

Dual frequency resonator for the correct determination of the in-field surface impedance frequency dependence of superconductors

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Abstract – In high frequency applications of superconductors, a relevant parameter used to synthetically describe their performances in terms of surface resistance (proportional to power losses) dependence on the operating frequency is the so-called pinning frequency ν_p . Customarily, high sensitivity resonant techniques, intrinsically operating at discrete frequencies, are used to estimate ν_p . By exploiting a dual frequency resonator, we show here that the inaccurate evaluation of ν_p in the common approach based on single-frequency measurements leads to heavy underestimations of the superconductor surface resistance.

I. INTRODUCTION

The low loss electrical conduction at high frequencies of superconductors is exploited in cutting edge research fields like high energy physics and dark matter hunt. Indeed, particle accelerators require superconducting cavities [1] in which the extremely low losses (i.e., very high cavity quality factor Q) allow to reach intense accelerating electric fields, of the order of tens of MV/m. The power handling capabilities of the next generation particle accelerators will be so demanding that also the beam screens, necessary to confine the electromagnetic radiation of the running high energy charged particles curving under static magnetic fields in the scale of 10 T, will require superconductors [2]. Axions, a possible candidate for the elusive dark matter, are predicted to interact with the electromagnetic field of high Q resonators, tuned to their mass frequency equivalent, within static magnetic fields, so that setups with superconducting cavities have been designed and tested [3].

Hence, the prediction, determination and optimization of superconductors surface impedance Z , which is the relevant electrodynamics quantity for high frequency regimes, and of their surface resistance $R = \text{Re}(Z)$, which governs the power losses, are mandatory. The connection of Z to the material (complex) resistivity ρ is represented by the bulk limit expression $Z = \sqrt{i2\pi\nu\mu_0\rho}$ [4] (with ν and μ_0 the frequency and permeability, respectively), applicable for (super)conductors with thickness d larger than both the normal δ and London λ penetration depths [5].

In particular, focusing on the applications within static magnetic flux density B , the dominant source of losses of the ubiquitously used type II superconductors is given by the motion of fluxons, nanometric magnetic flux tubes sustained by vortices of supercurrents [6]. Fluxons oscillate under the Lorentz force exerted by the high frequency currents, Faraday-inducing electrical fields with a component parallel to the currents, thus yielding power dissipation. Their motion is hindered by material defects, effectively acting as pinning centers and exerting elastic recall forces. The beneficial (from the point of view of limited power dissipation) effect of pins can be weakened by thermal activating depinning (often referred to as fluxon creep).

In overall, a full force balance for fluxons at high frequencies allows to describe them as damped harmonic oscillators, additional subjected to stochastic thermal forces [7, 8]. In this framework, a complex vortex motion resistivity ρ_{vm} can be written down [9]:

$$\rho_{vm} = \rho_{ff} \frac{\chi + i\frac{\nu}{\nu_c}}{1 + i\frac{\nu}{\nu_c}} \quad (1)$$

where $\rho_{ff} \propto B$ is the so-called flux flow resistivity, proportional to the number of fluxons and hence to B ; $\chi \in [0, 1]$ is an adimensional creep factor ($\chi = 0$ corresponds to no thermal depinning); ν_c is a characteristic frequency, depending on χ and on the so-called (de)pinning frequency ν_p through a function which depends on the specific model used [9]. At zero creep, $\nu_c \rightarrow \nu_p$. The pinning frequency is a very important quantity, often used to synthetically describe the performance of a superconductor for high frequency applications. Indeed, the higher ν_p the higher is the overall effectiveness of pinning centers: by varying the frequency range of operation, ν_p marks the separation between a low frequency, low loss regime and a high frequency, high loss regime. Creep effects decrease the sharpness of this separation, yielding a characteristic frequency $\nu_c < \nu_p$, and increase losses at low frequencies ($\rho_{vm} \rightarrow \chi\rho_{ff}$ for $\nu \rightarrow 0$).

