22nd IMEKO TC4 International Symposium & 20th International Workshop on ADC Modelling and Testing
SUPPORTING WORLD DEVELOPMENT THROUGH ELECTRICAL&ELECTRONIC MEASUREMENTS
IASI, ROMANIA, September 14-15, 2017

Electric Power Steering System Implementation using Generalized Predictive Control

Razvan C. Rafaila

Continental Automotive Romania, Iasi, Romania,
razvan.rafaila@continental-corporation.com

***Abstract* – Current research in the automotive industry is tending more and more to developing control systems for complete autonomous driving vehicles. In this paper our aim is to present a Generalized Predictive Control (GPC) approach for the Electric Power Steering (EPS) system for the classical purpose of assisting the driver in the steering manoeuvres and also for controlling the lateral vehicle dynamics in autonomous driving. The EPS system is currently employed for steering assistance in most of the vehicles due to its advantages - engine independence/ fuel economy energy, tunability of steering feel, modularity/quick assembly, compact size and environmental compatibility. First, the performance of the GPC control system is presented for driver assistance, and the performance in automatic steering manoeuvres is finally introduced. The results show that the proposed control system could show good performances in practical use.**

Keywords – EPS, Vehicle Dynamics, Generalized Predictive Control, single-track model

I. INTRODUCTION

Many new vehicles today already use Electric Power Steering Systems (EPS/EPAS) instead of classical hydraulic power steering systems, do to many advantages of the EPS, like engine independence, fuel economy, small size, and steering feel configuration using software control systems[1], [2]. Moreover, the EPS is very attractive for automated driving, for vehicle lateral dynamics control.

This paper will present a control system design using Generalized Predictive Control for the EPS, for both driver assistance and automated driving purposes.

The control system design for EPS it's a very challenging task already, because it must account for assistance torque generation, vibration attenuation, steering wheel reaturnability, and providing road information to the driver. It also needs to generate a proper steering feel for the driver, for safety and comfort. For a good steering feel, the assistance level at lower vehicle speeds needs to be higher, and at higher vehicle speeds the assistance level must be lower [3]. This is usually implemented using map or boost curves, that use the measured steering torque applied by the driver and the vehicle speed.

In automated driving, the goal of the steering control system is to control the vehicle lateral dynamics, for example in manoeuvres like turn taking, lane changing and obstacle avoidance[4], [5], [6]. Although the driver does not intervene and the steering feel is not as important as in the steering assistance control case, the passenger comfort and safety must be taken into account in all manoeuvres.

Predictive control techniques have been introduced mainly in order to deal with plants that have complex dynamics and plant model mismatch. They are of a particular interest from the point of view of both broad applicability and implementation simplicity, being applied on large scale in industry processes, having good performances and being robust at the same time. In order to address all the control system requirements that need to be fulfilled, a GPC control approach is proposed[7], [8]. This control method is very suitable for EPS control, as it incorporates optimal control – useful for assistance generation and comfort, and also it takes into account the "future" states and vehicle trajectories, which can be obtained by the information in front of the vehicle – for automated driving task.

In the following sections the plant mathematical modelling (EPS and vehicle) will be presented (Section II), a short introduction to GPC approach is provided in Section III, control system implementation and simulation results (Section IV) and finally the conclusions of our study (Section V).

II. SYSTEM MODELLING

In the following a vehicle setup with a column-type EPS is considered, equipped with a brushed DC electric motor. The mathematical modelling of the steering dynamics can be obtained by using Newton's second law of motion as:

$$J_c\ddot{\theta}_c = -K_c\theta_c - B_c\dot{\theta}_c + K_c\frac{\theta_m}{N} - F_c sign(\dot{\theta}_c) + T_d, \tag{1}$$

$$J_{eq}\ddot{\theta}_m = K_c\frac{\theta_c}{N} - K_{eq}\theta_m - B_{eq}\dot{\theta}_m + K_t i_m - F_m sign(\dot{\theta}_m) - \frac{r_p}{N}F_r, \quad (2)$$

