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# Experimental Validation of a Double D Coil Simplified Mathematical Model

Nicola Campagna[1], Rosario Miceli[2], Giuseppe Caravello[3], Antonio Cataliotti[4], Ciro Spataro[5]
*Department of Engineering, University of Palermo, Italy*
[1]nicola.campagna@unipa.it, [2]rosario.miceli@unipa.it, [3]giuseppe.caravello02@unipa.it, [4]antonio.cataliotti@unipa.it, [5]ciro.spataro@unipa.it

***Abstract* – The aim of this paper is the experimental validation of a Double D coil simplified mathematical model for both static and dynamic inductive power transfer applications. The model is based on the principle of superposition to valuate parameters such as the auto-inductance of an IPT antenna by considering the coil as a finite number of elements. Then, experimental measurements are carried out to validate the model.**

**Keywords: Inductors, Inductive Power Transfer, Double D antenna, modelling, wireless charging.**

## I. INTRODUCTION

The development of electric vehicles moves the action field of the research to innovative technologies and solutions in the automotive sector. Among the new enabling technologies, wireless charging is one of the burning issues because of the advantages introduced in terms of facilities and safety [1-3]. The core of this energy transfer method is the coupling system. The most used in the field of vehicle battery charging is the inductive coupling.

Literature shows how a lot of configurations and topologies are employed. Among them, Double D (DD) coupled inductors are preferred because they present low border effect, if compared, for example with the planar circular ones, and because under the same material employment, this configuration allows to have higher values of inductances and then better coupling effects can be obtained [4].

The efficiency of the wireless charging strongly depends on the coupling system, for this reason, a coil model is needed in order to evaluate preventively the performances and well design the entire system [5]. Besides, in order to maximize the power transfer, the inductive coupling is performed at the resonance frequency, standardly chosen equal to 85 kHz, by using a compensation network [6]. For this reason, a coil model is also useful in order to evaluate the reactive power has to be compensated. A lot of models are proposed in literature and in most of them, a FEM analysis is carried out [7].

Even though magnetostatic and magnetodynamic analyses through finite elements method are extremely accurate and useful, high computational efforts are required. In this paper a simplified mathematical model is presented and experimentally validated. The model allows to obtain a fast characterization of DD antennas, useful for the first design stages of an inductive power transfer (IPT) system.

## II. DD-COIL MODEL

### A. *Geometrical structure design*

Thanks to the principle of superposition, the entire DD antenna can be considered as a set of linear conductors. For this reason, the geometrical construction of the structure is carried out, in Matlab environment, through the implementation of a matrix with dimensions (N+1 × 3), whose elements represent the extreme space coordinates of the N linear conductors composing the antenna, linked by a continuity constrain.

One of the aspects that cannot be neglected in modelling is flexibility. For this reason, by varying parameters, such as the elementary turn length, the distance between turns or the number of turns, the algorithm can calculate the number of conductors composing the antenna and their space coordinates. An example of DD structure is shown in fig. 1.

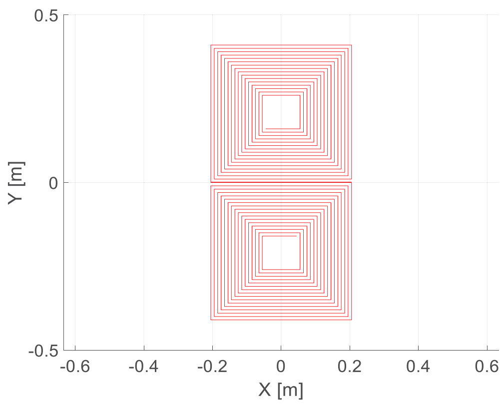



*Fig.1: Geometric structure of a DD coil*

### B. *Mathematical modelling*

The mathematical model of the described DD antenna is created in Matlab environment. It is based on the discretization of the antenna in a finite number of conductors, evaluating each single contribute of the potential vector in order to calculate the self-inductance value through the principle of superposition.

Flux Φ associate to the system can be valued through the following equation.

$$\Phi(B) = \int_S \vec{B} \cdot dS = \mu \int_S \vec{H} \cdot dS \qquad (1)$$

in which $B$ is the magnetic induction, $H$ is the magnetic field, $\mu$ is the magnetic permeability and S the integrating surface.
Appling the Stocks theorem for a homogeneous system, eq. (1) can be reformulated in:

$$\mu \int_S \vec{H} \cdot dS = \mu \int_S rot\vec{V} \cdot dS = \mu \int_l \vec{V} \cdot d\bar{l} \qquad (2)$$

in which $l$ is the surface contour and $V$ being the potential vector.

