 48 sine waveform, as shown in Fig. 6.

Table 1. Reconstruction sine waveform rms amplitude of differential sampler 1

N	80			100		
	A-B	B-A	Switching	A-B	B-A	Switching
10 traces RMS (V)	1.0000273	0.9999743	1.0000008	1.0000257	0.9999742	0.9999999
	1.0000274	0.9999741	1.0000008	1.0000271	0.9999742	1.0000007
	1.0000271	0.9999739	1.0000005	1.0000274	0.9999741	1.0000008
	1.0000275	0.9999741	1.0000008	1.0000268	0.9999738	1.0000003
	1.0000272	0.9999745	1.0000008	1.0000274	0.9999741	1.0000007
	1.0000274	0.9999741	1.0000007	1.0000275	0.9999740	1.0000007
	1.0000270	0.9999736	1.0000003	1.0000269	0.9999739	1.0000004
	1.0000272	0.9999740	1.0000006	1.0000269	0.9999741	1.0000005
	1.0000272	0.9999741	1.0000007	1.0000273	0.9999739	1.0000006
	1.0000272	0.9999735	1.0000004	1.0000269	0.9999742	1.0000006
Average (V)	1.0000272	0.9999740	1.0000006	1.0000270	0.9999740	1.0000005
Std (μV)	0.1	0.3	0.2	0.5	0.1	0.3

Table 2 Reconstruction sine waveform rms amplitude of differential sampler 1

N	80			100		
	A-B	B-A	Switching	A-B	B-A	Switching
10 traces RMS (V)	1.0000357	0.9999660	1.0000008	1.0000339	0.9999657	1.0000008
	1.0000359	0.9999656	1.0000007	1.0000355	0.9999656	1.0000007
	1.0000356	0.9999655	1.0000005	1.0000358	0.9999656	1.0000005
	1.0000359	0.9999657	1.0000008	1.0000353	0.9999654	1.0000008
	1.0000357	0.9999662	1.0000009	1.0000358	0.9999655	1.0000009
	1.0000358	0.9999658	1.0000008	1.0000359	0.9999657	1.0000008
	1.0000355	0.9999654	1.0000005	1.0000352	0.9999654	1.0000005
	1.0000355	0.9999657	1.0000006	1.0000352	0.9999656	1.0000006
	1.0000356	0.9999656	1.0000006	1.0000355	0.9999655	1.0000006
	1.0000356	0.9999650	1.0000003	1.0000354	0.9999658	1.0000003
Average (V)	1.0000357	0.9999656	1.0000007	1.0000354	0.9999656	1.0000007
Std (μV)	0.1	0.3	0.2	0.6	0.1	0.3

As can be seen from Fig. 6, the maximum change of RMS amplitude in the sampling window is less than 1 μV , calculate the mean value of the 48 RMS amplitude is 1.000027 V. as judgement N = 100, when N = 80, the retaining points is 68, Table 1 and Table 2 show the two differential samplers simultaneously measure the FLUKE 5720A RMS amplitude.

As can be seen from Table 1 and Table 2, although the standard deviation of the RMS amplitude measurement by differential sampler 1 and 2 is less than 1 μV. But the difference between 10 traces measured by differential sampler 1 and nominal value 1 V in A-B and B-A is bigger than 20 μV, and 30 μV in differential sampler 2. When calculated the mean of A-B and B-A, the difference between 10 traces measured by two differential sampler and nominal value 1 V is smaller than 1 μV, the experiment proves that switching measurement can effectively reduce the influence of circuit error on the accuracy of RMS amplitude measurement.As the FLUKE 5720A its own short-term changes is 1 μV ~ 5 μV. Therefore, it is impossible to determine the cause of the change in the RMS amplitude of the 10 traces.For this reason, we calculated the difference between the two samplers to measure the RMS amplitude, as shown in Table 3, and Table 4 shows the difference between the two samplers at different relative phases.