where θ_c is the steering column angle, θ_m is the electric motor angle, i_m is the electric motor current, F_r is the road reaction force which acts on the steering rack and T_d is the driver applied torque. The parameters used in the model have the following meaning:J_c is the steering column moment of inertia;B_c is the steering column viscous damping coefficient;$K_c$is steering column stiffness given by the steering torque sensor;F_c is the steering column friction;N is the electric motor gear ratio;r_p is the steering column pinion radius;K_t is the electric motor torque constant;J_{eq} is the equivalent inertia of the lower part of the EPS system, comprised from the electric motor inertia and the steering rack mass;B_{eq} is the equivalent damping coefficient of the lower part of the EPS system, comprised from the electric motor viscous damping and the steering rack viscous damping;$K_{eq} = \frac{K_c}{N^2} + \frac{K_r R_p^2}{N^2}$ is the equivalent stiffness of the lower part of the EPS system, comprised from the tire spring rate and steering column stiffness and F_m is the motor friction coefficient.

The DC motor dynamics can be described by:

$$L\frac{di_m}{dt} = -Ri_m - K_t\dot{\theta}_m + V, \quad (3)$$

where R, L, $K_t$are the motor resistance, inductance and torque constant, respectively. The voltage applied for controlling the DC motor is denoted by V.

The quantities J_{eq}andB_{eq} from (2) can be calculated as:

$$J_{eq} = J_m + \frac{r_p^2}{N^2}M_r, \quad (4)$$

$$B_{eq} = B_m + \frac{r_p^2}{N^2}B_r, \quad (5)$$

The motor angle can be also written as:

$$\theta_m = \frac{Nx_r}{r_p}, \quad (6)$$

Excluding the friction components in the model given by (1) and (2), and a linear state-space model of the steering system can be obtaned

$$\begin{aligned} \dot{x} &= Ax + B_1u_1 + B_2u_2 \\ z &= C_2x \\ y &= Cx \end{aligned}, \quad (7)$$

where $x = [\theta_c\dot{\theta}_c\theta_m\dot{\theta}_m i_m]^T$ is the system state vecor.

The vehicle model employed in this paper is a single track model, which can describe very whell the vehicle dynamics. Considering that the vehicle is traveling at constant velocity, the mathematical model can be described:

$$\begin{aligned} \dot{\beta} &= -\frac{(C_f+C_r)}{mv}\beta + \left(-1 + \frac{(bC_r - aC_f)}{mv^2}\right)\gamma + \frac{C_f}{mv}\delta_f \\ \dot{\gamma} &= -\frac{(b^2C_r + a^2C_f)}{J_v v}\gamma + \frac{(bC_r - aC_f)}{J_v}\beta + \frac{aC_f}{J_v}\delta_f \end{aligned} \quad (8)$$

where β is the side slip angle of the veicle, γ is the yaw rate, δ_f is the front wheel steering angle, v is the vehicle speed, m is the vehicle mass, $J_v$is the vehicle inertia, a is the distance from the front axle to the center of gravity, b is the distance from the rear axle to the center of gravity, and C_f and C_r are the front and rear tire cornering stiffness coeficients.

The connection between the EPS model described by (1), (2), (3) and the vehicle model (8) is given by the angle of the steering column, and the road reaction force generated by the tire-ground contact, which are described in what follows.

The steering angle of the front wheel is obtained as:

$$\delta_f = \frac{\theta_c}{G}, \quad (9)$$

where G is the ratio between steering column angle and the front wheels angle.

