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A new simple ECG signal model

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***Abstract* – A new mathematical model for generating ECG signal is presented in this paper. The generation is based on geometrical properties of real ECG signals instead of various artificial function used in commonly applicated models. Variation of parameters allows arbitrary setting of particular waves of PQRST complex. This way arbitrary PQRST signals typical for various heart diseases can be modelled and generated with high fidelity. Moreover, parameter variation allows generate different waveform for each subsequent heartbeat without mixing PQRST waves order. The elementary trigonometric function and Gaussian monopulse have been used for modelling each particular wave. Adding a noise and/or respiratory signal the final synthetized artificial signal can be applied for various signal processing method testing. The model was tested by comparison of synthetized patterns against patterns generated by LabVIEW Biomedical Toolkit. The model is tested against the real ECG records from MIT-BIH arrhythmia database as well, while the model parameters are found using a differential evolution algorithm.**

Keywords – synthetic ECG, ECG signal processing, ECG model, differential evolution, LabVIEW

I. INTRODUCTION

The heart generates spatio-temporal electric field during contracts within the cardiac pumping cycle. This electric field has been commonly measured in the form of electric signal by placing electrodes on the skin surface. The electric potential differences have been called the electrocardiographic signal (ECG) and it is one of the most important signals used in diagnostic methods for various cardiovascular diseases and for their compression which is required mainly for processing of long time ECG records. During development and experiments with various ECG signal processing methods it is essential for their testing to have suitable test signals, especially for compressed sensing or other longtime record processing methods. For such a purpose two approaches may be applied. The first one is using a database of real ECG recordings such as Phisionet [1]. However, as these recordings often contain lots of noise and artefacts they are not always suitable for testing and comparing the accuracy of some methods; it is difficult to measure how the particular methods react in case of the same basic ECG signal but with different characters and level of noise or digitizing method, e.g. level crossing ADC [2] or compressed sensing algorithms [3], [4], [5]. The second approach is to generate a synthetic ECG signal using suitable signal model producing precisely adjustable waveforms including addition of noise and/or other disturbances and distortion.

The well-known widely used ECG signal model for these purposes is the dynamical model [6] based on three coupled differential equations. Our approach presented in this paper significantly differs from that as it uses elementary trigonometric functions and linear function or a derivation of Gaussian pulse to model each particular wave.

II. ECG SIGNAL

An ECG signal consists of consecutive heartbeats. Every heartbeat is represented by five waves standardly labelled with the letters P, Q, R, S and T (Fig. 1). Each of these waves corresponds to depolarization and repolarization of heart muscles [7].

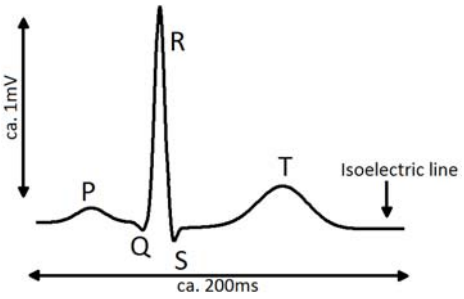

Fig. 1. PQRST complex of a normal lead II ECG recording

The P wave represents activation of atria and it is followed by relatively short isoelectric segment while the

cardiac impulse is passed through the A-V node and the His-Purkinje system. Once the muscles of ventricles are excited they get contracted. This contraction provides the main force for circulating blood to the body organs and creates the biggest wave in ECG signal. It consists of three components. First downward deflection is called the Q wave, consecutive upward deflection is called the R wave and the final downward deflection is called the S wave. Group of these three waves is referred to as the QRS complex. The polarity and presence of each particular wave of QRS complex may vary according to the position of sensing leads as well as body abnormalities. Following the QRS complex there is another isoelectric segment. Finally the ventricles return to their electrical resting state and this repolarization can be seen as a low-frequency wave called the T wave.

III. THE PROPOSED ECG MODEL

According to typical ECG morphology we decided to model each particular wave of a PQRST complex using elementary mathematical functions, which are concatenated and compose into a complete heartbeat. Parameters allow to set amplitude, width and position of each respective wave. In case of Q and S wave there are two variants introduced. The first one uses fixed shape of Q and S waves and allows setting their amplitude and width. The second one allows also changing their shape.

