

# Research on A New Active Power Measurement Algorithm

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**Abstract** – With the widely use of non-linear electrical equipment, the load signal in the smart grids exhibits distortion and dynamic characteristics. Simultaneously, non-integer-period sampling is inevitable in the power and energy estimation of the smart meter, which brings errors to the energy accumulation. In this paper, firstly, a new window is proposed based on the traditional filter power measurement algorithm and the frequency characteristics of the window are analyzed. Then, based on the window and structured model of smart meter, a new active power measurement algorithm is proposed. And then, in simulations, adopting different active power algorithms, the electric energy measurement errors are simulated and analyzed under non-integer-period sampling. These simulated waveforms include sinusoidal envelope and OOK envelope dynamic test signals based on fundamental wave and five kinds of distortion waveforms in IEC62052. Finally, the results show that the proposed filter can effectively improve the accuracy of active energy accumulation.

**Keywords** – window function, filter, active power measurement, non-integer-period sampling, energy accumulation

## I. INTRODUCTION

With the rapid development of smart grids, electric energy measurement appears an new requirements<sup>[1]</sup>. Active power is directly related to the measurement of electrical energy. The distortion of the signal and the limitation of dynamic fluctuation and non-integer-period sampling affect the measurement of smart meter and even cause the error of the energy measurement.

Various kinds of windows, such as the rectangular window<sup>[2]</sup>, Hamming window<sup>[3]</sup> and Blackman window<sup>[4]</sup>, have been proposed to improve the accuracy of spectrum analysis using FFT analysis<sup>[5]</sup>, but this method inevitably leads to spectrum leakage and attenuation. For dynamic signals, it is difficult to accurately measure electrical energy by these algorithms especially under non-integer-period sampling.

As elaborated above, the active power measurement algorithm of smart meters has become a focus of concern for researchers, and how to improve the active power filter and the accumulative accuracy of electric energy under non-integer-period sampling is a challenge in dynamic signals.

In this work, A new window function is proposed in section II. Section III provides analysis of frequency domain characteristics of new window. Section IV gives the active power measurement algorithm based on the new window and structured model of smart meter. Section V provides simulation analysis; and the conclusions appear in section VI.

## II. CONSTRUCTION OF SELF - CONVOLUTION WINDOW FOR PSA FUNCTION

The general form of Polynomial Sa window is expressed as the sum of  $K$  Sa functions with intervals of  $T$ :

$$w_{Sa}(t) = \sum_k^K \alpha_k \frac{\sin \frac{\pi}{T}(t-kT)}{\frac{\pi}{T}(t-kT)} \quad (1)$$

Through Fourier transformation, its spectrum function can be represented as:

$$W_{Sa}(\omega) = \begin{cases} T \sum_k^K \alpha_k e^{-j\omega kT} & |\omega| \leq \frac{\pi}{T} \\ 0 & |\omega| > \frac{\pi}{T} \end{cases} \quad (2)$$

When the continuous Polynomial Sa Function is discretized and truncated with a rectangular window, we obtain a finite length Polynomial Sa function window. (hereinafter referred to as PSa window)

$$w(n) = \sum_k^K \alpha_k \frac{\sin \frac{\pi}{T}(n-kT)}{\frac{\pi}{T}(n-kT)} \quad (3)$$

$$w_{psa}(n) = w(n)R_N(n) = \begin{cases} w(n) & n = 0, 1, 2 \dots N-1 \\ 0 & \text{others} \end{cases} \quad (4)$$

where  $R_N(n)$  is the rectangular window with the length of  $N$ , and the frequency domain expression of the PSA window function can be obtained by the frequency domain complex convolution Eq.as follows:

$$W_{PSa}(\omega) = W_{Sa}(\omega) * W_R(\omega) \quad (5)$$

where  $W_{Sa}(\omega)$  and  $W_R(\omega)$  express the frequency domain of the PSA window and rectangular window respectively,

$$W_R(\omega) = \sin\left(\frac{\omega N}{2}\right) / \frac{\omega}{2}.$$

The new window with the length of  $2N'-1$  can be obtained by using convolution for the two PSA windows with length of  $N'$ . According to the convolution operation theory, the PSA self-convolution window has the length of  $2N'$ , which can be obtained by zero-padding at the end of the self-convolution window:

$$w_{PSa}^2(n) = w_{PSa}(n) * w_{PSa}(n) \quad (6)$$

So, the frequency domain expression of the window  $w_{PSa}^2(n)$  can be expressed as:

$$W_{PSa}^2(\omega) = [W_{Sa}(\omega) * W_R(\omega)]^2 \quad (7)$$

Eq. (7) indicates that the narrower the main lobe, the faster the side lobe attenuation, and the better the side lobe attenuation after convolution.

