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A new CS method for ECG signal

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***Abstract* – Compressed sensing (CS) is a modern method reducing amount of transferred and stored data which have been applied for many types of signals fulfilling sparsity requirements. Electrocardiogram signals (ECG) is one of such signals. In this paper a completely patient-agnostic compressed sensing and reconstruction technique for ECG signals is proposed. A modified sensing method incorporating a QRS detector is used to guarantee the exact R wave positions in the reconstructed signal for any level of compression. For the signal reconstruction a novel method using a dynamic ECG signal model is described, in which the model parameters are found using the Differential Evolution optimization algorithm. Reconstruction quality is evaluated using the MIT-BIH arrhythmia database, and compared against wavelet dictionary reconstruction methods showing better reconstruction quality for compression ratios above 5.**

I. INTRODUCTION

Compressed Sensing (CS) is a compression technique which exploits the signal property called sparsity to represent the signal using much less samples than needed according to Nyquist-Shannon sampling theorem. It has recently emerged as a low-power alternative to the traditional lossy compression techniques [1]. Various methods for application of CS directly to analog signals were proposed [2]-[5] called the Analog to Information Converters (AIC).

Many CS and reconstruction methods were proposed also for ECG signals such as [6] - [9] where excellent reconstruction quality has been achieved. However, all of the mentioned methods use a trained reconstruction dictionary, where a large database of training records or even a patient-specific database is required. Patient agnostic methods which do not require a trained dictionary exploit a suitable wavelet or a spline basis for the signal reconstruction [10], [11]. Here the biggest disadvantage is that the reconstruction quality gets significantly worse for high compression ratios.

The main novelty of our proposed method described in this paper is application of signal reconstruction using a dynamic ECG signal model and our QRS detector to guarantee the exact R wave positions in the reconstructed signal for any level of compression.

The paper is organized as follow: after short Introduction and summarizing ideas of CS, the proposed compression and reconstruction methods are presented. The proposed methods were evaluated on real ECG signals and compared with Mexican hat method in the following chapter. Finally, the short conclusions are given at the end of paper.

II. ECG COMPRESSED SENSING BACKGROUND

The basic idea of CS comes from the fact that some signals can be well represented using linear combination of suitable basis functions. If only a small number of basis functions is needed, the signal can be called a sparse signal.

Consider $\mathbf{x}$ a N x 1 size vector of input signal, which we can write as:

$$\mathbf{x} = \mathbf{\Psi}\boldsymbol{\alpha}\ , \tag{1}$$

where $\mathbf{\Psi}$ is the N x L size basis matrix, containing L columns of basis functions ψ_l, l=1,2,...,L. The $\boldsymbol{\alpha}$ is a L x *1* vector of expansion coefficients. The sparsity of signal $\mathbf{x}$ is defined with the number of basis functions needed for its representation. If there are only s significant nonzero coefficients in the $\boldsymbol{\alpha}$, signal $\mathbf{x}$ is called the s-sparse signal. Sparsity can be fulfilled for any suitable arbitrary domain in which the signal of interest can be represented this way [2].

In CS, the sparse signal $\mathbf{x}$ is correlated with the measurement signal $\mathbf{y}$, which is represented in M rows of the M x N size sensing matrix $\mathbf{\Phi}$:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} = \mathbf{\Phi}\mathbf{\Psi}\boldsymbol{\alpha} = \mathbf{A}\boldsymbol{\alpha}, \tag{2}$$

The sensing matrix $\mathbf{\Phi}$ is incoherent with the basis matrix $\mathbf{\Psi}$ (so that the rows φ_m, m = 0, 1, 2, …, M of matrix $\mathbf{\Phi}$ do not sparsely represent the columns ψ_l of matrix $\mathbf{\Psi}$). Considering $s<M<<N$ this operation results in a measurement vector $\mathbf{y}$ of size M x *1* which contains enough information to fully reconstruct the signal $\mathbf{x}$ [12]. If the matrices Φ and Ψ forming a reconstruction matrix $\mathbf{A}$ are known, the vector α remains the only unknown needed for reconstructing the signal $\mathbf{x}$. However, the $\mathbf{A}$ is not a square matrix, thus it cannot be simply inverted, and an underdetermined system of M equations and L unknowns has to be solved. Among all possible solutions, the right solution is the sparsest one, while sparsity s of vector α is defined as number of its nonzero coefficients referred to as the ℓ_0 pseudo-norm denoted as $\|\alpha\|_0$ [13]. Therefore, the

correct solution can be found by optimization process, where the $\|\alpha\|_0$ is minimized.

