

The Dynamic Error Analysis of the Current Transformer

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Abstract – In order to solve the problem of the dynamic error analysis of the current transformer, firstly, the mathematical model of the current transformer is established. Secondly, based on the working principle of the current transformer, the causes of the dynamic error of the current transformer are studied. On this basis, the important parameters affecting the dynamic error of the current transformer are analyzed theoretically. Thirdly, different dynamic test signals under complex working conditions are selected. After that, a composite error analysis algorithm is proposed to give the dynamic error quantitatively. Finally, the influence of the above parameters on the dynamic error is given, and the relationship between these parameters and the dynamic error is discussed. The results show that, according to the composite error analysis algorithm adopted in this paper, the dynamic error of different dynamic test signals under complex working conditions can be calculated effectively and the specific influence of each parameter on the dynamic error can be obtained.

Keywords –Dynamic Test Signals, Current Transformer, Dynamic Error Analysis

I. INTRODUCTION

With the rapid development of the smart power grid, the working state of the high-power load changes rapidly, and its load current presents a fast and unsteady dynamic change. The current transformer (CT) provides a current collecting signal for the power energy data processing and measurement of the electricity meter, so it is urgent to analyze the influence of CT's transmission characteristics on electric energy metering. Error is an important index to measure its transmission characteristics, the error of CT is mainly divided into the steady error and the dynamic error. At present, the steady error has met the requirements of the electric energy metering. However, under complex working conditions, CT does not always operate in the steady situation, so its dynamic error also needs to be analyzed.

For many years, the error analysis of CT has used a

method which establishes a mathematical model of its basic excitation curve, using various types of functions to approximate the magnetization curve of CT. For example, the rational fraction has been applied to fit the magnetization curve to establish a CT model^[1], the exponential series has been used to fit the magnetization curve of the CT's core^[2], but the accuracy of these methods is not high. In 1984, D.C. Jiles and D.L. Atherton established a mathematical model of CT based on Langevin function in the form of differential equations according to the hysteresis characteristics of ferromagnetic materials^[3-4]. The Jiles–Atherton (J-A) Hysteresis Model has been widely used by scholars because of its clear physical concept and its mathematical model is easy to solve.

In the aspect of the dynamic error analysis of CT based on dynamic test signal models, the influence of the ratio and angle error of CT on the electric energy metering has been analyzed in [5], but only from the aspect of the steady error instead of the dynamic error and no specific dynamic test signal model is given. In summary, the lack of researches on the dynamic error is mainly reflected in the failure that the mathematical model is not established appropriately and the inability to accurately quantify the dynamic error of CT based on the actual dynamic signal model, as well as the inadequate analysis of the parameters that may affect the dynamic error of CT.

In view of the accuracy of the above mathematical model and the lack of dynamic test signal models, in this paper, the J-A model is adopted as the transmission function model firstly. According to the working principle of CT, a series of parameters affecting the dynamic error are determined. Secondly, for common dynamic test signals under complex working conditions, PSCAD and MATLAB are used to establish the composite error analysis algorithm. Meanwhile, the dynamic error of CT under complex working conditions is given quantitatively for the J-A model. Finally, the influence of the parameters on the dynamic error and their relationship with the dynamic error is given by changing the above parameters.

II. THE MODEL OF CT'S CORE

The J-A model is based on the analysis and description of the physical properties of magnetic materials. Langevin function is used to describe the magnetization characteristics of ferromagnetic materials, and then the mathematical model of CT is established. The J-A model represents the hysteresis loop by establishing the relationship between the magnetic flux density B , the magnetic field intensity H and the magnetization M

$$B = \mu_0(H + M) \quad (1)$$

The effective magnetic field intensity H_e is defined as

$$H_e = H + \alpha M \quad (2)$$

Where H is the magnetic field intensity of the external magnetic field. α is the average field parameter, which is used to characterize the coupling between magnetic domains. In this case, the hysteresis effect of ferromagnetic materials is neglected, and the relationship between the effective magnetic field intensity H_e and the anhysteretic magnetization M_{an} is expressed by Langevin function as follows

$$M_{an} = M_s \left[\coth\left(\frac{H_e}{a}\right) - \frac{a}{H_e} \right] \quad (3)$$