The lateral tire force which develops at the front wheel when the vehice is steering, can be calculated as:

$$F_y = C_f\alpha_f, \quad (10)$$

where $\alpha_f$is the front tire slip angle. The lateral tire force F_y described by (10) generates the self-alignment torque which tends to turn back the steered wheel to the center, and is expressed as:

$$T_{allign} = T_pF_y, \quad (11)$$

Finally the self-alignment torque is tranformed to the road reaction force wich acts on the steering rack trough the pinion whcih connects the steering column with the steering rack as:

$$F_r = \frac{T_{allign}}{r_p}, \quad (12)$$

It is important to note that the vehicle model described from (8) to (12) is liniarized assuming a nominal operating range, excluding the case of high lateral dynamics, or slippery roads do to ice or rain, where, for example, the tire force described by (10) is not valid, as it reaches the nonlinear range. Also the steering ratio in (9) is assumed constant, whereas in reality it is a nonlinear function of the steering angle. For more details regarding the system modelling, interested readers may refeer to [1] and [2]

In section IV it it will be shown how the model of the EPS and vehicle can be used in order to implement the GPC control system.

III. GENERALIZED PREDICTIVE CONTROL

Predictive control techniques have been introduced mainly in order to deal with plants with complex dynamics (unstable inverse systems, time-varying time delay, etc.) and plant model mismatch. They are of a particular interest from the point of view of both broad applicability and implementation simplicity [9].

The strategy for predictive control laws can be summarized as follows [10]:

i. it is assumed that the finite future values of the desired controlled states of the system are available at each instant of time.
ii. at the present discrete time instant k, the future states of the plant are predicted based on a system dynamical model using the present measured system states and the future control signal.
iii. the control vector is computed to minimize the given performance index along the entire future prediction horizon.
iv. the first sample of control signal u(k) is actually applied to the plant input and the whole procedure is repeated.

GPC belongs to the group of long-range predictive controllers representing a unification of many predictive control algorithms and it is based on a CARIMA (Controlled Autoregressive Integrated Moving Average) model [7], [8]:

$$A\left(q^{-1}\right)y\left(k\right)=q^{-d}B\left(q^{-1}\right)u\left(k-1\right)+\frac{e\left(k\right)}{\Delta}, \qquad (13)$$

with

$$A(q^{-1})=1+a_1q^{-1}+a_2q^{-2}+\dots+a_{n_A}q^{-n_A}$$

$$B(q^{-1})=b_0+b_1q^{-1}+b_2q^{-2}+\dots+b_{n_B}q^{-n_B}$$

where q^{-1} is the backward shift operator, Δ is the differencing operator $1-q^{-1}$, d is the delay of the process and $e(k)$ is a white noise. $A(q^{-1})$ and $B(q^{-1})$ are polynomials in the backward shift operator and n_A and n_B are the polynomials degrees.

GPC generates a sequence of (future) control signals within each sampling interval to optimize the control effort of the controlled system, but only the first element of the control sequence is applied to the system input. This is done by minimizing a rather complex cost function.

The prediction of the system output y is based on two different components [11]: the "free response" and the "forced response".

The "free response" represents the predicted behaviour of the output $y(k+j|k)$ (in the range from k+1 to k+N), based on old outputs $y(k-j|k)$ and inputs $u(k-j|k)$, assuming a future control action of zero, where N is the difference between the prediction horizon and the minimum prediction horizon.

The matrix form of the predictor is given by the following equation:

$$\hat{\mathbf{y}}=\mathbf{G}\mathbf{u}+\mathbf{F}\left(q^{-1}\right)y\left(k\right)+\mathbf{G}`\left(q^{-1}\right)\Delta u\left(k-1\right), \qquad (14)$$

where:

$$\hat{\mathbf{y}}=\begin{bmatrix}\hat{y}(k+d+1)\\ \hat{y}(k+d+2)\\ \dots\dots\\ \hat{y}(k+d+N)\end{bmatrix},\ \mathbf{u}=\begin{bmatrix}\Delta u(k)\\ \Delta u(k+1)\\ \dots\dots\\ \Delta u(k+N-1)\end{bmatrix},$$

$$\mathbf{G}=\begin{bmatrix}g_0 & 0 & \dots & 0\\ g_1 & g_0 & \dots & 0\\ \dots & \dots & \dots & \dots\\ g_{N-1} & g_{N-2} & \dots & g_0\end{bmatrix},\ \mathbf{F}\left(q^{-1}\right)=\begin{bmatrix}F_{d+1}\left(q^{-1}\right)\\ F_{d+2}\left(q^{-1}\right)\\ \dots\dots\\ F_{d+N}\left(q^{-1}\right)\end{bmatrix}, \qquad (15)$$