A. Fixed shape of Q and S waves

A heartbeat starts with a short isoelectric segment which is simply modelled by a function:

$$B(k) = 0; \text{ for } 0 \le k \le K_B, \quad (1)$$

where K_B is the width and k is the sample number within one wave segment. Ongoing P wave is modelled using a cosine function:

$$P(k) = -A_P \cos\left(\frac{2\pi k+15}{K_P}\right); \text{ for } 0 \le k \le K_P, \quad (2)$$

where A_P is the amplitude and K_P is the width of P wave. There is again a short isoelectric segment:

$$P_Q(k) = 0; \text{ for } 0 < k \le K_{PQ}, \quad (3)$$

where K_{PQ} is the width. The Q wave is modelled using a segment created by section of a Gaussian monopulse (a differentiated Gaussian pulse):

$$Q(k) = \frac{A_Q(\mathrm{k}-0.1\mathrm{K_Q}+0.1)19.78\pi}{K_Q} e^{-2\left(\frac{6\pi}{K_Q}(\mathrm{k}-0.1K_Q+0.1)\right)^2} \quad (4)$$

for $0 \le k \le K_Q$, where A_Q is the amplitude and K_Q is the width of Q wave. The R wave is modelled using a sinewave segment:

$$R(k) = A_R \sin\left(\frac{\pi k}{K_R}\right); \text{ for } 0 \le k \le K_R, \quad (5)$$

where A_R is the amplitude and K_R is the width of R wave. The S wave is modelled again using a segment of Gaussian monopulse:

$$S(k) = -A_S 0.1k \frac{19.78\pi}{K_S} e^{-2\left(\frac{6\pi}{K_S}0.1k\right)^2} \text{ for } 0 \le k \le K_S - K_{CS}, \quad (6)$$

where A_S is the amplitude, K_S is the basic width of the S wave. K_{CS} is a parameter which allows slight adjustment of S wave shape by cutting away a portion at the end. Now the transition between S and T wave is described as a linear function:

$$S_T(k) = -S(K_S - K_{CS})\frac{k}{s_m} + S(K_S - K_{CS}) \quad (7)$$

for $0 \le k \le K_{ST}$, where s_m is the slope parameter and K_{ST} is the width of transition segment. The T wave is modelled using a segment of cosine function:

$$T(k) = -A_T \cos\left(\frac{1.48\pi\mathrm{k}+15}{K_T}\right) + A_T + S_T(K_{ST}) \quad (8)$$

for $0 \le k \le K_T$, where A_T is the amplitude and K_T is the width of T wave. The final transition from T wave back to isoelectric line is modelled using function:

$$I(k) = T(K_T)\frac{s_I}{k+10}; \text{ for } 0 \le k \le K_I, \quad (9)$$

where s_I is the parameter for setting the transition slope between T wave and isoelectric line and K_I is the width of the ending section.

A complete heartbeat $H(^1b,n)$ is given as concatenation of all waves modelled by (1-8): the beginning of each segment is appended to the end of a previous one, while their order is always the same starting with (1) and ending with (8). Here ^1b denotes the set of heartbeat parameters for the first variant:

$$^1b = \{K_B, A_P, K_P, K_{PQ}, A_Q, K_Q, A_R, K_R, A_S, K_S, K_{CS}, s_m, K_{ST}, A_T, K_T, s_I, K_I\} \quad (10)$$

and n denotes the sample number within one generated heartbeat (n = 0, 1, 2, …, N_1-1). Each segment of a heartbeat has maximum length defined by width parameters (K_x). Thus, the sum of all segment lengths is $N_1 = K_B + K_P + K_{PQ} + K_Q + K_R + K_S - K_{CS} + K_{ST} + K_T + K_I$.