In practice, the ℓ_0 norm is highly unstable and finding the correct solution can be approached only using brute force methods. The noise contained in any real signal causes that α consists of few coefficients with large values while others are small, but non-zero. Because of that, the [14] suggested to minimize the ℓ_1 norm instead defined as:

$$\ell_1: \|\alpha\|_1 = \sum_{n=1}^{N} |\alpha_n|, \tag{3}$$

which guarantees to find a correct optimal solution if it exists. This is the most common method for CS signal reconstruction, because allows to use more efficient optimization algorithms such as the Orthogonal Matching Pursuit [15], Compressive Sampling Matching Pursuit [16] or Bayesian Learning [17].

For the purpose of application of CS the determination of suitable matrices **Ψ** and **Φ** is of great importance. Most of ECG signals are sparse in wavelet domain and various methods of using wavelet basis functions as the Ψ matrix were proposed [10], [11], [18]. Best results were obtained creating the **Ψ** matrix using dictionary training methods [7] or adaptive dictionaries [12]. The matrix **Φ** contains random numbers; usually a Bernoulli distributed pseudorandom sequence of ±1's [19].

III. THE PROPOSED COMPRESSION METHOD

The input signal is acquired using a conventional ADC (Fig.1) at the sampling frequency of f_s.

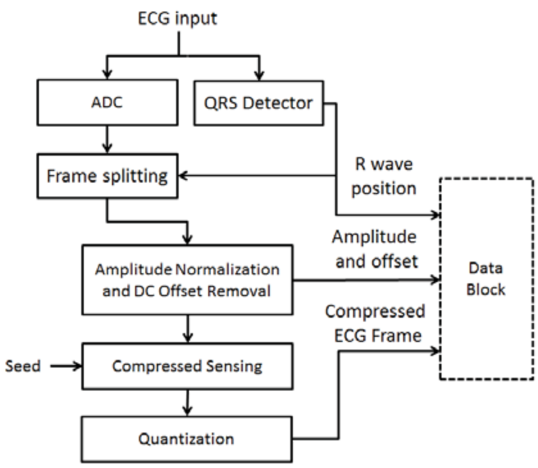

Fig. 1. Block diagram of the proposed CS method.

First of all, the ECG signal is split into frames with variable size. A QRS detector returns the exact positions of R waves n_{Ri} and the frames are cut fixedly at 2/3 of the subsequent heartbeat's R-R distance as it can be seen in Fig.2.

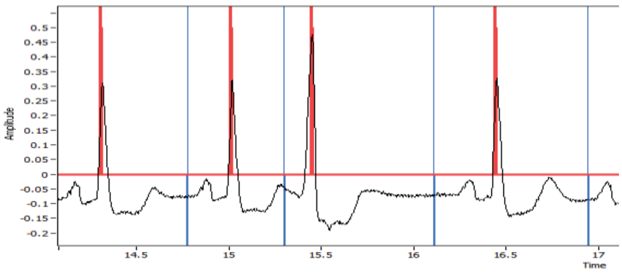

Fig. 2. Example of splitting the input ECG signal into frames with variable size.

Here a suitable QRS detection algorithm or a low power analog detection circuit such as [20] can be used. Even simple detection algorithms which pose high sensitivity at the cost of false-positive detections are suitable. Ideally each frame should contain one heartbeat at maximum. A high sensitivity QRS detection algorithm proposed in [21] and [22] was used for simulation and testing purposes in this work. Using such an approach of frame splitting has an advantage of knowing the R wave positions within the frame prior to the reconstruction. The precision of R wave position in the reconstructed signal then depends only on the QRS detection method used.