In order to represent the hysteresis loss and hysteresis characteristics of practical ferromagnetic materials to make the physical meaning of the magnetization M more clear, the magnetization M in the J-A hysteresis model can be divided into the irreversible component M_{irr} and the reversible component M_{rev} . Among them, M_{irr} will consume energy

$$M = M_{irr} + M_{rev} \quad (4)$$

Where the irreversible and reversible components of ferromagnetic materials have a certain proportional relationship, which can be expressed by the reversible magnetization parameter C

$$M_{rev} = c(M_{an} - M_{irr}) \quad (5)$$

It can be deduced from (4) and (5)

$$\frac{dM}{dH} = \frac{c \frac{dM_{an}}{dH_e} - \frac{M_{an} - M}{k\delta} - \frac{M_{an} - M}{\mu_0} \frac{1-c}{1-c}}{1 - \alpha c \frac{dM_{an}}{dH_e}} \quad (6)$$

Where K is the loss coefficient, δ is the directional coefficient. When $\frac{dH}{dt} < 0$, $\delta = -1$ and when $\frac{dH}{dt} > 0$, $\delta = 1$.

μ_0 is the vacuum permeability, the J-A mathematical model in the original form of differential equations can be obtained by combining (3) and (6).

III. THE PARAMETERS AFFECTING THE DYNAMIC ERROR OF CT

Based on the J-A mathematical model, it can be seen that the core excitation needs to consume energy and

generates excitation current, so combining the working principle of CT, the equivalent circuit of CT as shown in Fig. 1 is drawn [6]

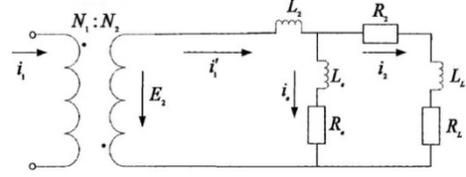


Fig. 1. The equivalent circuit of CT

Where I_1' is the secondary current, I_e is the excitation current, R_e is the excitation resistance, L_e is the secondary side reactance, R_2 is the resistance of the secondary winding, L_2 is the inductance of the secondary winding, R_L is the resistance of the secondary load, and L_L is the inductance of the secondary load, N_1 and N_2 are the numbers of turns of the primary and secondary sides respectively.

In CT, the relationship between the synthetic flux and the secondary induced electromotive force is as shown in the following equation [6]

$$\phi_e = \frac{\sqrt{2}E_2}{2\pi f N_2} \quad (7)$$

Where E_2 is as follows

$$E_2 = I_2 \sqrt{(R_2 + R_L)^2 + (\omega L_2 + \omega L_L)^2} = I_2 \sqrt{(R_2 + R_L)^2 + (2\pi f L_2 + 2\pi f L_L)^2} \quad (8)$$

According to the magnetic circuit law, the magnetic motive force balance equation of CT and equation (7), the following equation can be obtained

$$I_1 = \left(\frac{Z_2 L}{2\pi f \mu S N_1 N_2} - \frac{N_2}{N_1} \right) I_2 \quad (9)$$

Where S is the mean cross-sectional area of CT's core, μ is the magnetic permeability of CT's core material, L is the average path length of the magnetic circuit of CT's core. It can be seen that the mean cross-sectional area, the average path length of the magnetic circuit and the number of turns of the primary and secondary sides all have effects on CT's transmission performance.

IV. THE DYNAMIC ERROR ANALYSIS OF CT BASED ON DYNAMIC TEST SIGNALS

The research in this paper is mainly based on two kinds of common dynamic test signals under the actual complex working conditions: On-Off Key (OOK) dynamic test signal and M-sequence dynamic test signal. The model used in the simulation experiments is based on the circuit model described in the section III, which combines the mathematical model of CT's core described in the section II with the working principle of CT.