### III. ANALYSIS OF FREQUENCY DOMAIN CHARACTERISTICS OF PSA SELF-CONVOLUTION WINDOW

Let the weighting coefficient ratio of the three-term Sa function window be 1:2:1. The side lobe attenuation shows better attenuation characteristic, and the expression for  $w_{PSa}(n)$  is:

$$[\alpha_{-1} \alpha_0 \alpha_1] = [0.25 \ 0.5 \ 0.25] \quad (8)$$

$$w_{PSa}(n) = \sum_{k=-1}^1 \alpha_k \frac{\sin\left(\frac{\pi}{T}(n-kT)\right)}{\frac{\pi}{T}(n-kT)} \quad n = 0, 1, 2, \dots, N'-1 \quad (9)$$

Eq. (8) indicates the weighting coefficient for the three-term Sa function window. In Eq. (9),  $N'$  is the length of the window,  $N' = 2^i$ ,  $i$  is positive integer and the successive interval of the Sa function is  $T = N'/4$ . The PSA self-convolution window is constructed from a three-term Sa function window with a length of 64 ( $N' = 64$ ). The length of the constructed window is  $N = 128$ . Comparing the time domain waveform and frequency domain response of PSA self-convolution window with rectangular window, Hamming window and Blackman window are shown in Fig.1 and Fig.2.

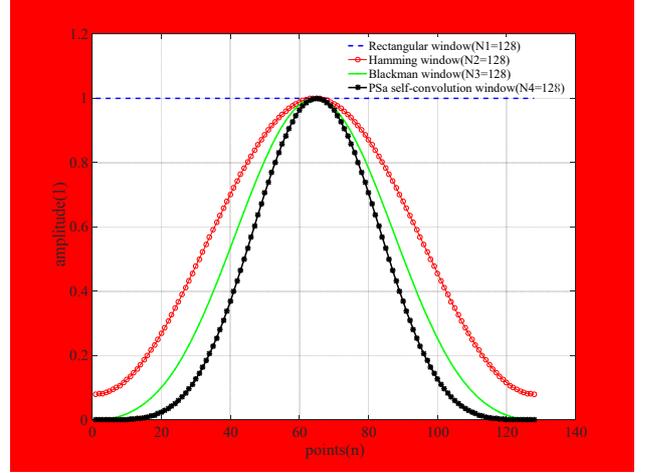


Fig. 1 The waveform of different window functions

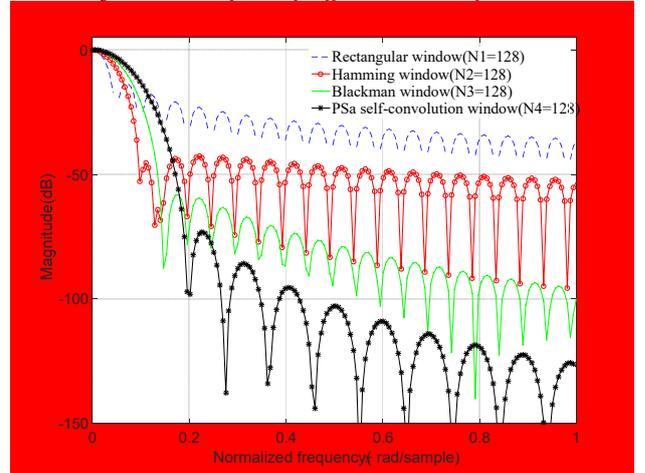


Fig. 2 The frequency domain response of different window functions

As can be seen from Fig.2, the PSA self-convolution window has the largest side lobe attenuation and the fastest side lobe attenuation speed.

### IV. ACTIVE POWER MEASUREMENT ALGORITHM OF PSA FILTER

As can be seen from Fig. 3, the instantaneous power signals can be obtained by digital multiplier and active power signal can be obtained by filtering the instantaneous power signals by low-pass filter. The active power signal is used as the output of the active power measurement part of the smart meter and is also the input of the electric energy accumulation unit, which converts the active power signal into electric energy pulses.

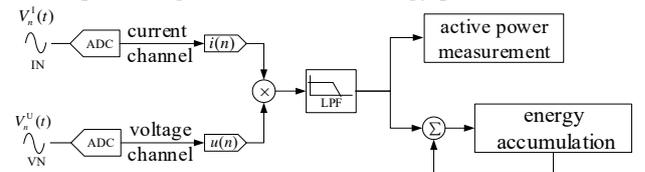


Fig. 3 Structured model of smart meter on active power measurement.