$$\mathbf{G}`\left(q^{-1}\right)=\begin{bmatrix}\left[G_{d+1}\left(q^{-1}\right)-g_0\right]q\\ \left[G_{d+2}\left(q^{-1}\right)-g_0-g_1q^{-1}\right]q^2\\ \dots\dots\\ \left[G_{d+N}\left(q^{-1}\right)-g_0-g_1q^{-1}-\dots-g_{N-1}q^{-(N-1)}\right]q^N\end{bmatrix}.$$

The "forced response" represents the additional component of the output y resulting from the optimization criterion:

$$J=\sum_{i=p_m}^{p}\left[\hat{y}\left(k+i\right)-r\left(k+i\right)\right]^2+\sum_{i=1}^{c}\lambda\left(i\right)\left[\Delta u\left(k+i-1\right)\right]^2, \qquad (16)$$

where $\hat{y}(k+i)$ are the predicted outputs and $r(k+i)$ are the future reference values. Beside the minimum prediction horizon pm, the prediction horizon p and the input horizon c, in the optimization criterion there is one more parameters represented by the gain sequence $\lambda(i)$, which usually is considered constant.

The horizon parameters are chosen using the following formulas:

$$c = N, \quad p_m = d + 1, \quad p = d + N . \tag{17}$$

The first term in the optimization criterion considers the predicted error and the second term considers penalized future control increments.

The optimization criterion can be rewritten to a matrix form:

$$J = (\mathbf{Gu} + \mathbf{f} - \mathbf{r})^T (\mathbf{Gu} + \mathbf{f} - \mathbf{r}) + \lambda \mathbf{u}^T \mathbf{u} , \tag{18}$$

where $\mathbf{f}$ is the “free response” and

$$\mathbf{r} = \left[r(t+d+1), r(t+d+2), ..., r(t+d+N) \right]^T . \tag{19}$$

Using the following notations:

$$\begin{aligned} \mathbf{H} &= 2\left(\mathbf{G}^T\mathbf{G} + \lambda \mathbf{I}\right) \\ \mathbf{b}^T &= 2(\mathbf{f} - \mathbf{r})^T \mathbf{G} \\ \mathbf{f}_0 &= (\mathbf{f} - \mathbf{r})^T (\mathbf{f} - \mathbf{r}) \end{aligned} \tag{20}$$

the optimization criterion can be rewritten as:

$$J = \frac{1}{2} \mathbf{u}^T \mathbf{H} \mathbf{u} + \mathbf{b}^T \mathbf{u} + \mathbf{f}_0 . \tag{21}$$

Minimizing J, when the control signal and the system states are not subject to constraints, it is obtained:

$$\mathbf{u} = -\mathbf{H}^{-1}\mathbf{b} = \left(\mathbf{G}^T\mathbf{G} + \lambda \mathbf{I}\right)^{-1} \mathbf{G}^T (\mathbf{r} - \mathbf{f}) , \tag{22}$$

and, as already stated, only the first element (sample) of the control sequence is sent to the system input.

IV. CONTROL SYSTEM DESIGN AND RESULTS

First, the control of the EPS for driver assistance is designed using the GPC approach. For this control purpose the electric DC motor torque is controlled, in order to provide the needed assistance torque, and also provide a good feedback to the driver trough the steering wheel. This proper feedback is generated using assistance or boost curves, which are functions of driver measured torque and vehicle speed. In this research, the boost curves presented in [3] are used, and the details of implementation are skipped here for the sake of simplicity. In order to obtain the CARIMA model of the DC motor from the EPS system, the transfer function from the input voltage V and the output current i_m in (3) is obtained as

$$G_m(s) = \frac{J_m s}{L J_m s^2 + R_m J_m s + K_t s^2} \tag{23}$$

The CARIMA model (13) can be obtained by discretizing (23) and obtaining $A(q^{-1})$and $B(q^{-1})$. The sample time chosen in the simulations is $T_s = 0.002$ $[sec]$.

