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Stochastic Two-bit On-line Temperature Measurement with RTD Pt-100 Sensor Operating in a Nonlinear Mode

Platon Sovilj[1], Bojan Vujicic[1], Dragan Pejic[1], Aleksandar Radonjic[2], Vladimir Vujicic[2]

[1]*Faculty of Technical Sciences, Novi Sad, Serbia,*
platon@uns.ac.rs, bojanvuj@uns.ac.rs, pejicdra@uns.ac.rs
[2]*Institute of Technical Sciences, The Serbian Academy of Sciences and Arts, Belgrade, Serbia,*
sasa_radonjic@yahoo.com, vujicicv@uns.ac.rs,

***Abstract*: The paper proposes a method and device for on-line temperature measurement with Pt-100 sensor operating in a nonlinear mode. The sensor measures the temperature in range of 0^0C to 200^0C, whereas the measured data are processed by applying a two-bit stochastic digital measurement method. The proposal is theoretically analyzed and tested experimentally. It is shown that the accuracy of the method depends on the accuracy of the sensor. At a sampling frequency of 1 MHz, the maximum accuracy is reached in 17.5 sec. and is equal to 0.015% of full scale (FS). On the other hand, when the measurement time interval is 0.1 sec. (the shortest measurement interval), the accuracy is equal to 0.1% of FS. Therefore, the proposed device can be used for both calibration and measurement purposes.**

Keywords–Two-bit stochastic digital measurements, Pt-100 sensor, nonlinear mode.

I. INTRODUCTION

Analog-to-digital converters (ADCs) are essential blocks in modern measurement systems [1]. They are used to digitize analog signals, thus allowing the microprocessor to calculate various signal parameters, such as amplitude, frequency and so on. According to the standard sampling method (SSM), the signal parameters will be calculated more accurately as the resolution of the ADC increases. However, it is known that the conversion speed of the ADC decreases with increasing resolution. Because of this, it is impossible to measure fast and/or noisy signals very accurately. This problem cannot be alleviated either by repeating or averaging the measurement results. On the other hand, the measurement over long intervals only increases the accumulated error.

One way to overcome all these problems is to use the stochastic digital measurement method (SDMM) [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. This method is a substantially different paradigm for digital measurements than the SSM. In the case of the SDMM, the quantization error of the ADC can be 100% of full scale (FS), but the measurement result will be very precise and accurate. In this regard, in [10] it was shown that the optimal resolution of the ADC is three bits. However, from the point of view of the overall system (the ADC + signal processing block), the optimal resolution is two bits [11]. This fact is demonstrated in many publications, including [12]. The authors of that paper also showed that the scope of application of the SDMM can be extended into the area of measurement of nonlinear quantities. In this regard, it should be noted that there are many sensors operating in a nonlinear mode [13]. In order to achieve greater accuracy, the calibration curve for these sensors is approximated by the calibration polynomial (CP). For the measurement purposes, it is important an inverse CP (ICP), which is obtained by finding the inverse function of the CP. In this paper, we will see how a two-bit SDMM and the ICP of the Pt-100 sensor can be used for on-line temperature measurement. In addition, we will also see that the accuracy of our method depends exclusively on the accuracy of the sensor.

II. AC RESISTANCE MEASUREMENT AND THE ICP OF THE PT-100 SENSOR

In Fig. 1 it is shown a linear converter which converts the resistance of the sensor into a sinusoidal voltage. The voltage amplitude on the sensor is proportional to the resistance of the sensor. More precisely, the following equations are valid:

$$u_{Pt} = -\frac{u_g}{R_g} \cdot R_{Pt} \tag{1}$$

$$\left|u_{Pt}\right| = \left|\frac{u_g}{R_g}\right| \cdot R_{Pt} \tag{1}$$

$$u_{Pt}^2 = \frac{u_g^2}{R_g^2} \cdot R_{Pt}^2 \tag{3}$$

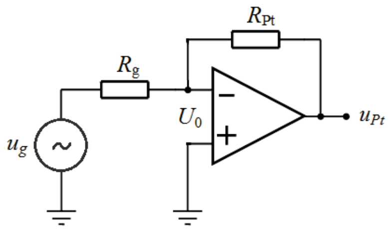

Fig. 1. A linear converter that converts the resistance of the sensor into a sinusoidal voltage.