B. PQRST complex with adjustable Q and S waves

This variant differs from the previous one by usage of two linear functions to model Q and S wave instead of a Gaussian monopulse, which allows adjusting the shape of

respective waves more precisely. Here all the modelling functions are the same as for the previous section, except of equations (4) and (6). The equation (4) is replaced by concatenation of two functions:

$$Q_1(k) = -A_Q \frac{k}{K_{Q1}}; \text{ for } 0 \le k \le K_{Q1}, \qquad (11)$$

$$Q_2(k) = A_Q \frac{k}{K_{Q2}} - A_Q; \text{ for } 0 \le k \le K_{Q2}, \qquad (12)$$

where K_{Q1} is the width of downward deflection, K_{Q2} is the width of upward deflection and A_Q is the amplitude of Q wave. Similarly, the equation (6) is replaced by concatenation of two functions:

$$S_1(k) = -A_S \frac{k}{K_{S1}}; \text{ for } 0 \le k \le K_{S1}, \qquad (13)$$

$$S_2(k) = A_S \frac{k}{s_s} - A_S; \text{ for } 0 \le k \le K_{S2}, \qquad (14)$$

where K_{S1} is the width of downward deflection, K_{S2} is the width of upward deflection, s_s is the slope parameter of upward deflection and A_S is the amplitude of S wave. The argument of $S(K_S\text{-}K_{CS})$ in equation (7) is now replaced by K_{S2}.

A complete heartbeat $H(^2b,n)$ is then adjusted using modified set of parameters ^2b for the second variant:

$$^2b = \{K_B, A_P, K_P, K_{PQ}, A_Q, K_{Q1}, K_{Q2}, A_R, K_R, A_S, K_{S1}, s_s, K_{S2}, s_m, K_{ST}, A_T, K_T, s_I, K_I\} \qquad (15)$$

here n denotes the sample number n = 0, 1, 2, …, N_2-1, and $N_2=K_B+K_P+K_{PQ}+K_{Q1}+K_{Q2}+K_R+K_{S1}+K_{S2}+K_{ST}+K_T+K_I$ is the sum of all segment width parameters.

C. Creating a custom ECG signal

For simplification let's denote the function of one heartbeat generated using any of the described variants as $H(b,n)$, where b is the set of parameters ^1b or ^2b and, n = 0, 1, 2, …, N=N_1-1 in case of ^1b or N=N_2-1 in case of ^2b respectively. Because the concatenation of functions forming a heartbeat obviously leads to unnatural sharp edges present in resulting signal, the generated heartbeat is filtered using a Savitzky-Golay smoothing filter with window size of 7:

$$H_f(b,n) = \frac{1}{21}(-2H(b,n-3) + 3H(b,n-2) + 6H(b,n-1) + 7H(b,n) + 6H(b,n+1) + 3H(b,n+2) - 2H(b,n+3)) \qquad (16)$$

for n=0, 1, 2, …, N, where $H(b,n\pm j)$=0 if the sum $n\pm j$ is out of the specified interval for n. The filter plays a significant role while using the second model variant, because it forms the smooth shape of Q and S waves.

A complete ECG signal now can be built by concatenating multiple heartbeats $H_f(b_i,n)$, where the set of parameters b_i can vary in each consecutive heartbeat. The beginning and ending segment of each heartbeat aligns smoothly with isoelectric line, thus there are no artifacts present at the position of their joint. This allows creating a completely custom-made ECG signal which can include various irregular heartbeats present at custom positions. Even mixing the heartbeats generated from both proposed model variants is possible. As an example, a short signal containing a premature ventricular contraction (PVC) is modelled in Fig.2 using the first model variant.

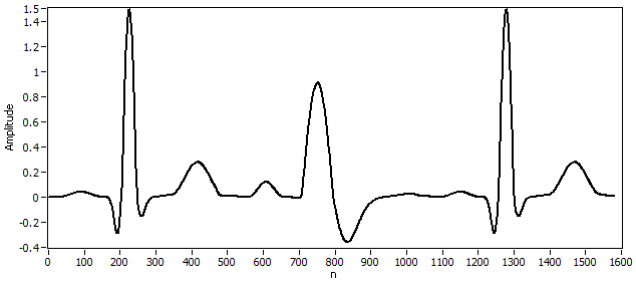

Fig. 2. A modelled ECG signal containing a PVC heartbeat.

D. Additive noise and the distortion by respiration

To make the modelled signal more realistic, white noise simulating measurement noise may be added into resulting signal. If needed, also powerline noise is added in the form of a small sinewave with powerline frequency 50Hz or 60Hz.

The respiration causes a baseline wander of ECG signal. It means that the isoelectric line relatively slowly periodically changes its position. A simple way to model this effect is to add a sinewave of respiration frequency into ECG signal, which may vary from about 0.2Hz to 0.5Hz (approximately 12 to 30 breaths per minute) [8].

