23rd IMEKO TC4 International Symposium
Electrical & Electronic Measurements Promote Industry 4.0
September 17-20, 2019，Xi'an, China

Applications of Programmable Josephson Voltage Standard on Magnetic Measurements

Zengmin Wang[1,2], Honghui Li[1,2], Yuan Gao, Yunfeng Lu[1,2], Qing He[1,2]

[1] *National Institute of Metrology, China*
[2] *Key Laboratory for the Electrical Quantum Standard of AQSIQ, Beijing 100029, China*

Abstract **–This paper describes the progress of magnetic applications on Programmable Josephson Voltage Standard (PJVS) at NIM. In past few years, we developed a PJVS based magnetic flux measurement system [1] and continued improving the magnetic flux and the mutual inductance measurement accuracy. This paper introduces the automatic flux and mutual inductance measurement system that we developed. Experimental results show that the type A uncertainty of the measurement of mutual inductances of 577 mH is less than 2 ppm, which demonstrates the feasibility and reliability of the flux and mutual inductance measurement system we proposed.**

Keywords – Magnetic flux, Mutual Inductance, Josephson Voltage Standard, Flux meter calibration.

I. INTRODUCTION

As a basic magnetic quantity, magnetic flux is very useful in a variety of magnet applications. According to the induction law, the change of flux in a measurement coil will induce a voltage across the coil terminals. It is the oldest of the currently used methods for magnetic measurements. Fluxmeters with precision analog integrators are commonly used for absolute measurements of magnetic flux. The relative measurement uncertainties of commercial fluxmeters are usually several parts in $10^3 \sim 10^4$ Wb/Wb.

A PJVS system based on a PTB SINIS array was built to synthesize stepwise-approximated ac voltage [2-4]. With a domestic multi-channel bias current source, this system is able to synthesize a variety of voltage waveforms including sine waves, rectangular waves, pulse waves, et al. In particular, it can produce rectangular pulses with given volt-second area (precise amplitude and pulse width), which can be considered as a volt-second generator (a magnetic flux standard).

We proposed a new approach of magnetic flux measurement by measuring the flux difference between rectangular pulse synthesized with a PJVS system and the flux under test. Different from the absolute measurement of magnetic flux with fluxmeters, this method only measure the flux difference. Another advantage of this method is the high accuracy of the volt-second signal driving from the PJVS. It is a new way of measuring magnetic flux with very high precision.

Different from voltage which can be continuously produced by a voltage source within a long duration, it is impossible to store flux or continuously produce flux. Quantitatively, the mutual inductance between two circuits may be defined as the flux linkage produced in one circuit by a current of one ampere in the other circuit. Therefore, a mutual inductance is used to produce magnetic flux in our measurement system. The measurement results of magnetic flux is also used to evaluate the mutual inductance.

In this paper, we describe the PJSV based system designed for magnetic flux and mutual inductance precision measurement.

II. METHODS AND ALGORITHMS

Fig. 1 shows the scheme of the flux and mutual inductance measurement system based on the programmable Josephson system. A mutual inductance and a current source are mainly used to generate the magnetic flux under test. The whole system is divided into two parts by the two coils of the mutual inductance. The circuit in the right illustration which consists of an exciting current source, a sampling resistance and the primary coil of the mutual inductance is considered as the exciting circuit. A sampling resistance, a Zener and a Keysight 34420A multimeter are used to measure the exiting current. The other circuit in the left illustration which consists of the programmable Josephson system, flux meter and the secondary coil of the mutual inductance is considered as the measurement circuit.

The measurement has two steps. Firstly, the fluxmeter is calibrated by the PJVS. Then, the fluxmeter is used to measure the flux difference between the rectangular pulse (volt-second signal) generated by the PJVS and the flux generated by the mutual inductance. In Fig. 1, the switch S1 and the switch S2 are firstly set to 1 and 1', and the rectangular pulse with a calculable volt-second area generated by the PJVS is used to calibrate the flux meter. Then, the switch S1 and the switch S2 are set to 2 and 2', and the calibrated fluxmeter is used to measure the flux difference generated by the PJVS and the mutual inductance. Finally the flux under test is reconstructed by adding the flux difference to the calculated volt-second area generated by the PJVS.