The measurement of the mean square of a sinusoidal voltage (an absolute value of sinusoidal voltage) using the SDMM is explained in detail in [2], [3].

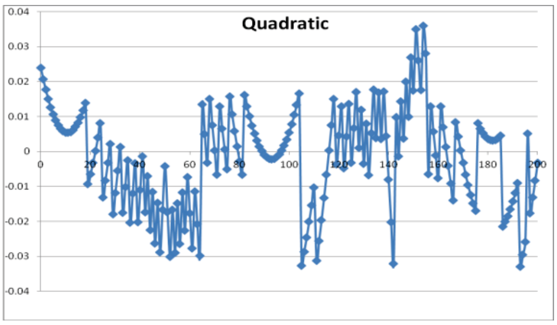

Fig. 2. Diagram of the error of a ES 100Ω RTD.

However, what is never explained is how the ICP of the Pt-100 sensor can be used for two-bit on-line temperature measurements. At the beginning, let us recall that the Pt-100 sensor measures temperatures between - 200°C and + 850°C. It is also known that the sensor's resistance (R_{Pt}) changes with temperature (T). One of the ICPs, which describes this relationship, is defined as

$$T = -244.83 + 2.3419 \cdot R_{Pt} + 0.0010664 \cdot R_{Pt}^2 \tag{4}$$

where $T \in \{0°C,\ 200°C\}$. This equation, known as a European standard (ES) 100Ω RTD, provides a maximum error of only 0.036°C (Fig. 2). Thus, it is often used to characterize the relationship between T and R_{Pt}.

III. ON-LINE TEMPERATURE MEASUREMENT USING A TWO-BIT SDMM

Before explaining our approach in detail, we will briefly recall a few theoretical results from [2]. The first one is related to the measurement of the mean value of the input signal $f_1(t)$. For that purpose it is necessary to add a dither signal h_1 to the input signal before its digitalization. In that case, the output value and the variance of the average error will be equal to

$$\bar{\Psi}(1) = \frac{1}{N} \cdot \sum_{i=1}^{N} \Psi_1(i) = \frac{1}{T} \cdot \int_0^T f_1(t)\, dt \tag{5}$$

$$\sigma_e^2(1) = \frac{1}{N} \cdot \left[\frac{\Delta}{T} \cdot \int_0^T \left| f_1(t) \right| dt - \frac{1}{T} \cdot \int_0^T f_1^2(t)\, dt \right] \tag{6}$$

where Δ is the quantum of a two-bit ADC. Similarly, if we want to measure the mean value of a product of two signals $f_2(t)$ and $f_3(t)$, we need to add two uncorrelated dithers h_2 and h_3. In that case, it will hold that

$$\bar{\Psi}(2) = \frac{1}{N} \cdot \sum_{i=1}^{N} \Psi_2(i) \cdot \Psi_3(i) = \frac{1}{T} \cdot \int_0^T f_2(t) \cdot f_3(t)\, dt \tag{7}$$

$$\sigma_e^2(2) = \frac{1}{N} \cdot \left[\frac{\Delta^2}{T} \cdot \int_0^T \left| f_2(t) \right| \cdot \left| f_3(t) \right| dt - \frac{1}{T} \cdot \int_0^T f_2^2(t) \cdot f_3^2(t)\, dt \right] \tag{8}$$

