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Resonant Column Testing Challenges

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Abstract – **The standard procedure to determine dynamic parameter of soils in the laboratory is the Resonant-Column test (ASTM D-4015). The objective of the present work is to overview alternative testing and interpretation methods based on small-strain measurements using continuous sinusoidal, pulse signals or random noise. The use of unconventional excitation signals and frequency domain methods extends the possibilities of testing. A technique, already applied to the analysis of nonlinear systems, combines sinusoidal signals and random noise showing a good potential for a quick evaluation of the dynamic properties of soils. Starting from this technique, we propose a procedure devoted to soil testing combining chirp with strain amplitude control and random noise. The aim is to improve the quality and to reduce the testing time for the evaluation of the dynamic properties of soils vs. frequency. The use of a dynamic simulator allows a quick testing of new analysis software devoted to RC testing systems giving greater confidence in the quality of work done to face this challenging test.**

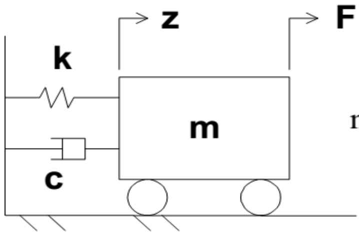
I. INTRODUCTION

The standard method to identify the behavior of approximated single degree of freedom (SDOF) system may benefit from elaboration borrowed from general dynamic system identification techniques. We will introduce these methods applied to RC test and to other applications.

The solution of a typical structural dynamics problem is considerably more complicated than its static counterpart is. This is due to the addition of damping and inertia to the elastic resistance and to the time dependency of all force quantities.

Any linear elastic structure subjected to an external dynamic excitation has physical properties related to its mass, elastic properties (stiffness), and energy-loss mechanism (damping).

The simplest model of a SDOF system assumes that each of these properties is concentrated in a single physical element. A sketch of such a system is shown in Fig.1 where the entire mass m of the system is included in the rigid block which can move only in simple translation and the single displacement coordinate z(t) defines its position.



$$m\ddot{z} + c\dot{z} + kz = F$$

Fig. 1 SDOF system

The elastic resistance to displacement is provided by the weightless spring of stiffness k, while the energy-loss mechanism is represented by the damper c. The external dynamic load producing the response of this system is the time-varying force F(t). There is a simple analogy between straight line and rotational motion changing acceleration, force and mass with angular acceleration, torque and moment of inertia.

The small strain stiffness of a soil is characterized by its initial shear modulus that is one of the most important parameters required to study problems related with dynamic and static movements of the ground and geotechnical structures.

The RC test is currently recognized as the reference laboratory method to evaluate dynamic properties of soils. In resonant column testing, a soil specimen is excited under torsional excitation at different frequencies to measure the transfer function between the applied force (torque i.e. current into the coils) and the measured response (angular acceleration or displacement).

The measured data are fitted with the theoretical transfer function of a SDOF system to determine the resonant frequency and the damping ratio of the material. The standard test requires that the input signal frequency be increased until resonance is reached. Then it is possible to compute the dynamic properties of the soil as derived from the dynamic equilibrium of the specimen.

By means of the RC apparatus, it is possible to perform other tests by modifying the characteristics of the excitation or using non-explicit relationships. Such methods may be used to assess the dynamic properties at loading frequencies away from resonance.

Certain dynamic characteristic of the soil can be measured better with a specific method and it is desirable to have a set of testing methods to obtain all the information from one specimen.

The methods considered are RC conventional test

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(RCS), the RC sine chirp test (RCC), the RC random noise test (RCN), the non-resonance test (NRC) and the combination of random noise with sine (FN and the proposed RCX). Fig. 2 shows a summary of the test methods considered and their acronyms.