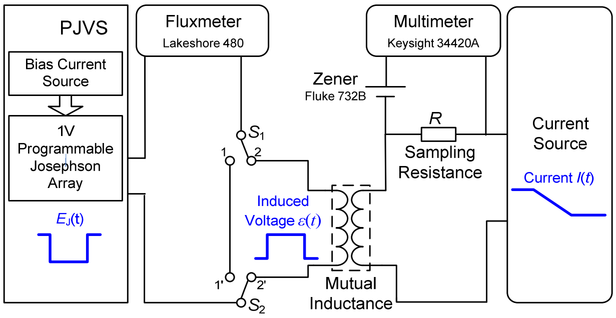

Fig. 1. Schematic diagram of the automatic flux and mutual inductance measurement system

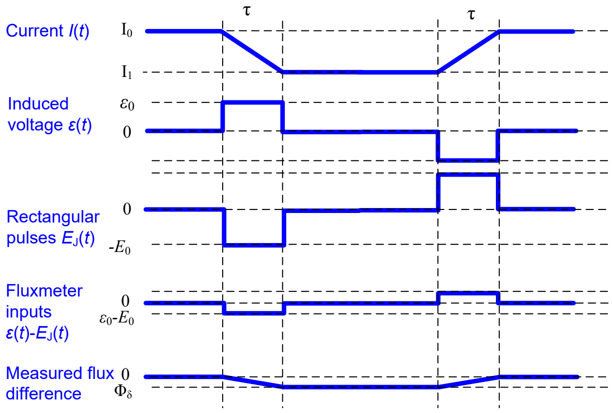

Fig. 2. Waveforms (one peroid) of the full-automatic flux and mutual inductance measurement system

In Fig. 1, the flux under test is provided by a mutual inductance with specific design. When the current in the primary coil of the mutual inductance changes, the flux generated in the secondary coil is proportional to the current variation. The proportional factor is determined by the mutual inductance of the two coils. The relationship among the mutual inductance M, the exiting current I, the induced flux and the induced electromotive force is as follows,

$$\Phi(t) = M \cdot I(t) = \int \varepsilon(t) \cdot dt \qquad (1)$$

Fig. 2 shows the waveforms of exciting current $I(t)$, induced voltage $\varepsilon(t)$, outputs of PJVS system $E_J(t)$ and the measured flux difference of the flux measurement system shown in Fig.1. During the first half period, the exciting current $I(t)$ rised linearly with time τ, and the induced voltage $\varepsilon(t)$ is a rectangular pulse with the amplitude ε_0, where,

$$\varepsilon_0 = -M \cdot \frac{dI(t)}{dt} \qquad (2)$$

The PJVS system is used to generate the rectangular pulse with its amplitude (E_0) and width close to $\varepsilon(t)$ in the opposite direction, therefore, the input of fluxmeter is a approximate rectangular pulse with the amplitude close to ε_0-E_0. So the measured flux difference is:

$$\Phi_\delta = -\int [\varepsilon(t) - E_J(t)] \cdot dt$$
$$= E_0 \cdot \tau - \int \varepsilon(t) \cdot dt \qquad (3)$$
$$= E_0 \cdot \tau - \Delta\Phi$$

Where $\Delta\Phi$ is the flux variation through the secondary coil caused by the exciting current $I(t)$ in the exciting circuit.

According to formula (2), the flux variation through the secondary coil is proportional to the current variation through the primary coil,

$$\Delta\Phi = M \cdot \Delta I \qquad (4)$$

Therefore, the mutual inductance measured in this system can be expressed as,

$$M = \frac{E_0\tau - \Phi_\delta}{\Delta I} = \frac{E_0\tau - \Phi_\delta}{I_1 - I_0} \qquad (5)$$

Where $E_0\tau$ is the calculable volt-second area of the rectangular pulse generated by the programmable Josephson system. Φ_δ is the flux measured by the flux meter. The amplitude of the falling ramp current generated by the exciting current source, as shown in Fig. 2, varies from I_0 to I_1.