Now we can go back to the description of the proposed instrument. From its block diagram (Fig. 3) we observe that the instrument calculates (in parallel) the linear and quadratic parts of the mentioned ICP. The input block of the instrument (Fig. 4) consists of the Pt-100 sensor, R/U_m converter, analog adder and two-bit ADC. The role of this block is to convert the temperature T (seen by the Pt-100 sensor) to a two-bit value proportional to the sinusoidal voltage. The absolute value of this signal ($|u_d|$) is further processed by the multiplier (Fig. 5) and both counters (Fig. 3). So, if all values are normalized to the full-scale (FS) of a 2-bit ADC ($f_1(t) = f_2(t) = f_3(t) = f(t) = U_m \cdot \sin(\omega t)$ and $\Delta = U_m = 1$), it is easy to show that the expressions (5)-(8) will reduce to

$$\bar{\Psi}(1) = \frac{2 \cdot R_{Pt}}{100 \cdot \pi} = k_1 \cdot R_{Pt} \tag{9}$$

$$\sigma_{\bar{e}}(1) = \sqrt{\frac{4-\pi}{2\pi \cdot N}} \tag{10}$$

$$\bar{\Psi}(2) = \frac{R_{Pt}^2}{2 \cdot 100^2} = k_2 \cdot R_{Pt}^2 \tag{11}$$

$$\sigma_{\bar{e}}(2) = \frac{1}{4} \cdot \sqrt{\frac{2}{N}} \tag{12}$$

The role of the computer (not shown in Fig. 3) is to calculate

$$R_{Pt} = \frac{\bar{\Psi}(1)}{k_1} \tag{13}$$

$$R_{Pt}^2 = \frac{\bar{\Psi}(2)}{k_2} \tag{14}$$

after which substitution in (4) one obtains the value of the temperature T. For the sake of completeness, it should be noted that there exist two categories of systematic errors caused by the imperfections of the instrument: the errors caused by comparators' offsets and the errors induced by the analog adder (AD). In [8] it was shown that using the cross-switching method the first group of errors can be suppressed by more than 80 dB. For this reason, they can be considered negligible. On the other hand, the errors induced by the analog adder cannot be eliminated, which means that they affect the accuracy of measuring the values of R_{Pt} and R_{Pt}^2. The concrete values of these errors can be determined from the diagram shown in Fig. 6.

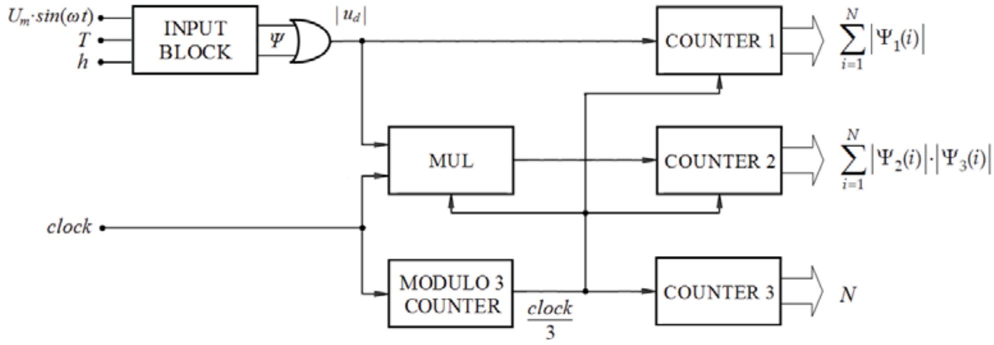

Fig. 3. The block diagram of the proposed instrument.