As could be seen from Table 3 and Table 4, the difference between the two differential sampling systems for measuring FLUKE 5720A is less than 0.1 μV. When the two samplers are subtracted each other, the measurement uncertainty introduced by the FLUKE 5720A is greatly reduced. It is proved that the differential sampling system has good stability and channel consistency by the experiment of changing the number of steps when measuring FLUKE 5720A. In order to further demonstrate the measurement accuracy of the differential sampling system, precision comparisons were performed between the differential sampling system and national AC voltage reference with FLUKE 5790A (the uncertainty is 7 μV/V) as transfer standard. Both of the 60 Hz waveforms RMS amplitude are 1 V. The sampling rate of the differential sampling system is 480 kHz, Fig. 8 shows the result of differential sampling system and national AC voltage reference.

As could be seen from Table 5, when using the differential sampling system and FLUKE 5790A to measure FLUKE 5720A simultaneously, the difference be-

Table 3 Difference of two differential sampler in different steps

N	40	50	80	100
CH 1 (V)	0.99999958	0.99999903	1.00000062	1.00000064
CH 2 (V)	0.99999951	0.99999901	1.00000054	1.00000066
Dif (μV)	0.07	0.02	0.08	-0.03

Table 5 Comparison of Differential Sampling System with National AC Voltage Reference

Measurement system	FLUKE 5790A	Differential Sampling System	
		CH 1	CH 2
2 traces (V)	1.0000006 0.9999999	1.0000002 1.0000002	1.0000002 1.0000001
Average (V)	1.0000003	1.0000002	1.0000002

Table 4 Difference of two differential sampler in different relative phase

Relative phase (°)	-1	-0.5	0	0.5	1
CH 1 (V)	0.99999932	0.99999970	0.99999988	0.99999979	0.99999938
CH 2 (V)	0.99999930	0.99999971	0.99999995	0.99999984	0.99999939
Dif (μV)	0.02	0.01	0.07	0.05	0.01

Table 6 RMS amplitude of PJVS measurements in different steps

N	**20**	**40**	**60**	**80**	**100**
RMS (V)	0.99998305	0.99998532	0.99997335	0.99997365	0.99995989
CH 1 RMS (V)	0.99998310	0.99998536	0.99997338	0.99997370	0.99995991
CH 2 RMS (V)	0.99998306	0.99998531	0.99997336	0.99997367	0.99995989
CH1 Error (μV/V)	0.05	0.04	0.03	0.05	0.02
CH2 Error (μV/V)	0.01	-0.01	0.01	0.02	0.00
Dif (μV/V)	0.04	0.05	0.03	0.03	0.02

-tween the differential sampling system and FLUKE 5790A is 0.1 μV. Although both sampler have goodconsistency on the RMS amplitude through the contrast testwith the national AC voltage reference, but it can only prove that the measurements of the differential system are accuracy in 10^{-6} because of the measurement uncertainly of FLUKE 5790A is 7 μV/V.

B. PJVS measurements

We connect the same output of PJVS to both input of differential sampler, taking one channel's PJVS as a reference signal to measure another channel's PJVS signal. Calculate the error and the standard deviation of the reconstructed stepwise RMS amplitude at different steps.

In Fig.7, when the number of steps varies from 20 to 100, variation of RMS amplitude error is less than 0.04 μV/V, RMS amplitude error is less than 0.06 μV/V, the standard deviation of RMS amplitude is less than 0.05 μV/V. Through this experiment, we can conclusion that the differential sampling system has very high measurement accuracy when measuring PJVS. Table 6 shows the measured RMS amplitude of both differential sampling system, measurement error and the difference between channel 1and channel 2.

V. CONCLUSIONS

In this paper, the physical model, which has been built, was used to analyze the sources and influence of differetial sampling systematic error. The influence of channel error can be reduced effectively through phase-changed measurement. System measuring error is less than 1 μV/V for selecting the appropriate PJVS staircases. By measuring the FLUKE 5720, the error of the measurement results is less than 4 μV/V and the standard deviation of the measurement results is less than 0.5 μV / V at 1V, 60Hz.

VI. ACKNOWLEDGMENT

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