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Calibration of lock-in amplifiers in the low-frequency range

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***Abstract* – A setup for the calibration of lock-in amplifiers in the low frequency range is presented. The setup is based on an uncalibrated voltage source, an ac voltmeter, and calibrated inductive and resistive voltage dividers. The measurand is the relative deviation of the lock-in amplifier reading from the calibrated applied voltage. Examples of calibration of a commercial lock-in amplifier in the 10 µV range, at frequencies of 17 Hz and 103 Hz, are given; the calibration uncertainty, relative to the instrument full-scale, is under evaluation and expected to be of the order of 10^{-4}.**

***Keywords* – Calibration, Linearity, Demodulation, Mesaurement techniques, Voltage measurement, Signal detection, gain error, Lock-in amplifier.**

I. INTRODUCTION

Lock-in amplifiers are instruments that can recover, by homodyne detection, the magnitude and phase of the component at frequency f of an input signal, where f is provided by a periodic reference signal to which the instrument locks in phase [1, 2]. Lock-in amplifiers are therefore widely employed in physics experiments to measure very small signals, down to the nanovolt range, in presence of noise. These are not precision instruments: typical specifications for the gain accuracy are of the order of percent. A gain calibration can reduce the measurement uncertainty in most experiments. To the authors' knowledge, the literature on the calibration of lock-in amplifiers is limited [3, 4, 5].

We describe here a simple setup for the calibration of the lock-in amplifier's gain in voltage mode. The setup synthesizes a sinewave signal having amplitude traceable

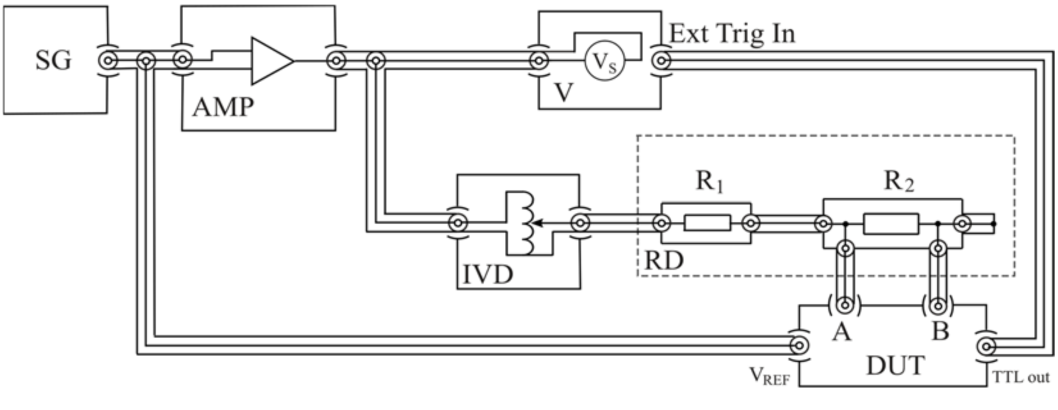

Fig. 1: Schematic diagram of the setup for the calibration of lock-in amplifiers at low frequency.

to the SI, in the voltage range from 1 µV to 100 µV, and in the frequency range from 10 Hz to 1000 Hz. These ranges are often employed in physics experiments. An example of setup performance for the calibration of a commercial lock-in amplifier is given.

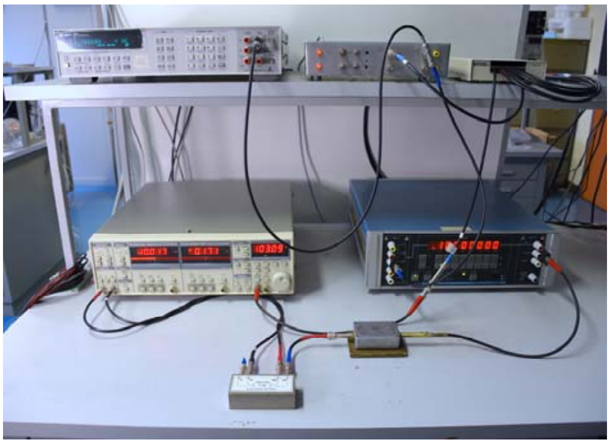

***Fig. 2**: Photograph of the calibration setup. From top-left: voltmeter V, 1:1 gain amplifier AMP, sinewave generator SG, DUT, inductive voltage divider IVD. In front position the resistive divider RD.*

II. SETUP AND OPERATION

The coaxial schematic diagram of the calibration setup is shown in Fig. 1, and a picture of the setup in Fig. 2. A signal generator SG, based on a digital-to-analogue converter board National Instruments PCI-6733, provides a large-amplitude, low-distortion sinewave V_S and an isofrequential reference signal V_{REF}. The signal V_S is buffered by the unity-gain amplifier AMP with differential input. The purpose of AMP is to provide electrical decoupling from SG and to provide the drive current for the following stages.