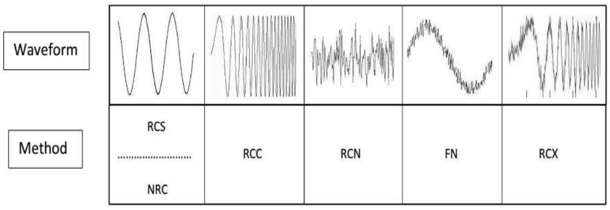

Fig. 2 Test methods and waveforms

Some of the methods are based on frequency domain analysis where the work is all on vibration modes, resonance frequencies and damping ratios.

Using noise instead of sine functions as the excitation signal reduces the limitations of the traditional tests with lower vibration levels and less undesirable influence on material.

The combination of sine and noise shows a good potential for the evaluation of dynamic properties of soils as a function of frequency in resonant column testing.

II. THEORETICAL BACKGROUND

Shear stiffness and shear damping are the key soil dynamic properties. These parameters exhibit linear and nonlinear changes in the strain ranges.

A typical scheme shows a decreasing trend of shear modulus G and an increasing trend of material damping D as strain increases. Fig. 3 represents graphically the different zones and threshold strains along the modulus and damping.

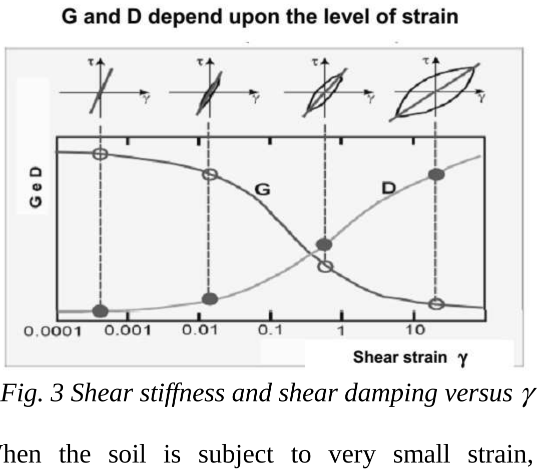

Fig. 3 Shear stiffness and shear damping versus γ

When the soil is subject to very small strain, its mechanical behavior is practically linear elastic with recoverable deformation and negligible energy dissipation and it does not degrade with the number of cycles.

Under such a situation, the appropriate method of analysis is linear elastic being the applicable strain-stress relationship defined by the initial shear modulus and a small damping ratio.

When the cyclic strain has an intermediate value, the soil behaves as an elasto-plastic material. In this case, the shear strain is not sufficient to generate permanent changes in the soil structure and the stress-strain curve during cyclic loading is a loop.

In such a situation, the appropriate method of analysis is whichever linear or nonlinear.

When the soil is subject to great shear strain, shear modulus degrades according to the number of cycles. In this situation, the analysis method should be nonlinear.

III. SYSTEM DESCRIPTION

In the RC testing system, the soil sample is placed in a triaxial stress chamber and is loaded with torque and isotropic load with the assumption of base fixity and free head.

An electromagnetic motor using current to cause torque generates the stress. In the apparatus, the drive system inducing torque has a quite significant inertia.

Changes in specimen length and volume are considered to adjust sample mass polar moment of inertia.

The apparatus can measure static and dynamic deformation characteristics of soil samples or soft rocks in the small and medium-sized amorphous deformation range (up to 0.1%).

Megaris has been developing and manufacturing since 1993 a number of RCTS apparatus for sample diameters ranging from 38 mm to 100 mm and maximum torque levels ranging from 1 Nm to more than 15 Nm.

Torque is proportional to the current flowing in the coils, controlled directly by a high accuracy electronic amplifier, minimizing the effect of counter EMF.

A control and data acquisition system supervises the entire test system, tracking the response of the soil sample by confining and pore pressure, axial displacement, volume change, head angle of rotation and acceleration.

An accelerometer and a couple of proximity sensors compose the measuring system of angular movement. An innovative micro positioning unit provides a zeroing of the proximity sensors. The use of multiaxial MEMS accerometers allows the detection of improper vibration modes and radiated components.

