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Design of Small Capacitance Standards Based on Analytical and Simulation Approaches

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***Abstract* – In the paper, analytical model and simulation are adopted to design of small capacitance standards based on fused silica. The small capacitance standards is designed with Kelvin equipotential protection structure. The stray capacitance introduced by the air gap between the capacitance eletrode and the Kelvin equipotential protection electrode(guard electrode) will affect the small capacitance value. The analytical model of the capacitor is established and the Schwarz-Christoffel transformation is used to analyze the stray capacitance. By adjusting the size of the air gap between the central electrode and the guard electrode, the electric field line collected by the guard electrode is controllable and usable to predict the final capacitance. The finite element analysis software Ansoft is used to simulate the small capacitance under different air gaps with limited ranges, and the appropriate air gap is recomended for implementing of small capacitance standards.**
Keywords – Small Capacitance Standard, Schwarz-Christoffel Transformation, Fringe Effect, Finite Element Analysis

I. INTRODUCTION

Capacitance is a basic electrical quantity, and its accurate traceability involves the requirement in energy, materials, aerospace and many other fields. With the rapid development of semiconductor technology, the need for traceability of small capacitance has grown rapidly.

To achieve high precision measurement of small capacitance, constructing of high-precision and high-stability small capacitance standards is essential. Small capacitance standards can be achieved with different structures. For example, the small capacitance can be made at chip size by MEMS fabrication process [1]. But due to the change of MEMS chip's critical dimensions, it is difficult to achieve high stability and accuracy small capacitance standards [2]. Using air dielectric to construct small capacitor which cannot be manufactured with small size, LNE made the Zickner air capacitors, which operates by interposing a screen having a small predefined aperture between its active electrodes [2]. As a good solid medium, fused silica is still the best dielectric material to make small capacitance standards with small size [3]. NIST developed the corresponding small capacitance standards based on fused silica which has high stability and high precision [4]. LNE also developed a programmable small capacitance fused silica standards [2] for multi-range calibration and traceability services.

By studying the capacitor's models, it is found that the smaller capacitance of the capacitor, the greater influence of stray capacitance caused by the fringe effect [5-6]. Kelvin equipotential protection electrodes can reduce the effects of stray capacitance. But when the capacitance is at the range of fF, or even sub-fF level, the stray capacitance can be close to or even exceed the designed capacitance. Therefore, the size of the air gap cannot be ignored for the capacitor, so it is important to determine the size of the air gap for designing small capacitance standards. The small capacitance standards of 100 fF is studied as an example in the paper, by using S-C transformation analytical and finite element simulation methods.

II. DESCRIPTION OF THE METHOD

A. *Small Capacitance Standards with Kelvin Protection Electrode*

The model of the small capacitance standards based on fused silica is shown in Fig.1. The shape of capacitance is divided into the following parts, including the electrodes, the Kelvin equipotential protection electrode (guard electrode) [6], and the metal shielding box.

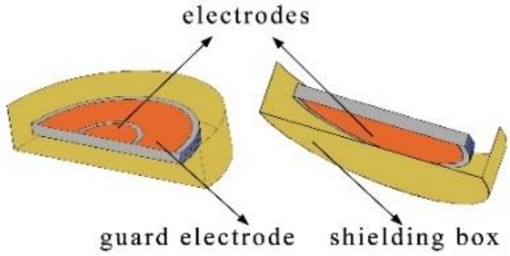

Fig.1. The Model of Small Capacitance Standards

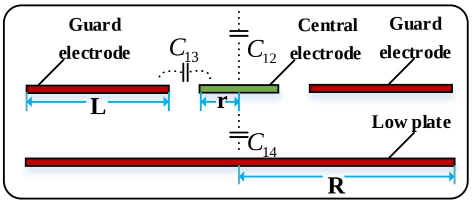

Fig.2. Simplified Diagram of the Capacitor

The Fig.2 shows the simplified schematic diagram of the capacitor. *L* represents the length of the guard electrode, *R* represents the radius of the lower plate, and *r* represents the radius of the central electrode. The capacitors include the capacitor C_{12} between the central electrode and the guard electrode, the capacitor C_{13} between the central electrode and the shielding box, and the capacitor C_{14} between the central electrode and the lower plate. In theory, the ideal standard capacitance is

$$C_{14} = \varepsilon_0 . \varepsilon_r . \frac{A}{d}, \quad (1)$$

where ε_0 is the vacuum dielectric constant, ε_r is the relative dielectric constant of the dielectric material, *A* is the area of central electrode, and *d* is the distance between the two main electrodes.