The determination of the three vortex parameters in (1) requires frequency dependent measurements, as those possible with wide-band techniques like the Corbino disk one [10, 11]. On the other hand, their low sensitivity and dif-

difficult calibration relegate them to niche studies, in favor of the more sensitive and widely used resonator-based methods [12–16]. The latter work intrinsically at fixed frequencies dictated by the resonant mode of operation. The consequent single-frequency measurement of ρ_{vm} yield an under-determined problem (two determinations - real and imaginary parts of the complex ρ_{vm} - vs three unknowns) which is usually solved by assuming a negligible creep and thus determining directly ν_p and ρ_{ff} . In Ref. [9], starting from physical and mathematical constraints, a discussion about the errors involved in such an approach was provided, showing, among the others, that both the obtained ρ_{ff0} and ν_{p0} (the additional subscript “0”, here, denotes the values obtained in the $\chi = 0$ assumption) are under-estimation of the correct values, with relative errors that can reach and exceed 100%. Alternatively, multi-mode resonators, i.e. capable of operating at multiple discrete frequencies, can be conceived [17, 18]. On superconductors, typically planar resonators are exploited which, on the other hand, require the patterning of superconducting sample in the thin film geometry.

Here we propose the exploitation of a specifically built dual mode Hakki-Coleman dielectric loaded resonator, used within the surface perturbation method (thus not requiring sample patterning) and operating on two modes with spaced resonant frequencies ~ 16 GHz and ~ 27 GHz.

This paper is organized as follows. In Sec. II the measurement method is briefly described, whereas sample measurements and results are reported in Sec. III. Short conclusions will be drawn in Sec. IV.

II. THE MEASUREMENT METHOD

The resonator is cylindrical and loaded with a single crystal sapphire rod (diameter (7.13 ± 0.01) mm, height (4.50 ± 0.01)) (see Fig. 1). It is used in the end-wall re-

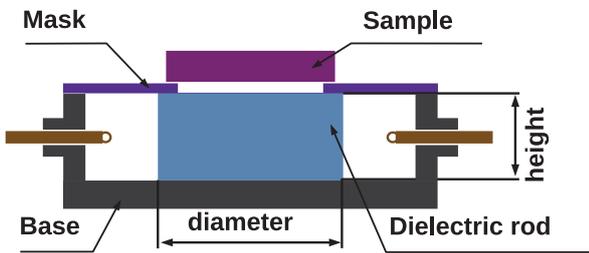


Fig. 1. Sketch of the dielectric resonator.

placement configuration with samples placed on a base and covered with a thin metal mask, having a central hole of diameter 6.50 mm in order to preserve the circular symmetry of the structure. The resonator operates on two transverse electric TE_{0i1} ($i = 1, 2$) modes, with resonant frequencies $\nu_{res,1} = 16.4$ GHz and $\nu_{res,2} = 26.6$ GHz. The resonator is operated in transmission through a two-ports coupling

with the external microwave circuit: by means of a Vector Network Analyzer, frequency sweeps around each of the resonant frequencies are performed and scattering coefficients measured.

With a fit procedure, taking care also of uncalibrated transmission line portions and other non-idealities [19, 20], the resonator Q factor and resonant frequency ν_{res} for each mode are determined. Within the small perturbation approach, the magnetic field H induced variations of the surface impedance $\Delta Z(H) = Z(H) - Z(0) = \Delta R(H) + i\Delta X(H)$, at fixed temperature T , are determined as follows [21]:

$$\begin{aligned} \Delta R(H) &= R(H) - R(0) = G \left(\frac{1}{Q(H)} - \frac{1}{Q(0)} \right) \\ \Delta X(H) &= X(H) - X(0) = -2G \frac{\nu_{res}(H) - \nu_{res}(0)}{\nu_{res}(0)} \end{aligned} \quad (2)$$

where G is a mode-dependent geometrical factor, determined through numerical simulations.

An example of actual resonance curves as measured in terms of the scattering coefficient $|S_{21}|$ for both modes is reported in Fig. 2. The resonator is loaded with the sample described in the next Section: hence its response, measured at fixed temperature $T = (10.00 \pm 0.05)$ K and at selected applied magnetic field intensities, changes in function of the applied field according to Eq. (2).

III. EXPERIMENTAL RESULTS AND DISCUSSION

Measurements are performed at fixed temperature, reached in zero-field cooling conditions, and then by applying a magnetic field H perpendicularly to the sample surface. The sample used for the present discussion is a $d = 240$ nm thick FeSeTe superconductor film, with critical temperature ~ 18 K and deposited on a CaF_2 square substrate 0.5 mm thick and 7 mm wide. Further details are reported in [22, 23].