Subsequently the DC offset is removed by subtracting the mean value within each frame and the original offset is stored in the O_i. Amplitude is normalized to 1, dividing each sample by the maximum within the frame and the original value is stored in the A_i. Then the CS is applied by means of multiplying the frame $\mathbf{x}_i$ by sensing matrix $\mathbf{\Phi}_i$:

$$y_i = \langle \Phi_i x_i \rangle, \tag{4}$$

where $I = 0,1,\ldots,I$ is the frame number and $\langle\ \rangle$ denotes the linear quantization. The rows of the matrix $\mathbf{\Phi}_i$ contain a Bernoulli distributed pseudorandom sequence of ±1's generated using always the same seed. Size of the matrix is $M_i \times N_i$ where the N_i is equal to the size of respective input frame $\mathbf{x}_i$ and M_i is given by the desired constant decimation factor D as:

$$M_i = \lfloor N_i / D \rfloor, \tag{4}$$

The output measurement vector $\mathbf{y}_i$ is linearly quantized to increase the compression gain and along with the A_i, O_i, N_i and n_{Ri} a single complex block of compressed data.

IV. THE PROPOSED RECONSTRUCTION METHOD

The reconstruction method slightly differs from the abovementioned theory and commonly used methods. Rather than using a dictionary it relays on the dynamic ECG signal model, which is suitable for generation of synthetic ECG. This is the main novelty of our proposed CS method for ECG signal. The principle of proposed reconstruction algorithm is shown in Fig. 3.

We used the dynamic ECG signal model, which was proposed in [21] and [22]. It relies on three coupled differential equations which produce a trajectory in 3D Cartesian coordinates. The model is well capable to capture the morphological features of typical ECG signals, and it can be used for removing the inherent noise [23].

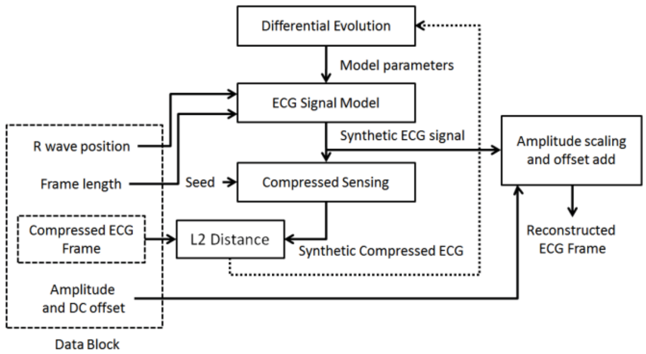

Fig. 3. Block diagram of the proposed reconstruction method.

PQRST waves are captured by a series of five exponential functions, each corresponding to respective wave and specified by the parameters of amplitude a_{wi}, width b_{wi} and position n_{wi} within a single cardiac cycle. We modified the model by adding a small amount of Gaussian noise to simulate the typical noise present in real signals, making the synthetic signal more realistic. From the CS point of view, it is obvious that using such a model the heartbeat is always represented by a sum of just 5 exponential basis functions which differ in their parameters. For the reconstruction of CS signal, it would be necessary to find only the optimal parameters' values.

Differential evolution (DE) optimization algorithm as a one from optimization methods has been used in the proposed reconstruction method. In order to use the DE algorithm first it is necessary to set the bounds within which the model parameters should be found. Because the duration of each heartbeat may vary, the optimal bounds of width and position parameters for PQRST would change according to the actual frame size. Due to parameter normalization, the DE algorithm can use invariably the same bound settings for input frame of any size.

V. EXPERIMENTAL RESULTS

The proposed method was simulated in LabVIEW programming environment, employing the MIT-BIH arrhythmia database [24] as a set of test signals. The database consists ECG records sampled at 360Hz with 11bit resolution. To assess the performance let us first define the average compression ratio (CR) as the reduction in the number of bits needed to describe the original I frames of heartbeats:

$$CR = \frac{1}{I}\sum_{i=0}^{I} \frac{N_i . B_0}{B_H + M_i . B_C} \qquad (1)$$

where B_0 is the bit resolution of the original input signal $\mathbf{x}$, B_C is the bit resolution of the quantized measurement vector $\mathbf{y}_i$ with the size of M_i, and B_H is the number of bits needed for a frame header, which carries the values of A_i, O_i, N_i and n_{Ri}. Considering the test database being used, the B_0 = 11bit. The frame header of B_H = 40bit includes 11 bits for A_i and O_i, 9 bits for N_i and n_{Ri} respectively.