In the capacitor, the low plate, guard electrode and shielding box are at the same potential. And *L* is set to greater than four times of the air gap size, the influence of the external divergent electric field to the central electrode can be ignored [7]. Due to existing of the air gap, the electric field line originally diverging to the guard electrode will leakage to the low plate, and the capacitance C_{14} will increase a little bit, so the actual capacitance value will change to

$$C_s = C_{14} + C_e, \quad (2)$$

where C_e is the leakage capacitance introduced by the air gap fringe effects. The leakage capacitance C_e will changed with the air gap size. We can study the C_e through studying the C_{12} with changing the air gap size. In this paper, an analytical model is built for the programmable capacitor, and then the relationship between the air gap and C_{12} is obtained.

B. *Analytical Model of Small Capacitance Standards*

Fig.3 shows the analytical model of small capacitance standards, the C_{12} can be divided into three parts: C_{S1} is the capacitance between lower surface of central electrode and the side of the guard electrode, C_{S2} is the capacitance between lower surface and guard electrode, C_{S3} is the capacitance between upper surface of central electrode and the guard electrode. *h* represents the thickness of the electrodes, δ represents the size of the air gap. It is worth noting that the medium of the capacitance C_{S1} is fused silica, and the medium of C_{S2} and C_{S3} is air.

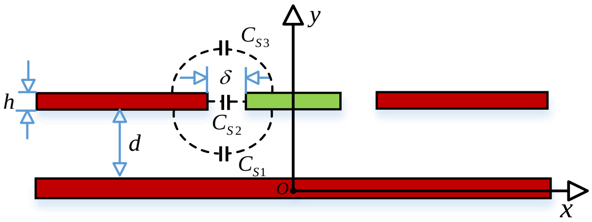

Fig. 3. Analytical model of Small Capacitance Standards

By using the Schwarz-Christoffel transformation (S-C transformation) to transform the complex model into a simple model, the capacitance of capacitor of any shape can be obtained, as the S-C transformation maps the non-circular boundary on *z=x+iy* plane to the boundary of the circle on the *w =u+iv* plane, which results in a simplified solution[8].Taking C_{S1} as an example to carry out S-C transformation, the two-dimensional model in the Z plane is transformed into a one-dimensional model in the W plane. Since the model is a symmetrical structure, the negative half-axis of the x-axis is used as a mapping object.

Fig.4 illustrates the model in the Z plane and W plane respectively. It should be noted that the S-C transformation only changes the shape of the model without changing the size of the proportion. Therefore, the physical characteristics of the capacitor have not changed, so the capacitance in the W plane is the actual capacitance in *Z* plane. *P*(0,∞) represents the ∞ point in the *Z* plane, and *Q*(0,0) represents the 0 point in the *Z* plane. Line segment x_1x_2 is guard electrode, line segment x_5x_6 is central electrode, line segment x_3x_4 is low plate. Point -b, -1, -a and a, 1, b represent the mapping point of x_1~x_6 in the W plane. Because the adopted S-C transform is a symmetric structure, so a×b=1.

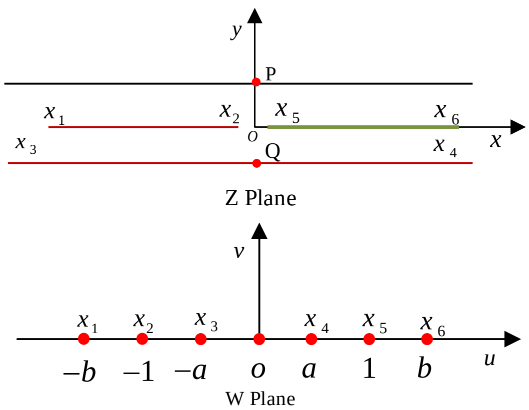

Fig. 4. Schwarz-Christoffel transformation model

Assuming that the air gap is infinitesimal, then the points of the Z plane can be mapped into the W plane according to Table 1.