Since $d \ll \max(\delta, \lambda)$, we resort to the so-called thin-film approximation [5] so that $Z = \rho/d$; moreover, since already in moderate fields $\mu_0 H \sim 0.2$ T field variations of ρ are negligible with respect to ρ_{vm} [24], one have $\Delta Z(H) \simeq \rho_{vm}(B)/d$, with $B \simeq \mu_0 H$ in the London limit. A sample measurement at $T = (10.00 \pm 0.05)$ K is reported in Fig. 3, where ΔZ_1 and ΔZ_2 for the two modes, respectively, are reported in terms of real and imaginary parts vs the field H .

By having two measurements of ρ_{vm} at the two frequencies $\nu_{res,1}$ and $\nu_{res,2}$, it is possible to easily solve the system of complex-valued equations based on Eq. (1), for each field point, and determine the corresponding parameters ρ_{ff} , χ , ν_c . Focusing on the latter two quantities, they come out nearly field independent with $\chi = 0.090 \pm 0.005$ and $\nu_c = (41.0 \pm 0.1)$ GHz at $\mu_0 H = 0.5$ T.

The obtained ν_c is within a factor of 2.5 with respect to

values reported in literature for ν_{p0} both in thin films [22] and single crystals [14]: considering the above recalled underestimating nature of ν_{p0} , together with the differences between distinct samples, this result is sensible.

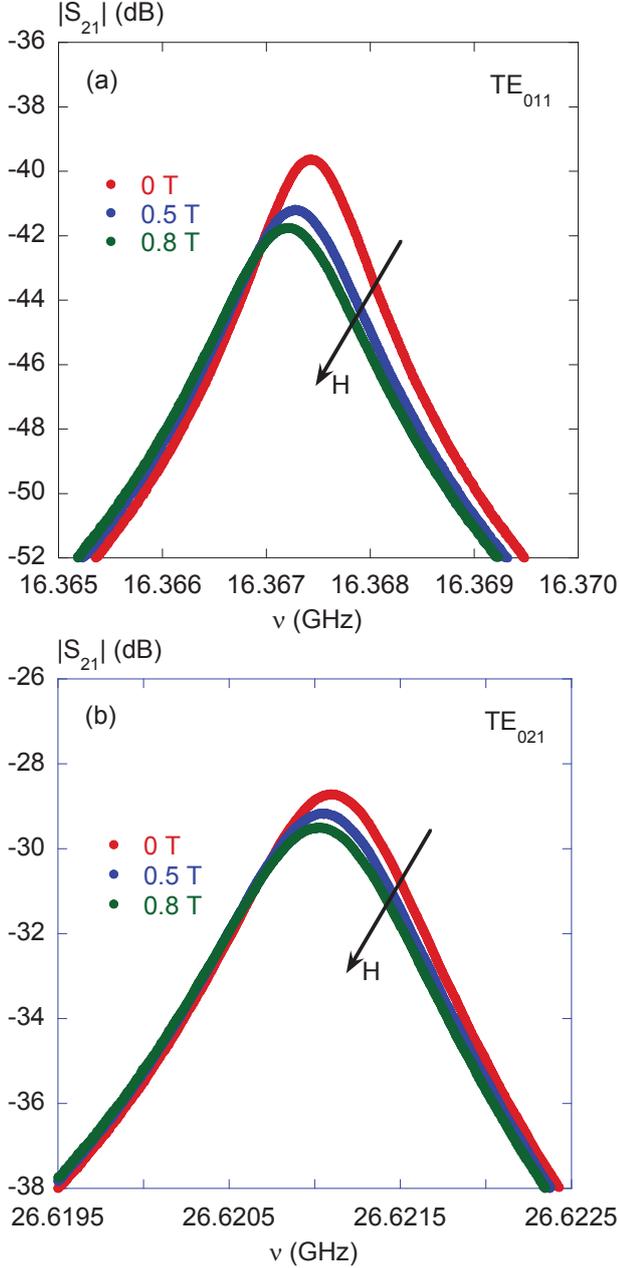


Fig. 2. Modulus of the scattering coefficient S_{21} of the resonator vs ν around the resonant frequencies of the two modes (TE_{011} in panel (a), TE_{021} in panel (b)), measured at fixed $T = (10.00 \pm 0.05)$ K and selected magnetic field H . It is apparent the change in the resonator response due to the sample $Z(H)$.