The mathematical expression of the voltage and current signals can be expressed as:

$$u(n) = \sum_{m=1}^M U_m \sin(2\pi m f_0 n / f_s + \varphi_m) \quad (10)$$

$$i(n) = v(n) \sum_{m'=1}^M I_{m'} \sin(2\pi m' f_0 n / f_s + \phi_{m'}) = v(n) i'(n) \quad (11)$$

In Eq.(10) and (11)  $f_0$  indicates the fundamental frequency as 50Hz,  $\varphi_m$ ,  $\phi_{m'}$  indicates the initial phase of the voltage and current signals from the fundamental wave ( $m, m' = 1$ ) and harmonics ( $m, m' \neq 1$ ), respectively.  $U_m$ ,  $I_{m'}$  indicates amplitude of the voltage and current signals from the fundamental wave and harmonics, respectively. the sampling frequency is  $f_s$ ,  $n=0,1,2 \dots$  and  $M$  indicates the highest harmonic obtained by sampling.

The expression of the instantaneous power test signal is as follows:

$$p_i(n) = u(n)i(n) = u(n)i'(n)v(n) \quad (12)$$

In Eq. (12),  $p_i(n)$  indicates an instantaneous power signal and the steady-state voltage and current signal is expressed as  $u(n)$ ,  $i'(n)$ . The steady-state test signal waveforms simulated in this paper are the five distorted waveforms specified in IEC62052<sup>[6]</sup> and the 50Hz power frequency fundamental wave. The five distorted waveforms include Quadriform Waveform, Peaked Waveform, Pulse Waveform, Multiple Zero Crossing Current Waveform and Multiple Zero Crossing Voltage Waveform.  $v(n)$  is an envelope signal and when  $v(n)=1$ ,  $p_i(n)$  expresses a steady-state power test signal. When  $v(n)$  expresses sinusoidal envelope, trapezoidal envelope<sup>[7]</sup> and OOK envelope signal<sup>[8]</sup>, it is a dynamic power test signal for  $p_i(n)$ .

The structural model of the active power filter is shown in Fig. 4. The instantaneous power signal  $p_i(n)$  passes through the low-pass filter and the output is the active power  $p_o(n)$ .

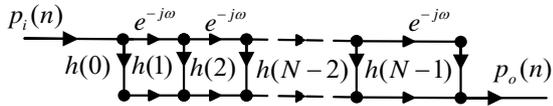


Fig. 4 structural model of active power filtering in which  $N$  is the length of the filter. the input signal is  $p_i(n)$ . The output  $p_o(n)$  which after the instantaneous power passing through the low-pass filter can be expressed as:

$$p_o(n) = p_i(n)h(N-1) + p_i(n-1)h(N-2) + \dots + p_i(n-N+1)h(0) \quad (13)$$

$$= \sum_{k=0}^{N-1} p_i(n-k)h(N-1-k)$$

It is noted that the active energy that before filtering for the infinite length of instantaneous power signal  $p_i(n)$  can be expressed as:

$$e_i = \sum_{n=-\infty}^{+\infty} p_i(n)T_s \quad (14)$$

Where  $T_s$  is the sampling interval, then the active power filtered can be expressed as:

$$e_o = \sum_{n=-\infty}^{+\infty} p_o(n)T_s = \sum_{n=-\infty}^{+\infty} \left( \sum_{k=0}^{N-1} p_i(n-k)h(N-1-k) \right) T_s \quad (15)$$

Since that the equality for the active power before and after filtering  $e_i = e_o$  in theory :

$$\sum_{n=-\infty}^{+\infty} p_i(n)T_s = \sum_{n=-\infty}^{+\infty} \left( \sum_{k=0}^{N-1} p_i(n-k)h(N-1-k) \right) T_s \quad (16)$$

$$= \sum_{k=0}^{N-1} h(k) \cdot \left[ \sum_{n=-\infty}^{+\infty} p_i(n)T_s \right]$$

It can be obtained that the filter coefficients requires meeting  $\sum_{k=0}^{N-1} h(k)=1$ , and still meet for the finite-length instantaneous power signal.

Then the filter can be constructed based on this conclusion through the PSa self-convolution window. Then the PSa filter can be expressed as:

$$h_{psa}(n) = \frac{1}{\sum_{n_k=0}^{N-1} w_{psa}^2(n_k)} \sum_{k=0}^K \alpha_k \frac{\sin \frac{\pi}{T}(n-kT)}{\frac{\pi}{T}(n-kT)} * \sum_{k=0}^K \alpha_k \frac{\sin \frac{\pi}{T}(n-kT)}{\frac{\pi}{T}(n-kT)} \quad (17)$$

where  $\sum_{n_k=0}^{N-1} w_{psa}^2(n_k)$  expresses to sum of the PSa self-convolution windows with the length of  $N$ .

## V. SIMULATION ANALYSIS

Taking the Quadriform Waveform as an example, the normalized waveform of the dynamic instantaneous power test signal is shown in Fig. 5, in which (a) expresses sinusoidal envelope instantaneous power signal, (b) expresses trapezoidal envelope instantaneous power signal, and (c) expresses the OOK envelope instantaneous power signal with an on-off ratio of 2:2.