IV. SYSTEM IDENTIFICATION

It is the basic knowledge area to build mathematical models of dynamical systems from measured data. It includes also the optimal design of experiments for efficiently generating informative data for fitting such models as well as model reduction.

System identification has received a growing interest over the last decades also to face the major challenge to reduce the cost, make it more user friendly and to optimize the experiment using less time and less energy to accomplish the task. The main goal of any system identification is to find parameters of a mathematical model representing a physical system (in our case soil and test system) by making use of the input and output data describing the excitation and response of the system.

The soil behaves actually as a non-linear material and may be perceived as a classification problem in which the non-linear parameters are extracted using the model of the underlying linear system.

It can also be perceived, as a problem of "model updating" and in this approach, the linear model of the structure is refined/corrected in order to match the experimental measurements.

In the general system identification field, method based on mapping of stiffness and damping at each frequency value is suggested by Mertens et al. (1989).

For a non-linear system, properties like superposition, reciprocity, homogeneity, which form the backbone of linear theory, are not valid.

For instance, the soil exhibits a non-linearity as for Fig.3 that has a similar stiffness variation as for the softening cubic non-linearity (Fig.4).

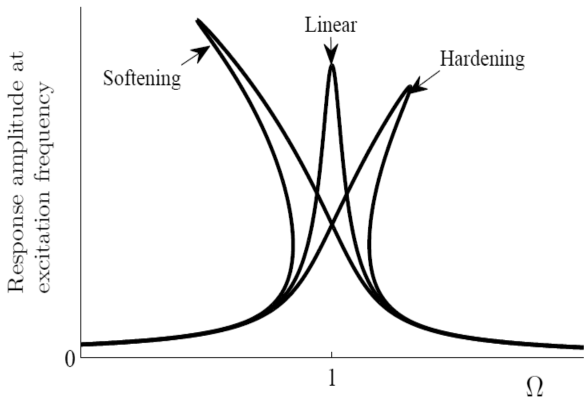

Fig. 4 Comparison linear and non-linear responses

In the hypothesis of linearity, the knowledge of the transfer function of the RC system allows the determination of the dynamic parameters of the system. Actually, the transfer function of the input towards the output isolates the inherent dynamic properties of a mechanical structure including the soil specimen.

The mathematical treatment in the frequency domain is similar to what is used in dynamic structural testing, in which the transfer function H, between input x(t) and output y(t) signals is computed from a number of data series.

The quality of the transfer function is assessed by the cumulative coherence function that is always between 0 and 1 and gives a measure of the linear dependence between two signals as a function of frequency.

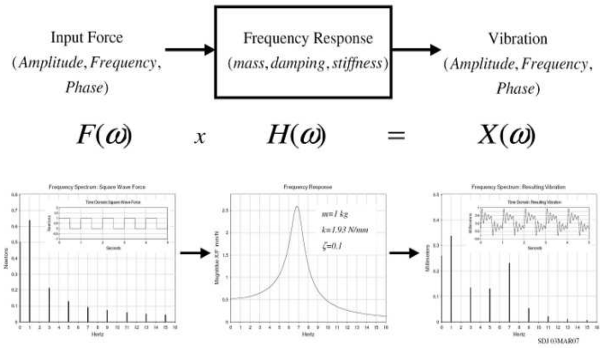

Fig. 5 The frequency response model

It is a valuable tool for the selection of an appropriate functional form for the nonlinearity. Frequencies with coherence function values close to unity mean perfect linear relationship between the two signals and not much contamination caused by noise or other non-linearities.

Each test is made of N samples, which are equally spaced by an interval Δt. The frequency resolution is Δf = 1/(Δt*N) and according to Shannon's sampling theorem the maximum frequency for a discrete signal is Fmax=1/(2*Δt).

Consequently, the time interval depends on the higher expected frequency in the logged signal, the number of samples per test depends on the desired frequency resolution and the number of tests affects the coherence level in the frequency of interest.