For the evaluation of reconstruction quality the average percentage root mean squared difference (APRD) has been calculated as:

$$APRD = \frac{1}{I}\sum_{i=0}^{I} \frac{\|x_i - \hat{x}_i\|_2}{\|x_i\|_2} \times 100\% \qquad (1)$$

To compare the results with other commonly used patient-agnostic methods which do not require a trained dictionary, the comparison against reconstruction using wavelet basis functions and orthogonal matching pursuit (OMP) [13] was done.

All the tests of the proposed method were performed using first 10 heartbeats of channel one MIT-BIH records (I = 10). DE algorithm uses uniform crossover method, population size of 500 and maximum number of 200 iterations. The compression ratio can be varied by decimation factor D (resulting in the change of M_i) as well as by altering the bit resolution of the measurement vector B_C.

Effect of the decimation factor on the reconstruction quality is compared in Fig.4. The dependence of reconstruction quality of proposed method on the measurement vector resolution B_C and various decimation factors D can be seen in Fig.5. The resulting APRD in figures is provided as an average of 48 MIT-BIH records.

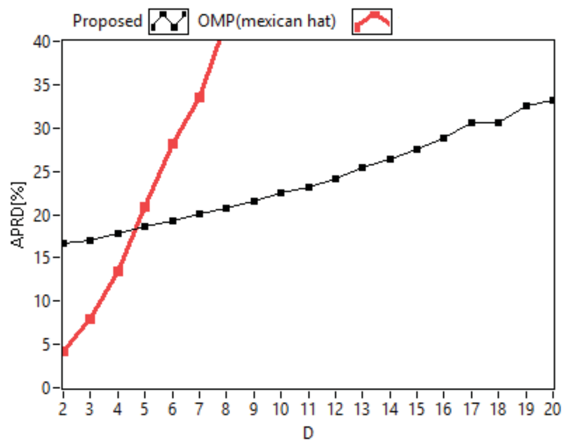

Fig. 4. Effect of decimation factor D on the reconstruction quality.

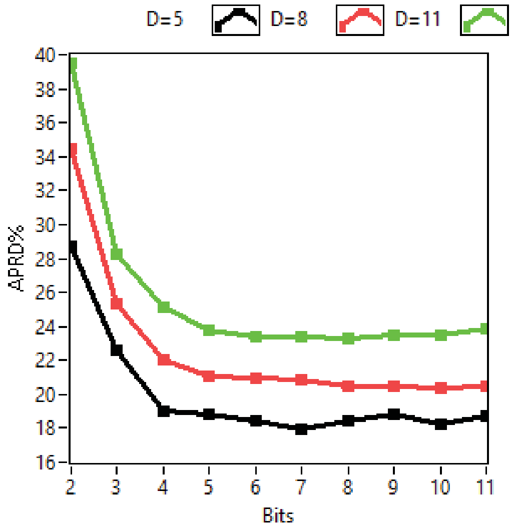

Fig. 5. Effect of measurement vector bit quantization B_C on the reconstruction quality at $D = 5$, $D = 8$ and $D = 11$

In the Fig.4 and Fig.5 it can be seen that the APRD depends almost linearly on chosen decimation factor *D*. In comparison with wavelet dictionary methods, the proposed method does not lead to better reconstruction results for small *D*, however it does not get significantly worse for higher compression ratios (*D*>5) as in case of the wavelet dictionaries. The APRD values start to significantly grow after the measurement vector $\mathbf{y}_i$ resolution is lower than 5 bits.

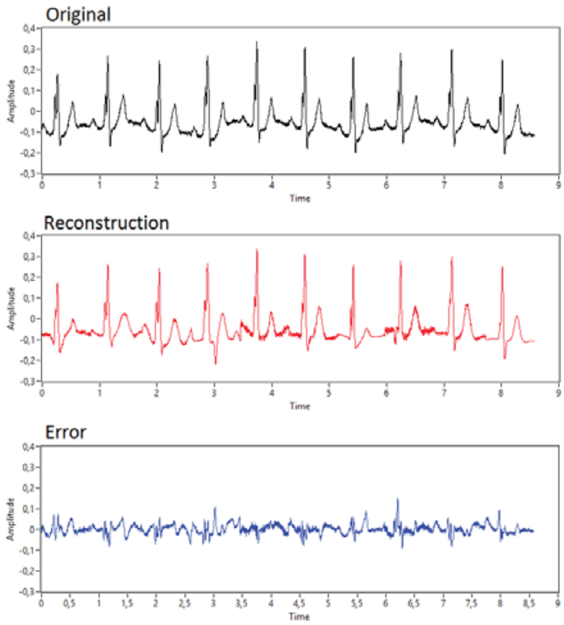

Fig. 5. Reconstruction of MIT-BIH №111 at CR = 9.81 corresponding to APRD = 27.52%.