If the measured flux difference Φ_δ is much smaller than the flux variation $\Delta\Phi$ generated by the mutual inductance, the impact of reading error on the measurement results will significantly reduce. For example, if the flux under test (generated by the solenoid) is around 200 mV·s, the flux difference can be reduced to less than 1 mV·s by adjusting the width and height of rectangular pulse $E_J(t)$. Assuming the relative error of indication of fluxmeter is several parts of 10^5, the reading error can be reduce to several parts of 10^7 which is an improvement of the flux measurement accuracy.

For instance, when measuring the flux around 1 Wb, the subarray of 2048 junctions is used to generate the rectangular pulse to compensate the flux under test. In theory, we don't need to care about the waveform of the compensation process as the integrator in the flux meter gives an accurate area measurement. However, it is still very important to synchronize both rectangular pulses as the asynchronous rectangular pulses may lead to flux meter overload during the compensation process. A digital start trigger is used to synchronize the output pulse of the PJVS with the exciting current generated by the current source. The digital start trigger is also used to synchronize data acquisition of the Keysight 34420A multimeter with both rectangular pulses.

According to formula (1), the width of the induced rectangular pulse waveform can be slightly adjusted by changing the width of the exciting current ramp. Since the height of the rectangular pulse produced by the subarray with 2048 Josephson junctions is quantized, we can slightly change the height of the induced rectangular pulse by adjusting the ramp slope of the exciting current in order to make them as similar as possible.

Fig.3 PJVS based flux and mutual inductance measurement system

A hand-making winding resistance of 0.5905379 Ω soaked in oil with an uncertainty of 3.0×10^{-6} (calibrated by the resistance standard) is used as a sampling resistance, as shown in Fig 4.

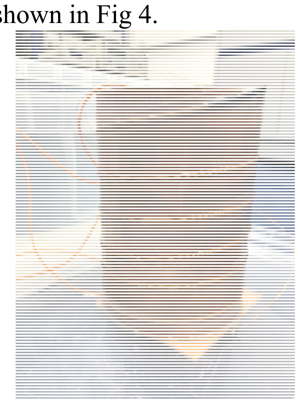

Fig. 4 Domestic sampling resistance

III. DOMESTIC BIAS CURRENT SOURCE FOR PJVS

We developed a system for precise rectangular waveform synthesis using a Josephson array. The PJVS system is based on 1 volt series array of Superconductor/ Insulator/ Normal metal/ Insulator/ Superconductor (SINIS) Josephson junctions provided by the PTB. The bias current circuits includes 16-channel bias source and comprehensive control software. Battery-powered channels are independent of each other. Each channel has a high speed switch used to control the state of output current. It is also able to synthesize pulse waves and square waves. Since the response time of the high speed switching is short. On each channel, the output drive current gives a rise-time less than 50 ns on 100 ohm non-inductive resistance. When the high speed switching turns off, the output current of this channel reduces to almost zero, which greatly reduces the risk of flux trapping due to current leakage. These characteristics are essential for extending the capabilities and features into magnetic applications through the use of the intrinsically stable and rapidly programmable quantum voltages.

As shown in Fig.5, the programmable Josephson array is split into 13 bits (subarrays), with the number of Josephson junctions in each bit increasing by a factor of 2. Each individual bit has its own connection to a channel of the bias current source. The bias source can control each bit independently of all the other bits. The bias current of each channel is provided by an individual current source with current output capability from 1.5mA to 6.5mA.

After measuring the operating margin for each subarray, the bias current is set to the margin centre for each subarray. This procedure ensures that the PJVS produces accurate quantized voltages every time it is programmed into a new state. To synthesize rectangular waves with this source, the Digital Waveform Generator (DWG) controlled by a computer generates a rapid sequence of states (“-1”, “0” and “1”) to control the bias current source. The high speed switching is able to switch the bias current in 5~8 ns after receiving the digital control signals of DWG, which ensures the fast settling time of bias current outputs (commonly less than 50 ns on 100 ohm resistant).