The calibrated voltmeter V measures the magnitude of V_S. This voltmeter is a Hewlett-Packard 3458A multimeter, set in the SETACV SYNC (Synchronously sub-sampled computed true RMS) mode, which allows the highest accuracy when performing measurements on low-frequency periodic waveforms, with an uncertainty of 8×10^{-5} [6]. To work in this mode, the voltmeter requires an external synchronous reference signal. As shown in Fig. 1, the lock-in TTL out is used for this purpose [6,7].

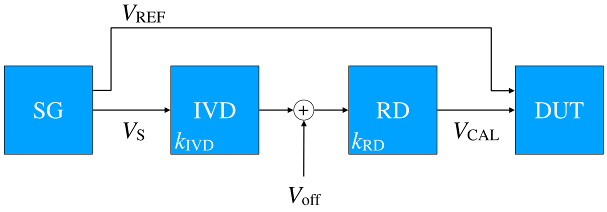

***Fig. 2**: Calibration signal path block diagram.*

Subsequently, the signal V_S is scaled to the amplitude V_{IVD} by an inductive voltage divider IVD set at the ratio k_{IVD}. The IVD is a TEGAM PRT73 Programmable Ratio Transformer. The 2.5 V/Hz low frequency option allows to achieve an accuracy of 10^{-6} in the 50 Hz - 400 Hz range [8]. The voltage V_{IVD} is further reduced in amplitude by the factor k_{RD} by the custom resistive divider RD, composed of the calibrated resistors R_1 and R_2. The resulting calibration signal V_{CAL} is fed to the lock-in under calibration DUT, to which the reference signal V_{REF} is also applied.

III. MEASUREMENT MODEL

A block schematic of the signal path, according to the description given in section II, is reported in Fig. 3. The quantity V_{off} that is added to the signal between IVD and RD represents possible interferences or feedthroughs that can alter V_{CAL}, most likely due to the magnetic coupling of V_S with the IVD cores. It is assumed that DUT is working on a fixed full-scale range and provides readings X and Y from the two channels.

The calibration signal V_{CAL} can then be expressed as

$$V_{CAL} = k_{RD}\,(k_{IVD}\,V_S + V_{off}), \qquad (1)$$

where k_{RD} can be computed from the knowledge of R_1, R_2, the DUT input resistance R_{in} and capacitance C_{in}.

We model the behaviour of the DUT, for a given frequency and full-scale range, around a working point, with a gain G (close to its nominal gain G_{NOM}), an offset O, and a phase shift ϕ (caused, among other effects, by the setting of the reference phase). Assuming offset quantities V_{off} and O, and by performing both a reading under V_{CAL} excitation (X, Y) and an additional reading (X_O, Y_O) by setting $k_{IVD} = 0$, we can express the gain error ϵ_G as

$$\epsilon_G = \frac{|(X-X_O)+j(Y-Y_O)|}{k_{RD}k_{IVD}|V_S|} - 1. \qquad (2)$$

IV. EXAMPLES OF CALIBRATION

In the following, we present preliminary results of the calibration of a commercial lock-in amplifier, a Stanford Research Systems SR830, serial number 81833. The calibrations were both performed on the 10 μV range, at the frequencies of 103 Hz and 17 Hz. These frequencies were chosen to reduce interference from power line harmonics (set as 50 Hz in Europe), as is typical during the normal operation of a lock-in amplifier in physics experiments.

The settings and the parameters of the calibration setup are as follows: $V_{S,RMS} = 1$ V, $R_1 = 100$ kΩ, $R_2 = 10$ Ω (so that $k_{RD} = 1 \times 10^{-5}$); k_{IVD} is then set from 0.1 to 1. The phase of V_{REF} with respect to V_S is set either to 0° or 180°. The nominal calibration voltage phasors obtained are thus V_{CAL} = -10 μV, -9 μV, …, -1 μV, +1 μV, …, +9 μV, +10 μV.

The relevant settings of the DUT are as follows: differential input with DC coupling, synchronous filter OFF, dynamic reserve Low Noise, power line notch filters off, time constant 1 s, with a filter slope of 18 dB/octave (103 Hz) or 24 dB/octave (17 Hz); the sampling frequency is 16 Hz.