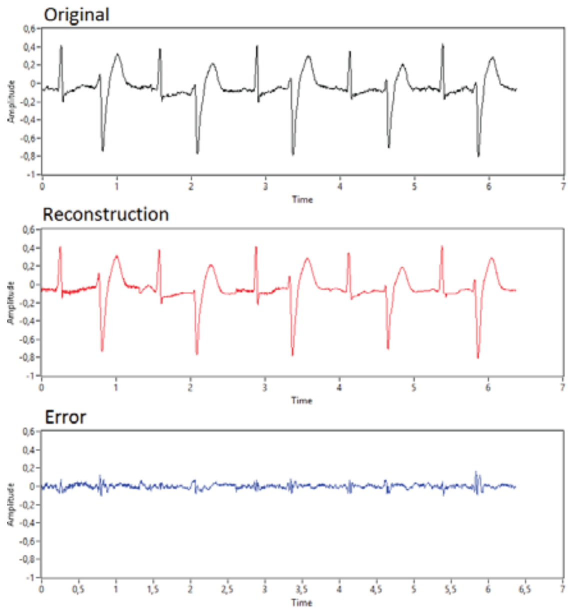

Fig. 6. Reconstruction of MIT-BIH №200 at CR =9.43 corresponding to APRD = 13.94%

Some examples of reconstructed signals, including error signals can be seen in Fig. 6 and Fig. 7. The error signal is defined as difference between the origin signal and its reconstruction.

As it can be observed in the reconstruction examples the structure of ECG is often preserved well but the shape of PQST waves may not be perfectly exact. The amplitude and position of R wave is always exact what is guaranteed by QRS detector used during acquisition. Another advantage of implementing the QRS detector at the acquisition node is that if a false-positive detection caused by artefact has occurred, the frame is split as if it was a usual heartbeat. Then such a frame is reconstructed as well as the artifact can be approximated by the model and as allowed by the bound setting.

The limitations of signal model to approximate the acquired signal have the greatest impact on reconstruction quality. Depending on the modeling functions used as well as the parameter bounds used during optimization, resulting APRD values for complete database are 20.61%. It was shown by [21] that the values of APRD<9% are evaluated as "good" or "very good" in terms of preserving the diagnostic content in the reconstructed signal. For some records in Tab. 1 the APRD is better than 9% at high compression ratios of *CR* ≈ 10, which cannot be achieved using wavelet dictionaries at similar conditions even with no quantization of measurement vectors.

Table 1. . Achieved values of APRD for chosen MIT-BIH database records for D = 8.

Record №	Proposed		Mexican hat basis	
	CR	APRD [%]	CR	APRD [%]
100	9.69	10.62	8	20.64
102	9.77	57.33	8	39.28
106	9.97	24.96	8	54.59
107	9.79	20.79	8	54.12
108	9.93	16.05	8	37.98
112	9.51	6.47	8	12.92
117	10.10	8.29	8	17.36
118	9.76	13.79	8	23.15
119	9.85	11.66	8	27.15
121	9.93	5.09	8	13.75
122	9.47	7.70	8	21.34
123	10.16	8.46	8	17.89
201	9.57	9.03	8	38.11
230	9.56	17.64	8	49.28
Average for all MIT-BIH	9.70	20.01	8	40.81

VI. CONCLUSIONS

The reconstruction of ECG signal acquired using a CS lossy data reduction can be done with no prior knowledge, patient-specific information or a large record database just using a suitable time-domain signal model. The proposed method keeps the structure of heartbeats preserved well including the exact positions of R waves and also works for noisy and interfered signals. An advantage is the relatively good reconstruction quality in comparison with wavelet-based dictionary methods for high compression ratios. The versatility of proposed method allows for reconstruction of some unexpected signal shapes including artifacts as well.

VII. ACKNOWLEDGMENT

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