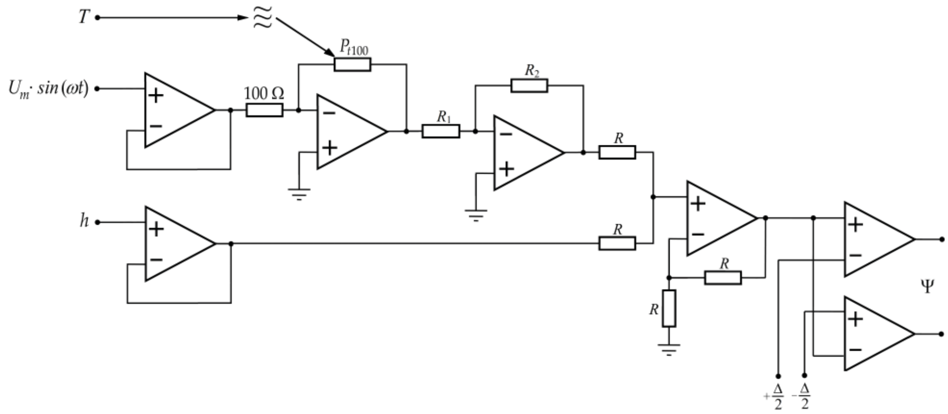

Fig. 4. The scheme of the input block.

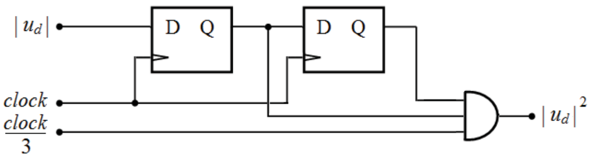

Fig. 5. The scheme of the MUL block.

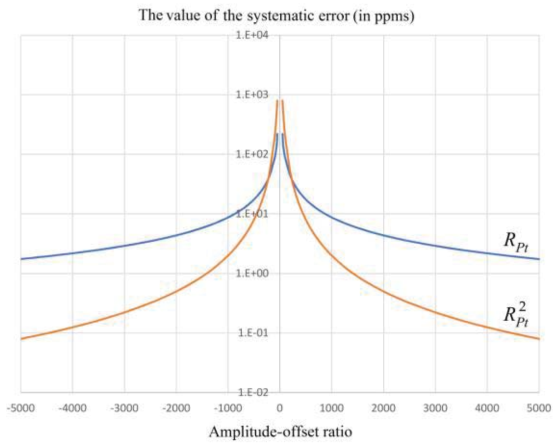

Fig. 6. Diagram of the systematic error caused by the AD.

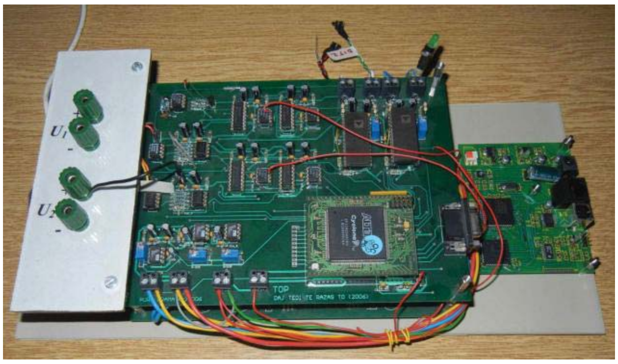

Fig. 7. Photo of the prototype instrument that measures the values of R_{Pt} and R_{Pt}^2.

IV. EXPERIMENT AND DISCUSSION

To validate the above theory, we have conducted several experiments. For that purpose, we used the prototype instrument for measuring the values of R_{Pt} and R_{Pt}^2 (Fig. 7). It is intended to measure the mean value of the product of two analog signals (e.g. voltage and current) with the accuracy of 40 ppm. In our experiments, this instrument was connected to the PC via RS-232 serial link. The sampling frequency was set to 25 kHz and each experiment lasted 100 seconds. The groups of three adjacent samples were treated as individual samples, which means that $25 \cdot 10^3 \cdot 100/3 = 833333$ samples per one experiment was processed. According to (10) and (12), the precision of the measurement of the values of R_{Pt} and R_{Pt}^2 is respectively equal to 0.04 % and 0.039 %.

Table 1. Experimental results in the case of the measurement of the value of R_{Pt} .