The measurement process is the following. k_{IVD} is first set to 0 and the offset (X_O, Y_O) of (2) is measured. Then k_{IVD} is progressively stepped up from the minimum (0.1) to the maximum value (1) to drive the DUT through the selected calibration points. The phase of V_{REF} is then reversed and a second measurement series is taken.

For each calibration point, the RMS value of V_S is measured by V as the average of consecutive readings (100 in these examples). Simultaneously, DUT reading samples (X, Y) are acquired and averaged. The acquisitions of V and DUT are asynchronous, with a sampling time dependent on each instrument setting.

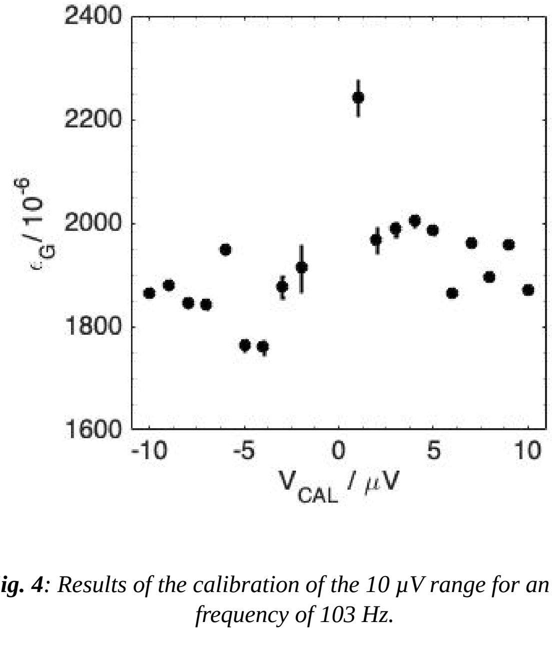

***Fig. 4**: Results of the calibration of the 10 μV range for an input frequency of 103 Hz.*

The readings (X_O, Y_O), Vs and (X, Y) are the input quantities of the model equation (2).

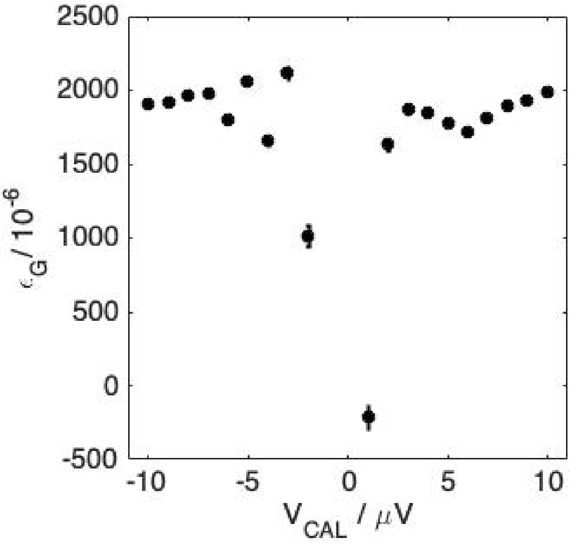

***Fig. 5**: Results of the calibration of the 10 μV range for an input frequency of 17 Hz.*

The outcomes of the two example calibrations are plotted in Fig. 4 (frequency 103 Hz) and Fig. 5 (17 Hz). For each calibration point, the uncertainty bar represents a preliminary Type A uncertainty evaluation.

Both measurements show that the nominal specifications of the DUT (1% gain accuracy) are met. In fact, the average gain error in both measurements is of the order of $+1.9 \times 10^{-3}$.

The type A uncertainty evaluation will require further refinements to take into account possible correlations between readings caused by the long time constant of the DUT and bias effects caused by the modulus function of (2).

The type B uncertainty is under evaluation, taking into account the voltmeter V uncertainty, the IVD uncertainty and the uncertainty of the calibration of R_1 and R_2.

V. CONCLUSIONS

We presented a system suitable for the calibration of the gain of lock-in amplifiers in the low frequency range. The system is presently under extensive testing. The target calibration uncertainty is expected to be of the order of

1×10^{-4}. More measurement examples, together with an expression of uncertainty, will be presented at the Conference.

This activity is framed in the context of EURAMET Project 1466 [9] that comprises an international comparison between INRIM and the Federal Institute of Metrology (METAS), Switzerland.

VI. ACKNOWLEDGMENTS

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