Considering independently each element composing the antenna, having length $L$ and radius $r$ and carrying the current $I$, the potential vector in a point $P$ with coordinates $(x,y)$ can be expressed as:

$$V(P) = \frac{I}{4\pi}\left(Arc\,sinh\frac{x+\frac{L}{2}}{y} - Arc\,sinh\frac{x-\frac{L}{2}}{y}\right) \qquad (3)$$

This expression must be integrated along each element of the antenna; obviously only those ones for which the scalar product between the generated potential and the induced element side path is not equal to zero can interact each other.
Generalizing, the potential vector referred to the midpoint of the induced *i-th* element can be evaluated as follow. Each potential is valued on the surface of the conductors in order to not involve the current distribution and then the internal potential vector.

$$iVii = \frac{I}{4\pi}\left(2LArc\,sinh\frac{L}{r} - 2\sqrt{L^2+r^2} + 2r\right) \qquad (4)$$

$$iV_{ji} = \int_{-\frac{L_i}{2}}^{\frac{L_i}{2}} V_j \cdot dl_i \cong V_j(i_m)\Delta l_{ij} \qquad (5)$$

While for the evaluation of the self-potential, the exact formula can be used as in (4), the calculation of the potential induced by the j-th element on the i-th element is approximated, as shown in (5) and in order to simplify the algorithm, by taking into account $V_j(i_m)$ which is the potential on the midpoint of the i-th element and $\Delta l_{ij} = l_i \cdot \cos(\alpha)$ that is the i-th element projection on the j-th element as graphically shown in fig.2. The angle α is determined through the difference between $\alpha_i$ and $\alpha_j$, formed by the i-th element and the j-th element with a common direction.

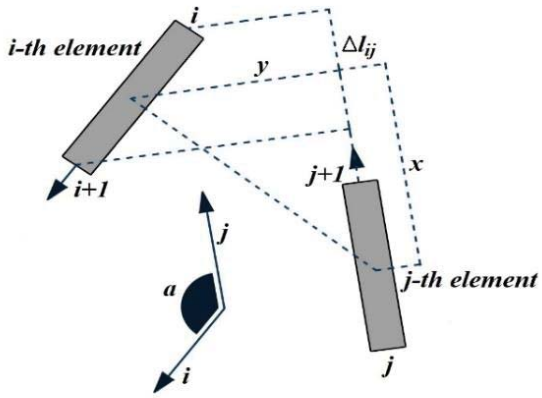



*Fig.2: Reciprocal position between two generic elements*

A matrix $iV$, with dimension (i × j) is created, having the contributes of the potential vectors acting on each element as rows. Then the self-inductance of the antenna can be assessed as follow by applying the principle of superposition:

$$L = \frac{\mu}{I}\sum_{i,j=1}^{N} iV_{i,j} \qquad (6)$$

in which N is the number of elements composing the DD antenna.

## III. EXPERIMENTAL VALIDATION

### A. Experimental test bench

Four coils with different geometric characteristics are simulated and built. In order to validate the model, the different values of the calculated self-inductance are verified by the use of the RCL Meter GW Instek 821, shown in fig.3.

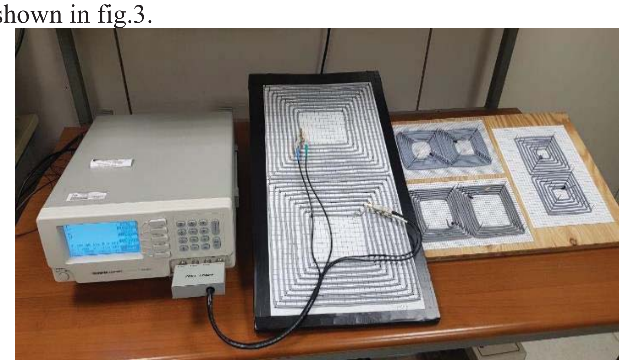


*Fig. 3: Test bench and instrumentation*

The geometric structures of the tested coils are shown in fig. 4, their characteristics are reported in table 1. For the measurement of the self-inductance, a series circuit and a voltage equal to 1 V are chosen. Measurement are carried out in the frequency range 1 – 200 kHz with steps equal to 5 kHz. For each frequency, five measurements are acquired and the average value is considered.