The creep factor, although small, is not negligible even at this small temperature, as often naively assumed: this

is consistent with result obtained with wide-band measurements in the same temperature range in other superconductors like Nb [25].

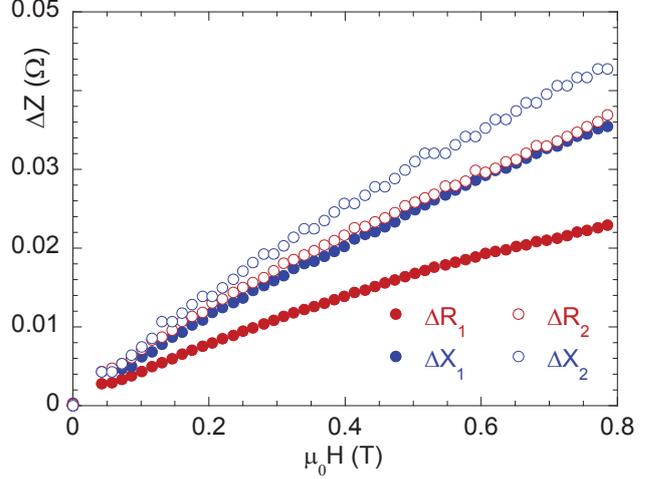


Fig. 3. $\Delta Z_i(H)$ at two frequencies, $\nu_{res,i} = \{16.4; 26.6\}$ GHz, vs field H at fixed $T = (10.00 \pm 0.05)$ K.

It is of interest to compare the characteristic frequency value ν_c , as obtained through the dual frequency analysis of the measurements at $\nu_{res,1}$ and $\nu_{res,2}$, to the value which would be obtained from a single frequency measurement, i.e. ν_{p0} from $\nu_{res,1}$ only. In the latter case, one obtains $\nu_{p0} = (24.9 \pm 0.1)$ GHz: it can be seen that ν_{p0} is a heavy underestimation of ν_c . This error is amplified if one tries to extrapolate to different, lower, frequencies the expected ρ_{vm} and hence $\Delta Z = \rho_{vm}/d$.

For example, at the selected field value $\mu_0 H = 0.5$ T, using ν_{p0} and the corresponding $\rho_{ff0}(\mu_0 H = 0.5 \text{ T}) = (1.4 \pm 0.2) \cdot 10^{-8} \Omega\text{m}$ and $\chi = 0$, through Eq. (1) $\Delta Z_0 = \rho_{vm}/d$ vs the frequency ν can be numerically computed. Analogously, the reconstructed frequency dependence of ΔZ_χ , where the “ χ ” subscript highlights that the experimentally obtained χ value is used, can be computed from the above reported values for ν_c and χ , and the corresponding $\rho_{ff} = (2.0 \pm 0.2) \cdot 10^{-8} \Omega\text{m}$. The results are reported in Fig. 4a, together with the relative errors $\varepsilon_z = (Z_{sim,\chi} - Z_{sim,0})/Z_{sim,\chi}$ in Fig. 4b. It can be seen that, in particular, the relative error on R can be as high as 100%, thus making the single frequency measurement completely unreliable in estimating the material behaviour at lower frequencies.

IV. SUMMARY

We have briefly discussed the main model and the relevant parameters governing the high frequency electro-dynamics response of a superconductor in static magnetic fields, namely the so-called pinning frequency ν_p , comple-

mented by the creep factor, which quantifies the intensity of thermal activated processes.