	1	2	3	4	5	6	7	8	9
	U	f	$\overline{\lvert u\rvert}_t$	$\overline{\lvert u\rvert}_m$	$\delta\cdot\overline{\lvert u\rvert}_{\max}$	$\sigma_{\bar{e}}(1)$	$2\cdot\sigma_{\bar{e}}(1)+\delta\cdot\overline{\lvert u\rvert}_{\max}$	$\left\lvert\overline{\lvert u\rvert}_m-\overline{\lvert u\rvert}_{ex}\right\rvert$	$\dfrac{\left\lvert\overline{\lvert u\rvert}_m-\overline{\lvert u\rvert}_t\right\rvert}{\overline{\lvert u\rvert}_{\max}}$
	[V]	[Hz]	[V]	[V]	[V]	[V]	[V]	[V]	[%]
	2.0	10	1.273	1.272	0.00127	0.0004	0.0021	0.0010	0.031

Table 2. Experimental results in the case of the measurement of the value of R_{Pt}^2 .

	1	2	3	4	5	6	7	8	9
	U	f	$\overline{\lvert u\rvert_t^2}$	$\overline{\lvert u\rvert_m^2}$	$\delta\cdot\overline{\lvert u\rvert_{\max}^2}$	$\sigma_{\bar{e}}(2)$	$2\cdot\sigma_{\bar{e}}(2)+\delta\cdot\overline{\lvert u\rvert_{\max}^2}$	$\left\lvert\overline{\lvert u\rvert_m^2}-\overline{\lvert u\rvert_t^2}\right\rvert$	$\dfrac{\left\lvert\overline{\lvert u\rvert_m^2}-\overline{\lvert u\rvert_t^2}\right\rvert}{\overline{\lvert u\rvert_{\max}^2}}$
	[V]	[Hz]	[V^2]	[V^2]	[V^2]	[V^2]	[V^2]	[V^2]	[%]
	2.0	10	2.000	2.001	0.00100	0.0004	0.0018	0.0012	0.010

In both cases, the effective precision is between 12 and 13 bits (Tables 1 and 2) (if the instrument were operating at a frequency of 1 MHz, this result would be obtained after 2.5 seconds). As for accuracy, it is completely independent of resolution. In the case when the offset value is equal to 5 mV, the accuracy of the measurement of R_{Pt} and R_{Pt}^2 is below 10 ppm (Fig. 7). In other words, the accuracy of measuring of both coefficients is better than 18 bits. Since the relative FS error of the the Pt-100 sensor is 0.015 %, the maximum accuracy is reached after 700 seconds (if the instrument were operating at a frequency of 1 MHz, the maximum accuracy would be reached after 17.5 seconds). The above facts indicate that the proposed method and device can be used for very accurate temperature measurements (in the temperature range from 0°C to 200°C). And not only that: from the above, we see that each member of the ICP is measured independently. Since the obtained measurement errors are random quantities, it is possible to apply the central limit theorem to their sum. The final result is that the overall accuracy is greater than the accuracies obtained for each individual member of the ICP. Moreover, the overall accuracy will increase with the square root of the number of ICP members, so that remarkably accurate results can be obtained. All these benefits cannot be achieved if the SSM were used.

V. CONCLUSION

This paper proposed a method and device for on-line temperature measurement with Pt-100 sensor operating in a nonlinear mode. More precisely, it has been analyzed and experimentally verified the case when the sensor measures the temperature in range of 0^0C to 200^0C, whereas the measured data are processed using a two-bit stochastic digital measurement method. Our analysis has showed that the maximum accuracy is 0.015% FS and that it can be is reached in 17.5 seconds if the instrument operates at a frequency of 1 MHz. On the other hand, when the measurement time interval is 0.1 seconds, the maximum accuracy is equal to 0.1% FS. Based on this it can be said that the proposed device can be used for both calibration and measurement purposes.

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