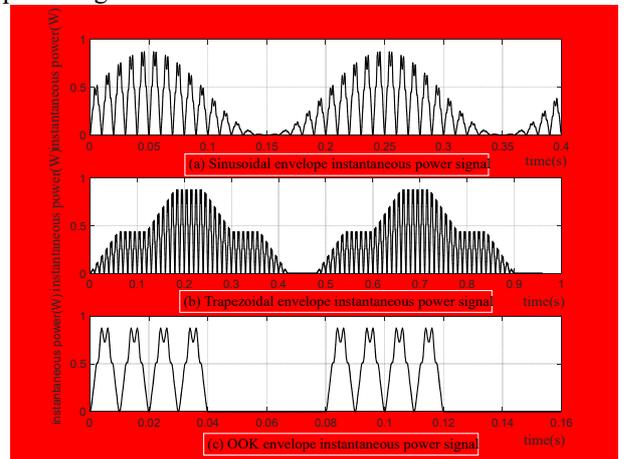


Fig.5 The dynamic instantaneous power test signal waveform for Quadriform Waveform

The electronic energy is measured with accumulated active power, and the expression for the accumulated error of electric energy calculated by formulas (14) and (15) can be expressed as:

$$\varepsilon_e = \frac{e_o - e_i}{e_i} \times 100\% \quad (18)$$

where  $e_i$  is the input electric energy before filtering and  $e_o$  is the accumulated output electric energy after filtering. As we can see from Table 1 and Table 2 that the results are the electric energy metering errors after filtering with different test signals by different active power algorithms. Table 1 shows sinusoidal envelope dynamic test signals, and Table 2 shows OOK dynamic test signals with an on-off ratio of 20: 20. In this simulation, the PSa filter is constructed by Eq. (17), the input signal length is two seconds, the sampling frequency is set as 3300Hz and the filter length is 128 which means non-integer-period sampling.

Table 1 Sinusoidal envelope different power algorithm energy measurement error

Sinusoidal envelope stabilization state waveform type	Rectangular window (%)	Hamming Window (%)	Blackman Window (%)	PSa Self-convolution window (%)
50Hz base wave	-1.24	-1.20	-1.19	$5.09 \times 10^{-14}$
Quadriform Waveform	-1.24	-1.20	-1.19	$4.03 \times 10^{-13}$
Peaked Waveform	-1.24	-1.20	-1.19	$1.19 \times 10^{-10}$
Pulse Waveform	-1.24	-1.20	-1.19	$5.61 \times 10^{-11}$
Multiple Zero Crossing Current Waveform	-1.25	-1.21	-1.20	$9.39 \times 10^{-08}$
Multiple Zero Crossing Voltage Waveform	-1.24	-1.20	-1.19	$3.19 \times 10^{-8}$

Table 2 OOK envelope different power algorithm energy measurement error

OOK envelope stabilization state waveform type	Rectangular window (%)	Hamming Window (%)	Blackman Window (%)	PSa Self-convolution window (%)
50Hz base wave	-1.61	-1.62	-1.62	$8.15 \times 10^{-13}$
Quadriform Waveform	-1.61	-1.62	-1.62	$-9.02 \times 10^{-13}$
Peaked Waveform	-1.61	-1.62	-1.62	$2.17 \times 10^{-10}$
Pulse Waveform	-1.61	-1.62	-1.62	$1.04 \times 10^{-10}$
Multiple Zero Crossing Current Waveform	-1.64	-1.64	-1.64	$1.76 \times 10^{-7}$
Multiple Zero Crossing Voltage Waveform	-1.62	-1.62	-1.62	$5.88 \times 10^{-8}$

In Table 1 and Table 2, the simulating experiment results show that the electric energy metering errors are as large as  $-1.64 \times 10^{-2}$ , when active power measurement

algorithm is based on rectangular window, Hamming window or Blackman window. On the contrary, the energy metering accuracy is **no more than**  $1.76 \times 10^{-9}$  when using the above PSa self-convolution window active power measurement algorithm.

## VI. CONCLUSION

In the paper, a new PSa self-convolution window is proposed, which has better side lobe attenuation characteristics than rectangular window, Hamming window and Blackman window. And based on the window, the PSa filter is constructed to measure active power and electric energy.

In the simulations, the energy measurement errors for fundamental wave and five kinds of distorted wave test signals are simulated under non-integer-period sampling with different active power measurement algorithms. The simulation results show that the PSa filter active power measurement algorithm improves the accuracy of electric energy measurement apparently.

The research results are important for the accurate measurement of electric energy of dynamic signals under non-integer-period sampling of smart meters.

## VII. ACKNOWLEDGMENT

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