An example of synthetized hypokalemia ECG signal with addition of equipment noise and effect of respiration can be seen in Fig.3.

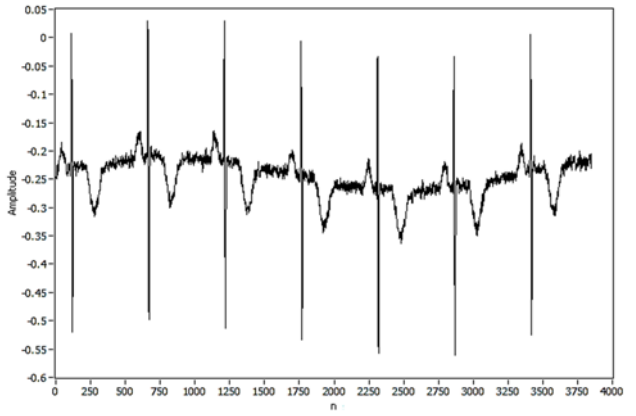

Fig. 3. An example of modelled hypokalemia ECG signal with noise and respiration frequency added.

IV. EVALUATION OF PROPOSED METHOD

The evaluation of proposed model was performed by two comparison ways. The first one is based on comparison of created artificial signals with those generated by LabVIEW Biomedical toolkit. The second evaluation method is based on comparison of modelled artificial signal with real signal taken from real patients' database.

A. *Evaluation by comparison with LabVIEW biomedical toolkit*

To test the possibilities of QRS pattern generation using proposed model its parameters were adjusted to fit typical irregular heartbeats. As a reference the irregular beats for various diagnoses were generated using ECG signal generator from LabVIEW Biomedical Toolkit. Each reference pattern $H_r(n)$ was extracted from 60bpm signal generated with sampling frequency of 512Hz and contains one QRS complex with length of 1s. The set of model parameters b was found using differential evolution optimization algorithm [9] (DE) solving the problem:

$$b = arg\min_{b} PRD \tag{17}$$

where *PRD* is the percentage root mean square difference between the reference and modeled heartbeat:

$$PRD = \frac{\|H_r(n) - H_f(b,n)\|_2}{\|H_r(n)\|_2} \cdot 100\%. \tag{18}$$

Initial bounds for b parameters were set in a following way: -0.2 to 2 for amplitude parameters, 0-150 for segment length parameters and 0-200 for slope parameters. After first iterations of DE the b parameter bounds were adjusted according to data found in Table 1.

Table 1. DE parameters bound setting for both model variants.

^1b	min	max	^2b	min	max
K_B	0	130	K_B	0	130
A_P	-0.2	0.1	A_P	-0.2	0.1
K_P	10	100	K_P	10	100
K_{PQ}	0	60	K_{PQ}	0	60
A_Q	0	0.5	A_Q	0	0.5
K_Q	10	150	K_{Q1}	0	70
A_R	1	2	K_{Q2}	0	50
K_R	10	150	A_R	1	2
A_S	0	1	K_R	10	150
K_S	10	200	A_S	0	1
K_{CS}	-5	150	K_{S1}	0	50
s_m	1	150	s_s	1	110
K_{ST}	0	110	K_{S2}	0	50
A_T	-0.5	1	s_m	1	150
K_T	50	200	K_{ST}	0	100
s_I	0	50	A_T	-0.5	1
			K_T	50	200
			s_I	0	150

Table 2. First variant model parameter sets for each pattern

^1b	Diagnosis							
	a	b	c	d	e	f	g	h
K_B	10	0	117	61	121	121	117	124
A_P	0.03	0	0.0045	0.053	0.035	0.035	0.05	-0.02
K_P	93	23	79	91	73	69	79	75
K_{PQ}	0	0	0	48	6	13	5	0
A_Q	0.135	0.325	0.065	0.04	0.04	0.02	0.03	0
K_Q	85	140	25	21	21	22	20	15
A_R	1.15	1.09	1.52	1.55	1.17	1	1.55	1.37
K_R	84	133	23	23	23	15	22	36
A_S	0.35	0.28	0.16	0.13	0.11	0.75	0.6	0.16
K_S	114	182	15	15	15	26	14	54
K_{CS}	61	100	5	2	4	-3	5	27
s_m	61	119	96	17	26	35	1	87
K_{ST}	52	57	101	52	56	64	6	42
A_T	0.13	0	0.19	0.132	0.685	-0.1	0.115	0.225
K_T	127	77	126	116	112	112	116	184
s_I	0	0	2	9	9	7	10	19
K_I	8	0	31	87	89	67	138	9
PRD [%]	7.8	7.61	11.12	11.3	14.0	9.89	13.4	7.65

Table 3. Second variant model parameter sets for each pattern.