A. Analysis of The Dynamic Error of CT Based on OOK Dynamic Test Signal

In this paper, the waveform of the secondary current is simulated by inputting the frequency of the sine wave generator and the frequency of the square wave generator based on the J-A current transformer model in PSCAD. The frequency of the sine wave generator is 50 Hz, the frequency of the square wave generator is 5 Hz, the on-state of the primary input current is 300 A, the off-state of the primary input current is 0 A, and the corresponding secondary current is 3A. Where the default parameters for the J-A model are: the primary turns is 4, the secondary turns is 400, the secondary resistance is 0.5Ω , the secondary inductance is $0.8\times 10^{-3}H$, the mean cross-sectional area is $2.6\times 10^{-3}m^2$, the average path length of the magnetic circuit is $0.6377m$ and the remnant flux density is 0T. After that, the primary current and the converted secondary current are imported into MATLAB. The process of conversion means that the secondary current is multiplied by the transformation ratio of CT. The primary and secondary current waveforms can be obtained by simulation as shown in Fig. 2 that the two curves coincide basically.

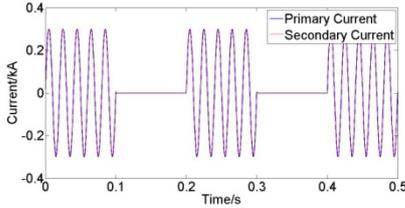


Fig. 2. The Primary and Secondary Current Waveforms of OOK Dynamic Test Signal

In order to reflect the transmission performance of CT under dynamic test signals better and quantify the dynamic error of the primary and secondary currents, the composite error is given to evaluate the dynamic error of CT in this paper.

The composite error of CT (Δ) is defined as

$$f(t) = K_1 I_2(t) - I_1(t)$$

$$\Delta = \frac{1}{I_p} \sqrt{\frac{1}{T} \int_0^T f(t)^2 dt} \times 100\% \quad (10)$$

Where $I_1(t)$ is the primary current, $I_2(t)$ is the secondary current, K_1 is the transformation ratio, I_p is the root-mean-square of the primary current, T is a cycle. Similarly, when the on-state of the primary input current is 100 A, the off-state of the primary input current is 0 A, and the corresponding secondary current is 1A, the composite errors of the above two cases are shown in Table 1.

B. Analysis of The Dynamic Error of CT Based on M-sequence Dynamic Test Signal

This paper uses a 5th-order M-sequence whose primitive polynomial is

$$f(x) = x^5 + x^2 + 1 \quad (11)$$

The initial state of the register is set as [0 0 0 0 1]. Based on the principle of M-sequence, the 5th-order M-sequence dynamic test signal is generated by using MATLAB then imported into PSCAD. The frequency of the sine wave generator is 100 Hz, the on-state of the primary input current is 300 A, the off-state of the primary input current is 0 A, and the corresponding secondary current is 3A. Where the default parameters for the J-A model are the same as those in the previous part. The primary and secondary current waveforms of the J-A current transformer model are shown in Fig. 3 that the two curves coincide basically.

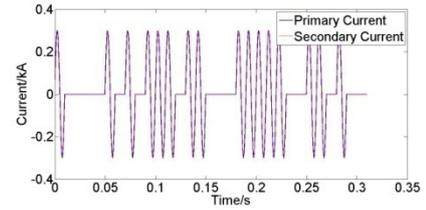


Fig. 3. The Primary and Secondary Current Waveforms of M-Sequence Dynamic Test Signal

Similarly, the composite error Δ (10) proposed in the previous section is used to quantify the dynamic error. Based on the above situation, when the on-state of the primary input current is 100 A, the off-state of the primary input current is 0 A, and the corresponding secondary current is 1A, the composite errors of the above two cases are shown in Table 1.