Table 1. Map corresponding points.

x_i	w_i	α_i
x_1	-b	0
x_2	-1	2π
x_3	-a	0
x_4	a	0
x_5	1	2π
x_6	b	0

Using the S-C transformation formula

$$\frac{\mathrm{dt}}{\mathrm{dz}} = \prod_{i=1}^{6}(z-w_i)^{\frac{\alpha_i}{\pi}-1}. \qquad (3)$$

By substituting the data in table 1 into equation (3), the mapping rule is obtained by integrating equation (3)

$$f(z) = R\left(ln\frac{z+a}{z-a} + ln\frac{z+b}{z-b}\right) + S. \qquad (4)$$

R and S are both unknown parameters. In this case, the boundary conditions P, Q and x_5 are substituted into the formula (4), then the formula (4) can be rewritten as

$$f(z) = \frac{d}{z}\left[\ln\left(\frac{z^2+\left(tanh\frac{\pi\delta}{8d}+cotn\frac{\pi\delta}{8d}\right)z+1}{z^2-\left(tanh\frac{\pi\delta}{8d}+cotn\frac{\pi\delta}{8d}\right)z+1}\right)\right] + dj. \qquad (5)$$

In the further calculation of capacitance, the electric field intensity is first calculated according to Laplace equation, then the electric charge is calculated according to Gauss's Law, and finally the specific capacitance value is obtained according to $C=Q/U$ [9].

In order to ensure that all electronic lines of the central electrode can be included, x_3 and x_1 should be respectively set to the point ∞ and the point 0. So all points in Fig. 4 need to plus a, then be taken reciprocal, finally subtract $1/(a-b)$. After the above mathematical transformation, the place order of x_i also changes. W Plane of Fig.4 can be changed to the new one as shown in Fig.5.

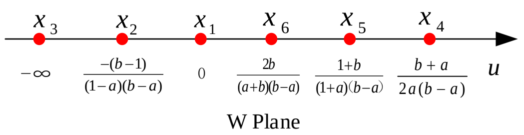

Fig. 5. W plane model after changing the coordinate system

In the cylindrical coordinate system, the formula (6) is used for calculating the electric field intensity.

$$E = -\frac{V_0}{\alpha\cdot\rho}a_{\emptyset}. \qquad (6)$$

α represents the angle between the plates, ρ represents the air gap, V_0 represents the potential of central electrode, a_{Φ} represents the radial amount.

By using Gauss's law can get Q

$$Q = -\frac{\varepsilon_0 V_0}{\pi}\int_{x_6}^{x_4}\frac{1}{\rho}d\rho. \qquad (7)$$

Then

$$C = \frac{Q}{V_0} = \frac{\varepsilon_0}{\pi}\left(ln\frac{a+b}{2a} - ln\frac{2b}{a+b}\right). \qquad (8)$$

Because the medium of C_{S1} is fused silica so the final result is

$$C_{S1} = \varepsilon_0\varepsilon_r\left(r+\frac{\delta}{2}\right)\ln\left(\frac{\tanh\left(\frac{\pi\delta}{8d}\right)^2+1}{2tanh\frac{\pi\delta}{8d}}\right). \qquad (9)$$

And the C_{S2} is similar to the C_{S1}, but the spacing and the medium are changed, so the result is

$$C_{S2} = \varepsilon_0\left(r+\frac{\delta}{2}\right)\ln\left(\frac{\tanh\left(\frac{\pi\delta}{8d}\right)^2+1}{2tanh\frac{\pi\delta}{8d}}\right). \qquad (10)$$

Similarly, the C_{S3} can also be obtained by S-C transformation, so the formula of C_{S3} can be easily obtained as follows

$$C_{S3} = \frac{\pi\varepsilon_0 h}{\ln(r+\delta)-\ln(r)}. \qquad (11)$$

Then, the overall capacitance

$$C_{12} = C_{S1} + C_{S2} + C_{S3}. \qquad (12)$$

It can be seen from the formula that the air gap size, the thickness of the plate and the distance of the central and low plates all have influence on the stray capacitance C_{12}. However, the thickness and the distance of plates can be predetermined by the calculation and the requirements of the manufacturing process of the capacitors. Therefore, the size of the air gap which is major parameter of the stray capacitance should be analysed and determined.

In order to verify the relationship between the air gap size and the stray capacitance, the finite element analysis software Ansoft is used to establish a 3D model to compare simulated values with theoretical values. For the theoretical and simulated values are not identical at a certain degree, the relative variation of the capacitance values is more accurate and used for simulated and theoretical values comparison.