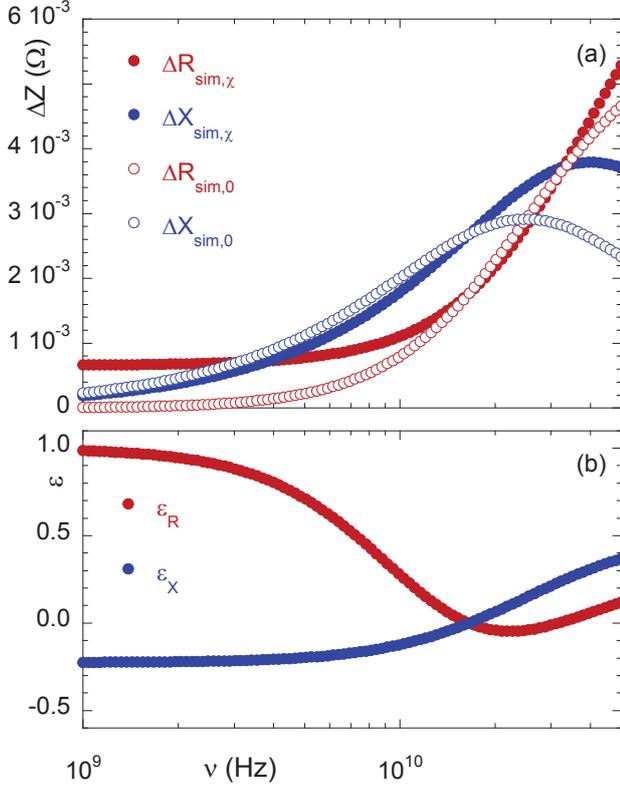


Fig. 4. Numerically computed - through Eq. (1) - ΔZ at $\mu_0 H = 0.5$ T extrapolating the results of a dual frequency measurement (subscript “ χ ”) and single frequency measurement, (subscript “0”). Panel (a): ΔZ vs frequency. Panel (b): relative error on R and X vs frequency.

We have recalled how the superconductor surface impedance Z , and its real part, the surface resistance, proportional to the power losses, has a frequency dependence mainly governed by ν_p , hence used as a synthetic measure of the superconductor performance (the higher, the better). We have shown that single frequency measurements of Z , as often done through electromagnetic resonators, lead to heavy underestimations of ν_p and that the often neglected creep has to be taken into account, together with a correct determination of ν_p , to correctly extrapolate the Z frequency dependence.

V. ACKNOWLEDGMENTS

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REFERENCES

- [1] S. Posen and D. L. Hall, “Nb₃Sn superconducting radiofrequency cavities: fabrication, results, properties, and prospects”, *Supercond. Sci. Technol.*, vol.30, 2017, Art. no. 033004.
- [2] A. Abada, M. Abbrescia, S. S. AbdusSalam, I. Abdjukhanov, J. A. Fernandez et al., “FCC-hh: The Hadron Collider. Future Circular Collider Conceptual Design Report Volume 3”, *Eur. Phys. J. Spec. Top.*, vol.228, 2019, pp. 755–1107.
- [3] D. Alesini, C. Braggio, G. Carugno, N. Crescini, D. D. Agostino et al., “Galactic axions search with a superconducting resonant cavity”, *Phys. Rev. D*, vol.99, 2019, Art. no. 101101.
- [4] R. E. Collin, “Foundation for Microwave Engineering, 2nd ed”, McGraw-Hill International Editions, Singapore, 1992.
- [5] N. Pompeo, K. Torokhtii and E. Silva, “Surface impedance measurements in thin conducting films: Substrate and finite-thickness-induced uncertainties”, *IEEE Int. Instrum. Meas. Technol. Conf.*, 2017, pp. 1–5.
- [6] M. Tinkham, “Introduction to Superconductivity”, McGraw-Hill, Inc., New York, NY, USA, 1996.
- [7] M. Golosovsky, M. Tsindlekht and D. Davidov, “High-frequency vortex dynamics in YBa₂Cu₃O₇”, *Supercond. Sci. Technol.*, vol.9, No.1, 1996, pp. 1–15.
- [8] N. Pompeo, A. Alimenti, K. Torokhtii and E. Silva, “Physics of vortex motion by means of microwave surface impedance measurements”, *Fiz. Nizk. Temp.*, vol.46, 2020, pp. 416–421.
- [9] N. Pompeo and E. Silva, “Reliable determination of vortex parameters from measurements of the microwave complex resistivity”, *Phys. Rev. B*, vol.78, No.9, 2008, Art. no. 094503.
- [10] J. C. Booth, D. H. Wu, and S. M. Anlage, “A broadband method for the measurement of the surface impedance of thin films at microwave frequencies”, *Rev. Sci. Instrum.*, vol. 65, No. 6, 1994, pp. 2082–2090.
- [11] E. Silva, N. Pompeo, K. Torokhtii and S. Sarti, “Wideband Surface Impedance Measurements in Superconducting Films”, *IEEE Trans. Instrum. Meas.*, vol.65, No.5, 2016, pp. 1120–1129.
- [12] M. Perpeet and M. Hein, “Enhanced electron-phonon coupling of Ti-doped superconducting Nb₃Sn films investigated at microwave frequencies”, *Phys. Rev. B*, vol.72, No.9, 2005, Art. no. 094502.