In Figure 1, a step responce of the closed loop system is shown, where it can be seen that the GPC controller obtaines very good performances, in terms of settling time and reference tracking.

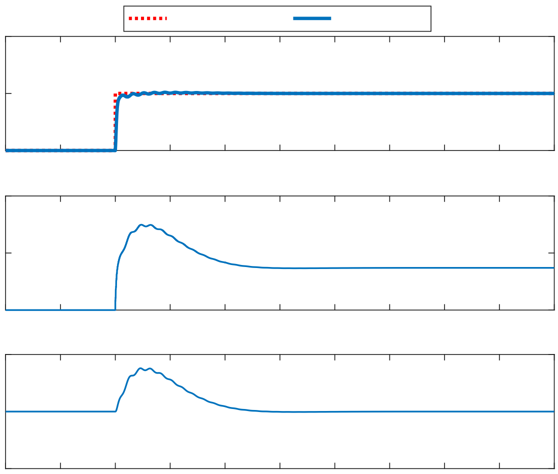

Fig 1. Step response of the EPS control system for assistance generation

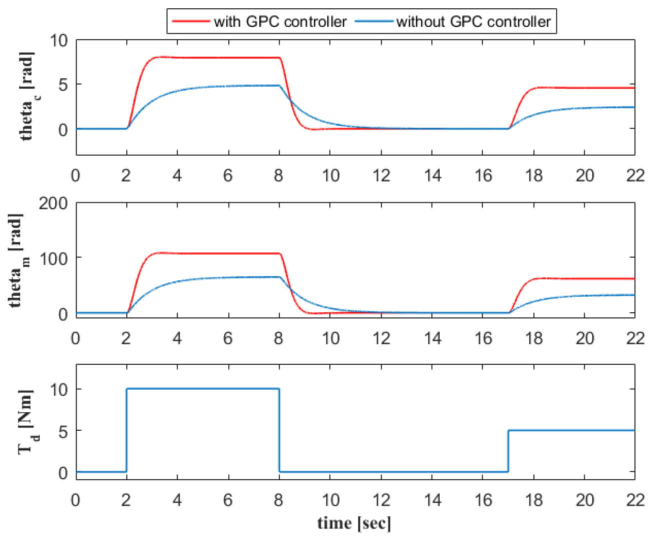

Figure 2. Comparison between EPS with and without GPC

Figure 2, shows how the EPS system assists the driver while steering by making a comparison between the column and DC motor angles $\theta_c$and θ_m, when the control system is active and when it is not active. It can be seen that when the GPC control is active, the column and motor angles θ_c and θ_m are higher, when the driver is applying the same steering torque T_d. In this simulation it is important to note that the vehicle speed is considered constant $v = 72\ km/h$.

From the above analysis, the GPC controller shows to be a good choice for implementing the assistance generation algorithm for the EPS.

In order to extend the proposed GPC control algorithm for vehicle lateral dynamics control, the CARIMA model needs to be extended to consider besides the dynamics of the DC motor, the dynamics from the motor current i_m to the steering column angle θ_c and further to the steering angle δ_f which will be the control signal of the yaw rate γ.

The transfer function is obtained by making use of the equations (8), (9), and considering the previously obtained transfer function (23). Thus, the transfer function which describes the dynamics from $i_m$toθ_c is obtained as:

$$G_c(s) = \frac{K_c K_t/N}{(J_c s^2+B_c s+K_c)(J_{eq} s^2+B_{eq} s+K_{eq})-(K_c/N)^2} \quad (24)$$