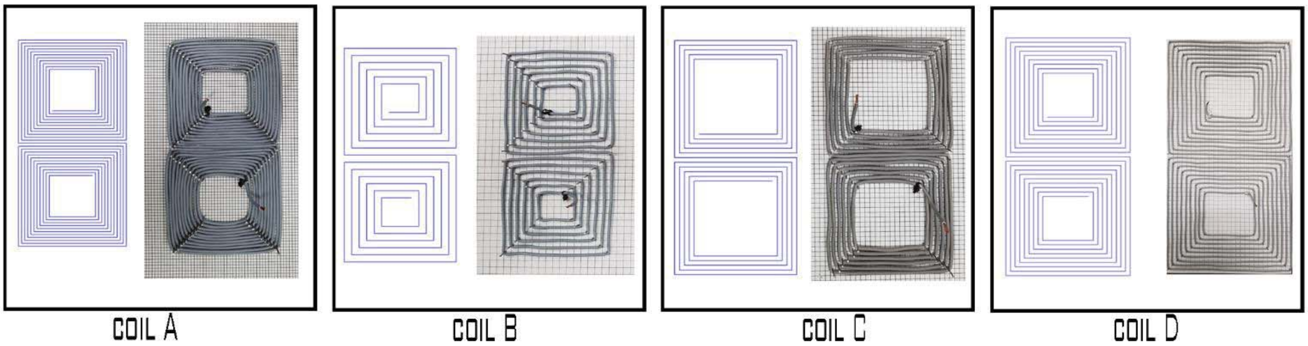


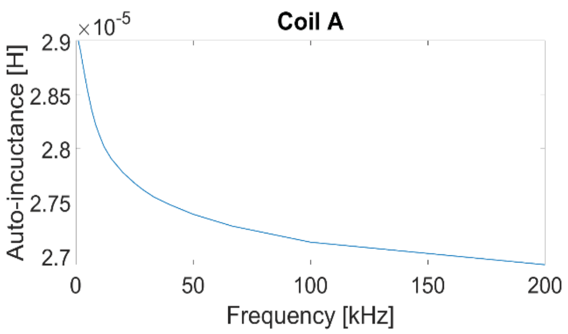

*Fig 4: Structure of the simulated DD coils and the built ones.*

*Table 1. Geometrical characteristics of the coils*

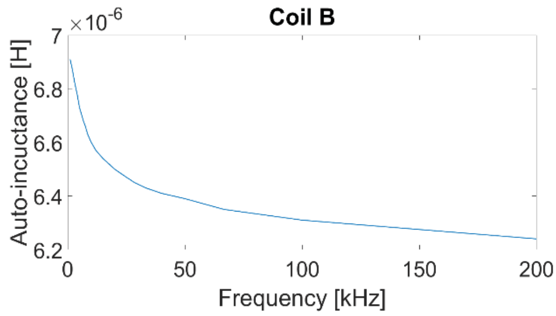
| Coil | Total length | Dimensions | N of turns | Distance between turns | Copper wire section |
|---|---|---|---|---|---|
| Coil A | 6,57 m | 10,5 cm × 21 cm | 11 | 0,3 cm | 1,5 mm$^2$ |
| Coil B | 3,29 m | 10,5 cm × 21 cm | 6 | 0,7 cm | 1,5 mm$^2$ |
| Coil C | 4,22 m | 12 cm × 24 cm | 5 | 0,5 cm | 2,5 mm$^2$ |
| Coil D | 13,95 m | 27 cm × 54 cm | 9 | 1 cm | 2,5 mm$^2$ |

## B. *Results*

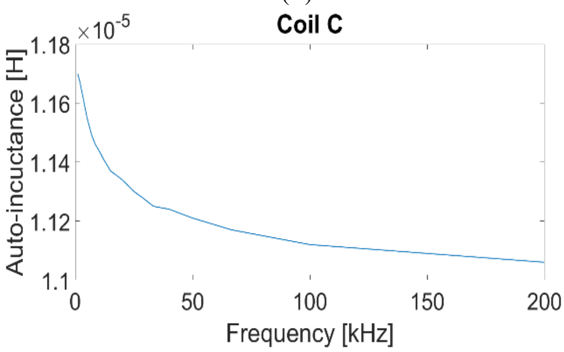
The values of self-inductance obtained by the model described in the previous paragraph are respectively equal to 2,8011×10$^{-6}$ H for coil A, 6,7545×10$^{-6}$ for coil B, 1,2228×10$^{-5}$ for coil C and 4,7834×10$^{-5}$ for coil D, while in fig. 5 the measured self-inductance values are reported. It can be noticed how the self-inductance is function of the frequency because of several factors that are examined better in the next section.