C. Analysis of the Influence of the Parameters on the Dynamic Error

In order to study the relationship between the parameters which are mentioned in the section III and the dynamic error, the default parameters of CT in simulation are changed in PSCAD according to the practical parameters of CT. For both OOK and M-sequence dynamic test signals, each parameter in case (1)-(3) is changed independently on the basis of the default parameter, the other parameters remain unchanged at the same time. In case (4), the three parameters are changed simultaneously on the basis of the default parameters.

Case (1): The mean cross-sectional area increases from $S = 2.6\times 10^{-3}m^2$ to $S' = 3.6\times 10^{-3}m^2$

Case (2): The average path length of the magnetic circuit decreases from $L = 0.6377m$ to $L' = 0.5024m$

Case (3): The number of turns of the primary side is changed to 8, the number of turns of the secondary side is changed to 800, while the transformation ratio remain unchanged.

Case (4): The mean cross-sectional area increases from $S = 2.6\times 10^{-3}m^2$ to $S' = 3.6\times 10^{-3}m^2$, the average path length of the magnetic circuit decreases from $L = 0.6377m$ to $L' = 0.5024m$, the number of turns of the primary side is changed to 8, the number of turns of the secondary side is changed to 800, while the transformation ratio is

maintained. The composite errors calculated according to the above four cases for OOK and M-sequence dynamic test signals are shown in Table 2.

V. RESULTS AND DISCUSSIONS

In Table 1, when the dynamic test signal is OOK, the composite error is large when the steady current is large, when the dynamic test signal is M-sequence, the composite error is large when the steady current is small. Meanwhile, the composite error of M-sequence dynamic test signal is larger than OOK dynamic test signal. In Table 2, for both OOK and M-sequence dynamic test signals, the dynamic errors are affected by the changes of the three parameters individually, of which the turns ratio has the most conspicuous effect. When multi-parameters change simultaneously, the composite error is 0.004% for OOK dynamic test signal and 0.360% for M-sequence dynamic test signal. The composite errors of M-sequence dynamic test signal are larger in all cases. The reason is that M-sequence dynamic test signal changes more frequently than OOK dynamic test signal in one cycle, so the composite error of M-sequence dynamic test signal is larger than that of OOK dynamic test signal, which will

Table2. The composite error (%) for different dynamic test signals when the parameters are changed

The parameters	The default parameters	Changing a single parameter	Changing a single parameter	Changing a single parameter	Changing Multi-parameters
		$S = 2.6 \times 10^{-3} m^2$ $L = 0.6377m$ Turns ratio 4:400	$S' = 3.6 \times 10^{-3} m^2$	$L' = 0.5024m$	Turns ratio 8:800
The composite error for OOK (%)	0.031	0.021	0.024	0.007	0.004
The composite error for M-sequence (%)	2.050	1.510	1.630	0.600	0.360

VI. CONCLUSIONS

In this paper, the mathematical model of the current transformer is established. At the same time, according to the working principle of the current transformer, the parameters of the current transformer which affect the dynamic error are obtained. Then a composite error analysis algorithm is given to quantify the dynamic error based on different dynamic test signals. Furthermore, the influence of the above parameters on the dynamic error is given and analyzed, and the relationship between the parameters and the dynamic error is studied by changing these parameters. The research results show that this paper provides an effective method for calculating the dynamic error of the current transformer based on different dynamic test signals under complex working conditions.

VII. ACKNOWLEDGMENT

have a prominent impact on electric energy metering.

The above simulation results show that the composite error of CT is proportional to the average path length of the magnetic circuit of CT's core and inversely proportional to the mean cross-sectional area of CT's core. Besides, the composite error is also inversely proportional to the turns ratio when the transformation ratio is kept constant. Meanwhile, the influence of various parameters on the composite error is different for diverse dynamic test signals.

Table1. The composite error (%) of CT based on OOK dynamic test signal and M-sequence dynamic test signal

The parameters of the power	on-state of the primary input current is 300 A	on-state of the primary input current is 100 A
The composite error for OOK (%)	0.031	0.027
The composite error for M-sequence (%)	2.050	2.300

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