^2b	Diagnosis							
	a	b	c	d	e	f	g	h
K_B	10	0	117	61	121	121	117	124
A_P	0.03	0	0.0045	0.053	0.035	0.035	0.05	-0.02
K_P	93	23	79	91	73	69	79	75
K_{PQ}	1	1	14	50	13	43	14	17
A_Q	0.13	0.33	0.065	0.065	0.047	0	0.06	0
K_{Q1}	55	55	12	12	12	0	11	0
K_{Q2}	31	37	7	7	7	0	7	0
A_R	1.15	1.09	1.52	1.55	1.17	1	1.55	1.37
K_R	77	137	22	22	22	12	23	32
A_S	0.38	0.27	0.18	0.16	0.12	0.48	0.1	0.19
K_{S1}	32	44	9	7	7	12	5	15
s_s	62	108	4	4	6	6	2	8
K_{S2}	33	31	4	4	6	7	4	4
s_m	53	97	138	64	14	1	29	55
K_{ST}	52	52	100	56	53	74	1	62
A_T	0.12	0	0.2	0.13	0.665	-0.1	0.12	0.23
K_T	119	87	137	126	126	126	133	183
s_I	17	0	0	128	32	0	23	0
K_I	9	45	11	76	72	48	118	0
PRD [%]	9.5	10.6	9.18	9.59	6.69	13.9	10.6	11.4

The length parameter of last segment K_I is calculated so that the total sum of length parameters $N=N_1$-1 or $N=N_2$-1 is always equal to 512 and thus it is excluded from the set of parameters being optimized. Signal output of the model with such a configuration does not need resampling.

Irregular QRS patterns used as a reference for which the model was tested correspond to atrial tachycardia (a), ventricular tachycardia (b), junctional tachycardia (c), atrioventricular block (d), hyperkalemia (e), hypokalemia (f), hypercalcemia (g) and hypocalcemia (h).

Corresponding parameter setting ^1b found by DE and resulting PRD is listed in Table 2. For the second model

variant the found parameters and PRD is listed in Table 3. Results were obtained using simulation in LabVIEW programming environment. DE optimization used uniform crossover method, population size of 500, and maximum number of 200 iterations for all the testing results provided. The PRD values listed in all the tables are given as an average for 10 runs of DE algorithm.

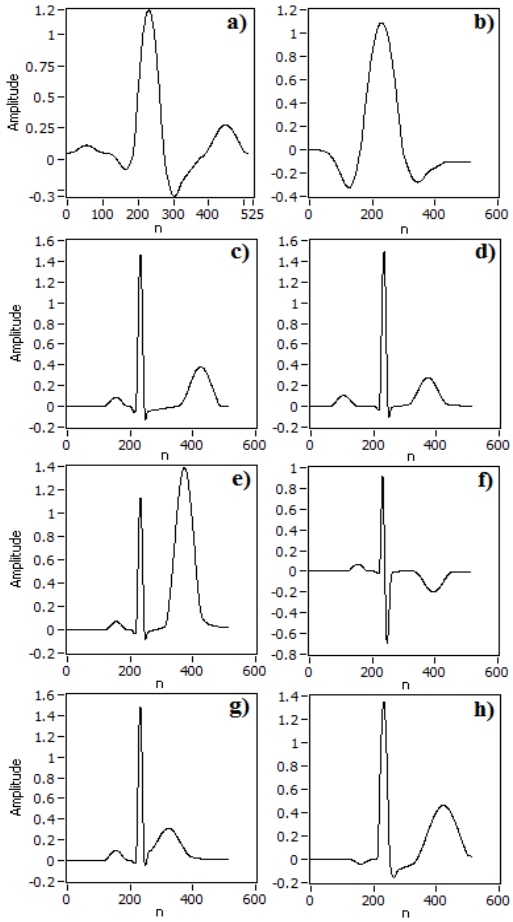

Fig. 4. ECG QRS patterns generated by the first model variant: a) atrial tachycardia; b) ventricular tachycardia; c) junctional tachycardia; d) atrioventricular block; e) hyperkalemia; f) hypokalemia; g) hypercalcemia; h) hypocalcemia.