- [13] N. Pompeo, K. Torokhtii and E. Silva, “Dielectric Resonators for the Measurements of the Surface Impedance of Superconducting Films”, *Meas. Sci. Rev.*, vol.14, No.3, 2014, pp. 164–170.
- [14] T. Okada, F. Nabeshima, H. Takahashi, Y. Imai and A. Maeda, “Exceptional Suppression of Flux-Flow Resistivity in $\text{FeSe}_{0.4}\text{Te}_{0.6}$ by Back-Flow from Excess Fe Atoms and Se/Te Substitutions”, *Phys. Rev. B*, vol.91, 2015, Art. no. 054510.
- [15] N. G. Ebensperger, M. Thiemann, M. Dressel and M. Scheffler, “Superconducting Pb stripline resonators in parallel magnetic field and their application for microwave spectroscopy”, *Supercond. Sci. Technol.*, vol.29, No.11, 2016, Art. no. 115004.
- [16] A. A. Barannik, N. T. Cherpak, Y. He, L. Sun, X. Zhang et al., “Microwave response of a cavity resonator with thin superconductor film depending on film temperature and orientation”, *Low Temp. Phys.*, vol.44, No.3, 2018, pp. 247–251.
- [17] Y. Kobayashi and H. Yoshikawa, “Microwave measurements of surface impedance of high- T_c superconductors using two modes in a dielectric rod resonator”, *IEEE Trans. Microw. Theory Tech.*, vol.46, No.12, 1998, pp. 2524–2530.
- [18] J. Powell, A. Porch, R. Humphreys, F. Wellhöfer, M. Lancaster and C. Gough, “Field, temperature, and frequency dependence of the surface impedance of $\text{YBa}_2\text{Cu}_3\text{O}_7$ thin films”, *Phys. Rev. B*, vol.57, No.9, 1998, pp. 5474–5484.
- [19] N. Pompeo, K. Torokhtii, F. Leccese, A. Scorza, S. Sciuto and E. Silva, “Fitting strategy of resonance curves from microwave resonators with non-idealities”, *IEEE Int. Instrum. Meas. Technol. Conf.*, 2017, vol.21, pp. 1–6.
- [20] K. Torokhtii, A. Alimenti, N. Pompeo, and E. Silva, “Uncertainty in uncalibrated microwave resonant measurements”, 24th IMEKO TC4 International Symposium, 2019, pp. 1–5.
- [21] D. H. Staelin, A. W. Morgenthaler and J. A. Kong, “*Electromagnetic Waves*”, Prentice-Hall, Inc., 1994.
- [22] N. Pompeo, A. Alimenti, K. Torokhtii, V. Braccini and E. Silva, “Microwave properties of $\text{Fe}(\text{Se},\text{Te})$ thin films in a magnetic field: pinning and flux flow”, *J. Phys. Conf. Ser.*, vol. 1559, 2020, Art. no. 012055.
- [23] N. Pompeo, K. Torokhtii, A. Alimenti, G. Sylva, V. Braccini, and E. Silva, “Pinning properties of FeSeTe thin film through multifrequency measurements of the surface impedance”, submitted for publication, preprint: arXiv:2006.08186 [cond-mat.supr-con], 2020.
- [24] E. Silva, N. Pompeo, and O. V. Dobrovolskiy, “Vortices at Microwave Frequencies”, *Phys. Sci. Rev.*, vol.2, No.10, 2017, Art. no. 20178004.
- [25] E. Silva, N. Pompeo and S. Sarti, “Wideband microwave measurements in $\text{Nb}/\text{Pd}_{84}\text{Ni}_{16}/\text{Nb}$ structures and comparison with thin Nb films”, *Supercond. Sci. Technol.*, vol.24, No.2, 2011, Art. no. 024018.