The system resonance frequency is experimentally obtained during the test. If the laboratory procedure is performed without any kind of automatic closed-loop control, it becomes a time-consuming process.

An automatic system may solve this problem in different ways:

- Application of a chirp (within a preset range of frequencies and interval). The response amplitude varies with frequency and its peak is the system resonant frequency
- Application of few cycles at a fixed frequency with the measurement of the phase lag between the input and output signals
- Application of a random noise with a frequency content adjusted after a preliminary broadband test. This is possible at low strains level.

V. INFLUENCING FACTORS

The RC test is done under the assumption of linear behavior but in most cases the soil behavior is far from that condition and the transfer function is characterized by a non-symmetric shape.

Nonlinearity is generic in nature, and linear behavior is an exception. In certain cases, the phase lag between input and output signals in resonance does not have the same value as if the system were linear. The theory of non-linear oscillation could be a good framework to develop this type of analysis.

Among the factors affecting soil dynamic response are stress/strain rate, duration of excitation, moisture content, confinement and frequency. Method based on mapping stiffness and damping at each frequency is a classic to detect the presence of non-linearity in a single-degree-of-freedom system.

For the case of the initial shear modulus, it is necessary to exclude from the table the strain amplitude because this factor is constant and kept at the smallest value possible. In addition, the number of cycles of loading is not important, because shear modulus is unaffected by this variable when dealing with small strain ranges.

In general, for soils, increasing the frequency, the shear modulus has a slight trend to increase while the damping ratio does not exhibit a clear tendency. Lai et al. (2001) and Khan et al. (2008) noticed a change of the dynamic properties of soils and evidenced frequency dependence implicit into the dispersive nature of soils.

Biot (1956) first studied the influence of the frequency in a liquid-saturated porous solid introducing the concepts of apparent masses and dynamic coupling between water and solid to explain the variation of shear wave velocity as the wave frequency increases.

At low vibration frequency, the solid particles and the water move together due to the viscosity of the water. As the frequency increases, there is a relative motion between solid and water since the viscous forces are less important in comparison with the inertial forces.

The shear wave velocity increases as the input frequency increases and this effect is more important when the soil permeability is relatively high. The boundary of this region is the "characteristic frequency" defined as the frequency in which the viscous skin depth of the flow matches with the radius of the pores.

About frequency related factors, a work from Shibuya et al. (1995) on cohesive soils, explains that the damping ratio increases at frequencies below 0.1 Hz for the creep effects of soil skeleton, remains constant between 0.1 and 10 Hz due the dominant hysteretic damping and increases above 10 Hz because of the pore fluid viscosity.

Moreover Meza-Fajardo and Lai (2007) demonstrated that phase velocity and damping ratio are inter-dependent and therefore should be simultaneously determined during testing, in opposition to the conventional procedure in which each value is obtained by mean of separated techniques.

VI. STANDARD AND CHIRP METHODS

RC conventional test (RCS) and RC sine chirp test (RCC) were the first methods used for soil characterization.

In the RCS frequency increases in discrete steps with a defined sequence and the specimen resonant frequency is the one with the maximum response amplitude.

The test usually consists of two steps, one with a large range and the following with a small one around the resonance. Each frequency is applied for a fixed number of periods.

RCC uses a chirp linear modulation i.e. a continuous modulation where frequency increases within the limits set by the user.

To improve the performance, the spectrum ripple levels can be reduced introducing short rise and fall times (R-F) at the signal ends. As a result, there is very little power outside the band of interest.

Fig. 6 shows the spectrum of a linear chirp with 4% rise and fall and time-bandwidth of 250.