III. RESULTS AND DISCUSSIONS

As a finite element analysis software, Ansoft can make 3D models and obtain accurate capacitance under electrostatic field conditions. The theoretical and emulational parameter are set to d=0.4 mm, r=0.56 mm, ΔR_{0}=10-(r+δ) mm, R=10 mm, h=0.025 mm, V_0=10 V.

The air gap is changed at the range of 0.01~0.2 mm with 20 points, and then the column vector can be got

$\{C_i\}^T$.

$$\{C_i\}^T = \{C_1, C_2, \ldots, C_i\}^T \; i=1, 2, \ldots, 20 \qquad (13)$$

the variation of capacitance column vector $\{\triangle C\}^T$ is obtained as follows.

$$\{\triangle C\}^T = \{C_2 - C_1, \ldots, C_i - C_{i-1}\}^T \qquad (14)$$

In combination with (13) and (14), the simulation and theoretical numerical trend of $\{\triangle C_{12}\}^T$ can be given. The results as shown in Fig.6, the simulation trend is consistent with the theoretical variation trend of $\{\triangle C_{12}\}^T$ .In the range of 0.1 mm~0.2 mm is below 0.2 fF. Therefore, the air gap range of 0.1 mm~0.2 mm is in the allowable range of the error.

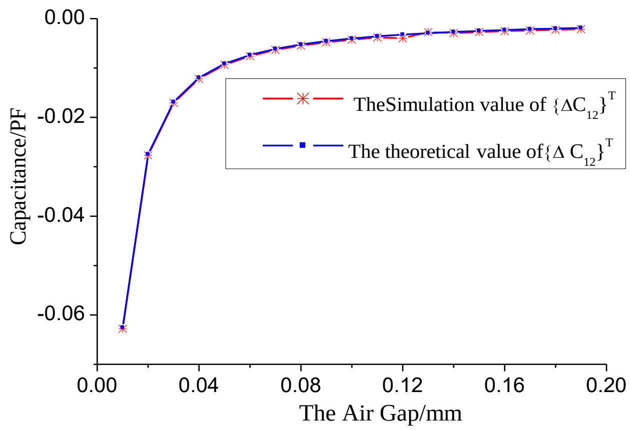

Fig. 6. The variation of $\{\triangle C_{12}\}^T$ in simulation and theoretical

In order to study the relationship between the variation trend of C_{12} and C_e in the range from 0.1 mm to 0.2 mm, the curves was made based on the analytical and simulation results, as shown in Fig.7.

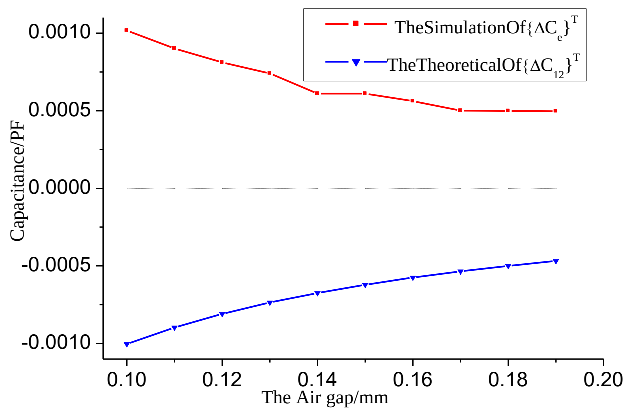

Fig.7. .The variation of $\{\triangle C_e\}^T$and $\{\triangle C_{12}\}^T$

It can be seen from the Fig.7 that reducing the C_{12} will make the C_e to increase, and the variation range of the two capacitances is within 1 fF in the air gap range from 0.1 mm to 0.2 mm. The variation tendency of $\triangle C_e$ is inversely proportional to the $\triangle C_{12}$. Based on the above results, when the air gap δ is recommended as 0.14 mm, for every 0.01 mm change in air gap which is determined by manufacturing process, the change of C_e is within 0.0027 fF, and the final capacitance can be predicted with accuracy better than 0.01% for 100 fF small capacitor. Based on the recommended air gap size and other parameters, the final simulated capacitance is 100.0016 fF.

When making standard capacitors of the fF to sub-fF level, the recommended size of the air gap can improve the accuracy and consistency of the small capacitor standards.

IV. CONCLUSIONS

A reasonable air gap can reduce the errors introduced by manufacturing process .The design method proposed in the paper is also very suitable for design of programmable capacitor to achieve high linearity changing of small capacitance units on a same fused silica plate.

V. ACKNOWLEDGMENT

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