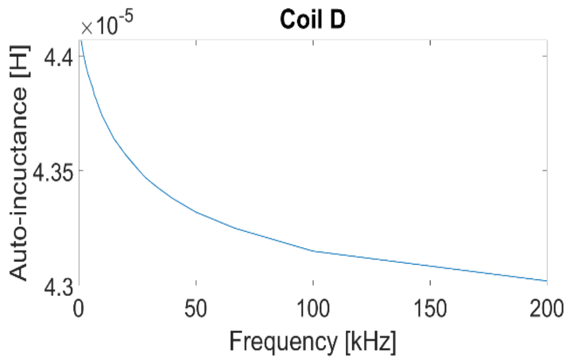

(a)




(b)




(c)




(d)

*Fig 5: Measured auto-inductances vs frequency*

The accuracy of the measurements $A_e$ is calculated with the following formula (7), how indicate in [6].

$$A_e[\%] = 2\ counts \pm 0{,}03\% + 0{,}02\%[X/Y_{max}] \qquad (7)$$

In which $X$ is the measured value and $Y_{max}$ the limit range reported on the LCR Meter Manual [8]. No voltage factor is needed, because the test voltage is less than 1,25 V, and the temperature factor can be neglected being the environmental temperature during tests equal 22°C.
In the worst case, accuracy is equal to ±0,045%

The uncertainty, with a confidence interval of 98% is calculated as follow:

$$\mu_e[\%] = \pm\left(2{,}58 * \frac{A_e}{\sqrt{3}}\right) = \pm 0{,}067\% \qquad (8)$$

After the self-inductance measurement of each coil the recalibration of the instrument was performed in order to keep the instrument in the condition needed to give accurate readings. In order to evaluate the frequency dependence of the self-inductance, different factors such as the skin effect, the parasitic capacitance effect and so on should be considered and implemented, but they would complicate the model, which concept is just a preliminary design of DD coils. For this reason, in order to take into account the self-inductance variation with frequency, the following simplified formula is considered.

$$L = \frac{L_0}{f^{10^{-2}}} \qquad (9)$$

In which $L_0$ is the model obtained value of auto-inductance and $f$ is the frequency.

Fig. 6 shows how the previous formula (9) can well describe the phenomenon. The blue trend expresses the variation with frequency of the measured self-inductance values and the red trend represents the model obtained one taking into account the frequency dependence.

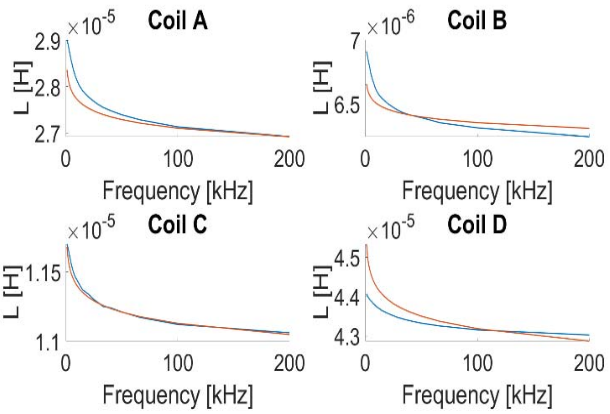



*Fig 6: Measured auto-inductances vs calculated ones*

Fig. 7 shows in red the deviation between the single calculated self-inductance value and the measured ones by varying frequency. It is simple to notice, how including the frequency dependence of the self-inductance in the model, even with a very simple method, the deviation between the calculated ones and the measured ones is reduced a lot (blue trend).

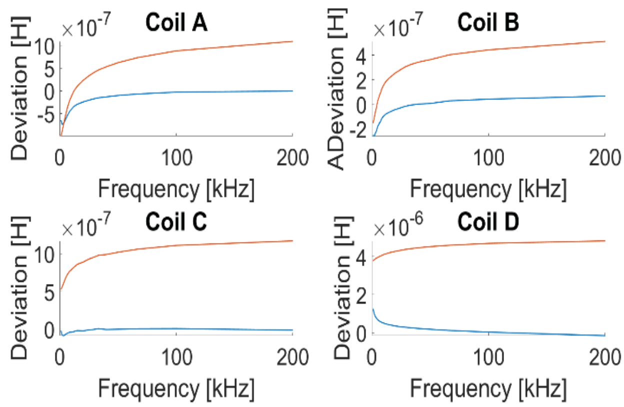



*Fig. 7: Deviation between model calculated auto-inductances and measured ones*

## 1. SUMMARY

In this paper a simple mathematical model of DD coils for IPT applications is presented. The model is based on the effects superposition principle to valuate parameters such as the auto-inductance of an IPT antenna by considering the coil as a finite number of elements inducing each other a potential vector. The goal of the model is a simple evaluation in the early design stage, avoiding high computational efforts usually required by complex FEM models. Four typologies of DD coils were simulated and built. The self-inductance of the coils was measured thanks to an RLC meter in the frequency 1 - 200 kHz. The frequency dependence of the self-inductance is considered in the model by the use of an empirical relation. The measurements confirmed the model-obtained auto-inductance values and the high potentialities of the model for fast and simple analyses.

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