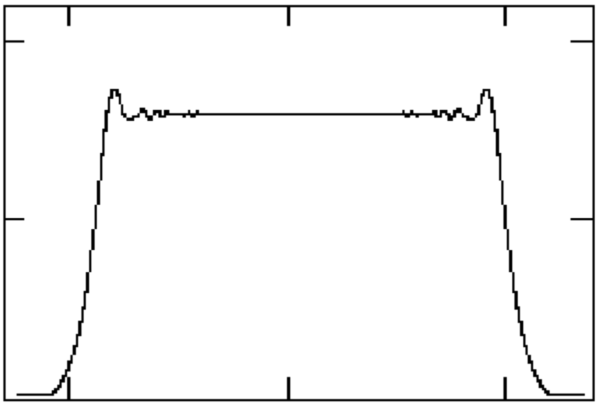

Fig. 6 The chirp spectrum with $T.\Delta f$ =250 and 4% R-F

It is possible to clean up the transfer function representation, that may appears very noisy especially when low strain amplitude are used. The technique used may be a simple non-causal moving average filter applied to FFT samples. As common practice, a generous zero padding maximizes the FFT resolution in the frequency range of interest.

Fig. 7 shows what can be the noise difference between the spectral response using RCC with (the black line) and without the filtering, function (the gray pattern on background).

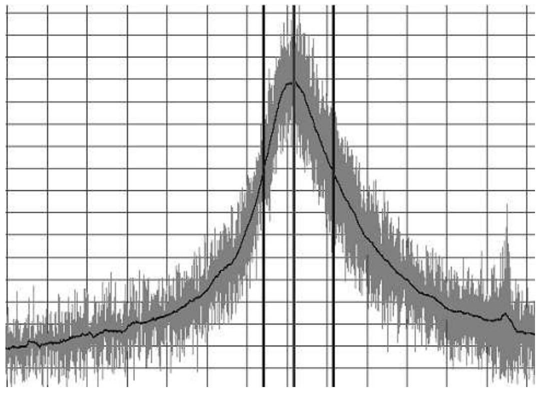

Fig. 7 A low strain filtered chirp RC

VII. RANDOM VIBRATION

The transfer function of a system may be obtained using random noise as excitation. Random vibrations cannot be used for higher levels of strain because the system becomes non-linear, which is the premise of the frequency domain analysis of linear systems.

The test should last an adequate amount of time and requires the adjustment of the input signal by modifying its frequency content according to the first mode of the system.

A torque narrow band random signal applied to the specimen is able to obtain dynamic properties at very small strain where RCS and RCC tests are not accurate. Usually, RC are able to impose shear strain levels from 10-6 to 10-4 and with the use of random noise this limit may be even reduced.

Amini (1995) conducted a study showing that the time effect is less pronounced during random than sinusoidal vibration at the same level of strain. This method could correct the error in the estimation of the damping ratio caused by a low frequency resolution.

Cascante and Santamarina (1996-98) studied the random noise technique and presented a relationship to compute the representative strain level considering that in random vibrations there is not a representative peak of vibrations like in sinusoidal excitation testing.

The representative shear-strain level is still a questionable issue: should be used the RMS, the peak or any other intermediate value? Nevertheless, this subject does not prevent its utilization because soil properties obtained by this method are under the linear range. The shear modulus obtained by random vibrations may be up to 15% greater than values measured by conventional resonant-column testing.

VIII. NON-RESONANT TEST (NRC)

To obtain the dynamic properties the resonant condition is not necessary.

If a harmonic excitation with known frequency (ω) and amplitude (T0) is applied to the system, and the resulting rotation (θ) and phase lag are measured, the only unknown variable is Ω*, since the apparatus constants are previously determined by calibration.

Applying the Newton-Raphson method to the complex values of variables, it is possible to solve the transcendental equation and to compare the dynamic properties at different frequencies.

The Non-Resonance method was first used on the determination of the dynamic properties of polymers and composites (Read and Dean, 1978). In geomaterials, Lai et al. (2001) firstly applied the NRC method finding good agreement if the frequency point is the resonance.

To verify how the dynamic properties are related to the excitation frequency and taking into account the non-linear behavior of soils we need to keep the shear strain constant and lower than the “linear cyclic” threshold.