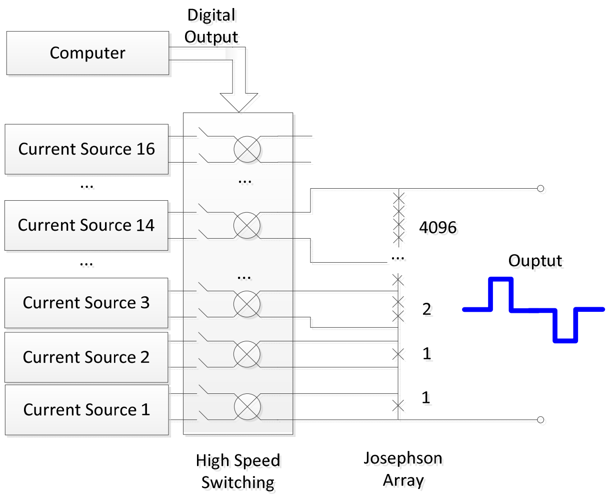

Fig. 5 Setup of PJVS bias current system

IV. FLUXMETER CALIBRATION WITH PJVS

In this measurement system, an integrating fluxmeter is used to make DC flux measurements and the measured field changes in a non-periodic way.

Integrators are used in magnetic measurements because of the physical relationship between coils of wire and magnetic flux (Φ).

As a volt-second generator, the binary Josephson system is able to produce rectangular pulse with calculable volt-second area. This kind of rectangular pulse can be considered as a precision standard of magnetic flux which is useful to calibrate integrating fluxmeters. Fig. 6 shows the setup using the PJVS to calibrate the fluxmeter. The synthesized Josephson waveform $u(t)$ is a rectangular pulse with the height of E_J (constant voltage steps) and the width of τ, giving the volt-second area $E_J \cdot \tau$ used to calibrate the fluxmeter on a given range. Output of the PJVS is directly connected to the fluxmeter. After generating the rctangular pulse, final readings of the fluxmeter is recorded and used to compare with the value $E_J \cdot \tau$. Take the Lake Shore Model 480 Fluxmeter as an example, each rectangular pulse present at its input results in a volt second reading and the relative error of indication is around several parts in 10^5.

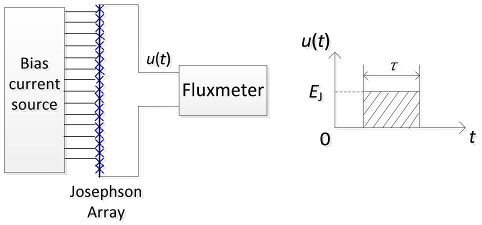

Fig. 6 Schematic diagram of fluxmeter calibration with PJVS

V. RESULTS AND DISCUSSIONS

In our flux and mutual inductance measurement system, the flux under test generated in the secondary coil of the mutual inductance is proportional to the height of the falling edge of the exciting current in the primary coil. According to the formula (5), during measuring the flux under test Φ via the fluxmeter, the height of the falling ramp of the exiting current I is also measured through a sampling resistance.

The ratio of Φ to I is the mutual inductance. Table 1 gives a group of the measurement results of the flux under test, the exiting current and the mutual inductance. The flux under test is around 1 Wb. The measured flux difference is less than 4 mWb, which is 0.4 percent of the flux under test. As mentioned in Section II, the flux under test is related to the mutual inductance and the current variation in the primary coil. Since we could accurately measure the current variation in the primary coil via a sampling resistance, the measurement result of the mutual inductance can be used to evaluate the accuracy of the flux measurement. As shown in Table 1, the standard deviation of the mutual inductance measurements is 9.4×10^{-7}.

Table 1 Flux and mutual inductance measurement results

	Flux difference Φ_δ (mV·s)	Exciting current variation I (A)	Flux under test Φ (mV·s)	Mutual inductance (mH)
1	1.214269	1.718675	986.6069	574.0508
2	1.307284	1.718521	986.5139	574.0482
3	1.408753	1.718335	986.4124	574.0512
4	1.506309	1.718164	986.3148	574.0516
5	1.623609	1.717963	986.1975	574.0504
6	1.715909	1.717807	986.1052	574.0490
Average (mH)				574.0502
Type A uncertainty (mH/mH)				9.4E-07

Fig. 7 shows the flux compensation process between the induced rectangular pulse and the rectangular pulse produced by the PJVS. It is measured by the Lakeshore 480 flux meter and recorded by the Keysight 34420A multimeter at NPLC of 2. The microwave frequency of the PJVS system is 71 GHz, so the height of the rectangular pulse generated by the subarray of 2048 Josephson junctions is 0.3006796 V and the width of it is around 3.327 s. The flux reference produced by the PJVS system is 1000.3 mWb. The falling edge height of the current ramp is around 1.73 A and the mutual inductance is 578 mH. The flux under test is around 999.9 mWb. The flux difference between the two rectangular pulses is less than 0.2 mWb, which is shown at the end of the readings in Fig. 7.