As it can be seen from higher PRD for (a) and (b) diagnoses, the second model variant is problematic when it comes to modelling patterns where the Q and S waves have low frequencies and thus are too wide. In this case the linear functions used for modelling create sharp edges present at the peaks of each wave and the filter (16) window size is too low to smooth these artefacts. Thus, at modelling low frequency Q and S waves the first model variant performs better. For some situations where the Q and S waves are of high frequency e.g. diagnoses (e) and (g), the second model variant performs better. However, because the second model variant has more adjustable parameters and they have been more difficult to set, for general purposes of simulating an ECG signal the first model variant is much suitable and simpler to use.

Resulting QRS patterns modelled using first model variant for each diagnose can be seen in Fig. 4.

B. Evaluation by comparison with real ECG patterns

The evaluation for real ECG patterns is similar as in the previous subsection. However, now the reference pattern is made by extraction of a single heartbeat from the beginning of the first channel of each MIT-BIH database record [6]. Because the database records use a 360Hz sampling frequency the extracted pattern is resampled to a duration of 1 second at sampling frequency of 512Hz and after that it is used as the reference pattern $H_r(n)$. By such an approach the parameter bounds of DE algorithm can remain the same as listed in the Table 1 for the previous experiments. The resulting values of PRD for real ECG patterns of MIT-BIH database are listed in the Table 4.

Table 4. DE parameters bound setting for both model variants.

Record No.	*PRD* [%]		Record No.	*PRD* [%]	
	Variant 1	Variant 2		Variant 1	Variant 2
100	16.23	15.09	201	14.56	14.35
101	15.36	15.16	202	20.01	41.62
102	31.09	31.61	203	26.68	22.96
103	12.13	21.79	205	8.73	8.81
104	38.73	40.89	207	41.01	44.82
105	19.37	15.11	208	42.65	24.79
106	18.83	19.41	209	11.03	9.84
107	31.37	19.33	210	34.98	24.75
108	32.25	21.88	212	31.71	48.62
109	16.19	10.85	213	17.41	16.71
111	34.42	33.86	214	36.57	29.05
112	8.56	7.66	215	11.71	9.09
113	24.91	23.36	217	33.06	14.69
114	18.11	39.85	219	19.02	16.11
115	12.99	13.19	220	5.93	7.46
116	6.77	13.29	221	16.25	15.58
117	7.02	19.22	222	24.72	27.21
118	11.17	10.49	223	15.84	14.59
119	5.73	6.13	228	20.91	19.39
121	11.19	12.4	230	15.69	11.59
122	5.88	4.87	231	17.57	12.39
123	7.19	6.79	232	20.18	22.15
124	15.37	10.53	233	35.43	28.58
200	21.11	20.89	234	21.88	19.43
			Average	20.11	19.54

As it can be seen, each model is suitable for different types of patterns. The average PRD is better for the second model variant although it should be noted that the extracted heartbeats may possibly contain also noise and artifacts which were not included into the signal model during this test.

V. CONCLUSIONS

Two ECG model variants were introduced for generation of artificial ECG signals. Both models allow creating a custom-made signal including irregularities occurring at custom positions and at custom time instances as well as addition of equipment noise and respiration effect. The possibility of various irregular heartbeats generation was tested against reference QRS patterns generated by LabVIEW Biomedical Toolkit and against real ECG records from MIT-BIH. Differential evolution optimization algorithm was used to find the model parameters fitting the reference QRS patterns corresponding to various diagnoses and the PRD was evaluated for each case.

The performed evaluation showed that the first model is suitable for modelling low frequency Q and S waves, while the second model variant using linear functions performs better in some cases of high frequency Q and S waves. However, because the first model uses fewer setting parameters it is much simpler to use and can be used universally. Second model variant is less suitable for generating QRS patterns where the Q and S waves are too wide; however, it shows better average results for imitating the real signal patterns. The relatively complex setting of heartbeat frequency of long-term signals by adjusting the length parameters of each wave segment is its complication. It can be solved by resampling the generated signal into suitable form.

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