By using (23), (24) and (9) the dynamics from V to δ_f is obtained as:

$$\delta_f = \frac{1}{G} G_c(s) G_m(s) \quad (25)$$

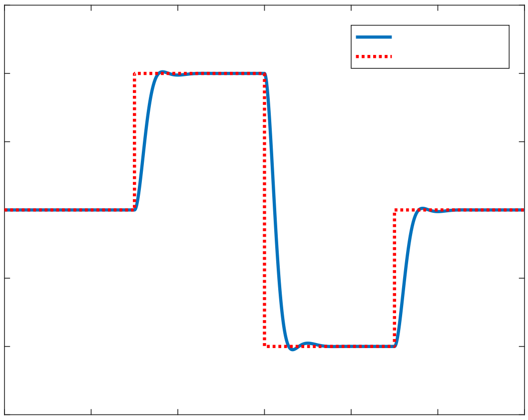

Figure 3. Yaw rate tracking using GPC

Having the above, itonlyremains to obtain the transfer functionwhichdescribesthedynamicsbetween$\delta_f$andγ, which can be obtained using the state space model (8) of the vehicle, whichwill be skippedhere for simplicity.In Figure 3, a typical lane change maneuver is simulated in terms of the yaw rate of the vehicle, and it can be seen that the GPC controller, using the CARIMA model obtained using (23), (24), (25) and (8) performs very well in controlling the yaw rate γ of the vehicle. With this algorithm, the lateral dynamics of the vehicle can be optimally controlled in terms of the yaw rate.

V. CONCLUSIONS

In this paper, a Generalized Predictive Control algorithm was developed for the Electric Power Steering system, for driver assistance and automated driving. The proposed control strategy was applied to a column-type EPS, and the system modelling and control design were discussed. Very good performances were obtained in simulation scenarios, which show that the proposed approach can provide a suitable solution for real-time implementation.

REFERENCES

[1] Marouf, A, Sentouh, C, Djemai, M and Pudlo, P, *Control of an Electric Power Assisted Steering System using reference model*, IEEE Conference on Decision and Control, Orlando, December 2011.

[2] Xin, L, Xue-Ping, Z and Jie, C, *Controller design for Electric Power Steering system using T-S Fuzzy Model approach*, International Journal of Automation and Computing, May 2009, pp. 198-203.

[3] Zhao Xue-Ping, Li Xin, Chen Jie, Men Jin-Lai, *Parametric design and application of steering characteristic curve in control for electric power steering*, Elsevier, Mechatronics, Vol 19, 2009, pp. 905-911.

[4] Keviczky, T, Falcone, P, Borrelli, F, Asgari, J and Hrovat, D, *Predictive Control Approach to Autonomous Vehicle Steering*, American Control Conference, Minneapolis, 2006, pp. 4760-4675.

[5] Carvalho, A, Lefévre, S, Schildbach, G, Kong, J and Borrelli, F, *Automated driving: The role of forecasts and uncertainty - a control perspective*, European Journal of Control, 2015, pp. 14–32.

[6] Falcone, P, Tufo, M, Borrelli, F, Asgari, J and Tseng, H,E, *A linear time varying Model Predictive Control approach to the integrated vehicle dynamics control problem in Autonomous Systems*, 46th IEEE Conference on Decision and Control, New Orleans, LA, USA, 2007, pp. 2980 - 2985.

[7] Clarke, D, W, Mohtadi, C and Tuffs, P, S, *Generalized Predictive Control - Part I. The basic algorithm*, Automatica, vol. 23, 1987, pp. 137-148.

[8] Clarke, D, W, Mohtadi, C and Tuffs, P, S, *Generalized Predictive Control - Part II. Extentions and Interpretations*, vol. 23, 1987, pp. 149-160.

[9] Rusnak, A, Fikar, M, Najim, K and Meszaros, A, *Generalized Predictive Control based on neural networks*, Neural Processing Letters, 4, 1996 pp. 107-112.

[10] Byun, D, Predictive control: a survey and some new stability results, Preceedings of ISL Winter Workshop, 22-24 Feb, 1988.

[11] Kennel, R, Linder, A and Linke, M, Generalized Predictive Control (GPC) – ready for use in drive applications?. In: 32nd IEEE Power Electronics Specialists Conference (PESC), vol 4, 2001, pp. 1839 – 1844.