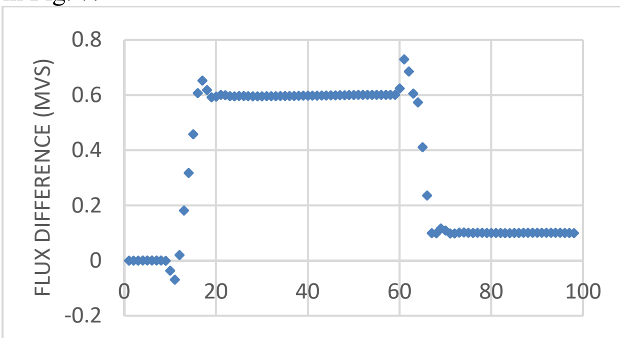

Fig. 7 Compensation of rectangular pulses measured by the flux meter

Fig. 8 gives a group of the 37 measurement results of the mutual inductance. The flux under test is around 1 Wb. The measured flux difference is less than 4 mWb, which is 0.4 percent of the flux under test. The maximum exciting current is around 1.7 A.

As mentioned in Section II, the flux under test is related to the mutual inductance and the current variation in the primary coil. Since we could accurately measure the current variation in the primary coil via a sampling resistance, the measurement result of the mutual inductance can be used to evaluate the accuracy of the flux measurement. As shown Fig. 8, the type A uncertainty of mutual inductance measurements is 1.1×10^{-6}, which is able to prove the accuracy of the flux measurement method.

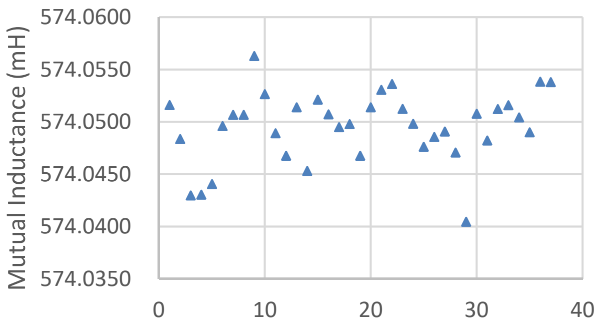

Fig. 8 Mutual Inductance Measurement results

VI. CONCLUSIONS

Experimental results show that the flux and mutual inductance measurement repeatability is several parts in 10^6 which is a further improvement of flux measurement accuracy.

VII. ACKNOWLEDGMENT

This work is supported by National Key R&D Program of China under Grant No. 2016YFF0200401 and National Natural Science Foundation of China under Grant No. 51477159 and 51307156.

REFERENCES

[1] Yuan Gao, Honghui Li and Zengmin Wang. Magnetic Flux Measurement with Binary Josephson System. CPEM 2012 Conference Digest, 2012.

[2] Wang Zengmin, Gao Yuan and Li Honghui. Establishment of New Type Josephson Voltage Standard in China - Synthesis of AC Quantum Voltage. Chinese Journal of Scientific Instrument. 31(9): 1965-1971.

[3] Wang Zengmin, Gao Yuan and Li Honghui. Research of Josephson Arbitrary Waveform Synthesizer. Acta Metrologica Sinica. To be published.

[4] Zengmin Wang, Yuan Gao, Honghui Li. [C], Synthesis of Precise AC Voltage with SINIS Josephson Junction Arrays, CPEM 2010 Conference Digest, 2010, 207-208.

[5] Curtley, M, Prince, O.A. et al., *A synchronous detector with improved parameters*, Proc. of Int. Symp. on Circuits and Systems, Barcelona, July 15-17, 2009, pp. 255-260.