Fig. 8 presents the ratio between the strain (angle) and the stress (torque) at constant strain versus frequency.

A peak in the ratio appears at the resonance frequency of the system, because at this point it is necessary a small torque to produce the target strain.

To work at different frequencies, the excitation amplitude should be modified to obtain the target strain.

To guarantee constant shear strain, Khan et al. (2008) used a NRC at a single-frequency sinusoidal excitation with initial amplitude of a previously performed RCS, but different frequency.

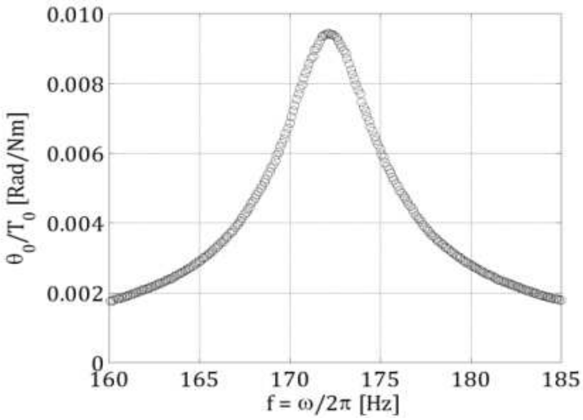

Fig. 8 The ratio between the strain (angle) and stress (torque) at constant strain [7]

They found, in the small strain range, a good agreement in shear modulus and damping ratio results and for large strains that the RCS and RCC method overestimates the damping ratio because the shear strain level is not constant for all the frequencies.

Some studies using non-resonance techniques focus on the effect of the frequency on the initial shear modulus. According to these studies, the shear modulus has a slight trend to increase with frequency probably due to the dispersive nature of wave propagation on soils, which implies that velocity and damping ratio are frequency dependent.

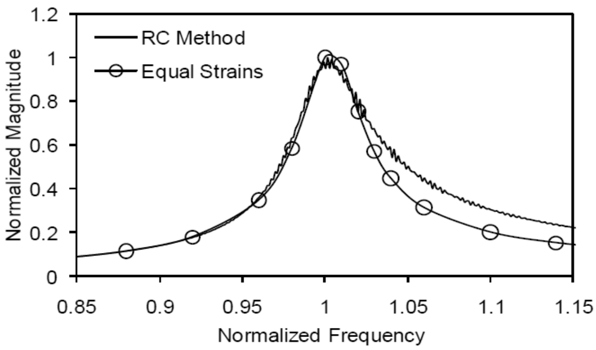

Fig. 9 Transfer function of RC (RCS or RCC) and equal strain (NRC) points [7]

This method consents anyway measurements of shear modulus and damping ratio of soils over a broad frequency range by using a RCTS (combined resonant-column and torsional-shear device).

NRC should be used with caution, because it is very sensitive to small changes in the torque and the phase angle measurements. For high strains, the degradation produced by the repeated cycles contributes to distort the shape of the transfer function.

IX. THE FN METHOD

A new methodology (FN) was proposed by Moayerian (2012) to evaluate the dynamic properties from an equivalent constant-strain transfer function. It consists in a simultaneous excitation with a sine (carrier signal) at the required strain amplitude added with a small amplitude narrow band random noise. Similar techniques were used for the identification of non-linear structural systems.

To obtain variation due to non-linearity in structural testing, Feldman (1994) proposed a method based on Hilbert transform to analyze the systems with excitation under narrow and wide band random excitations and slow and fast sweep sine tests.

In the RC, the strain level induced by the fixed sine controls the resonant frequency and damping coefficient of the specimen whereas the random noise controls the shape of the transfer function.

The representative shear strains estimated from RCS and RCC method are less accurate than the NRC and the FN methods. The variation of dynamic properties with frequency is better estimated by FN method compared to NRC method because this one is sensitive at low strain to slight variations in phase difference and the results become highly unreliable.

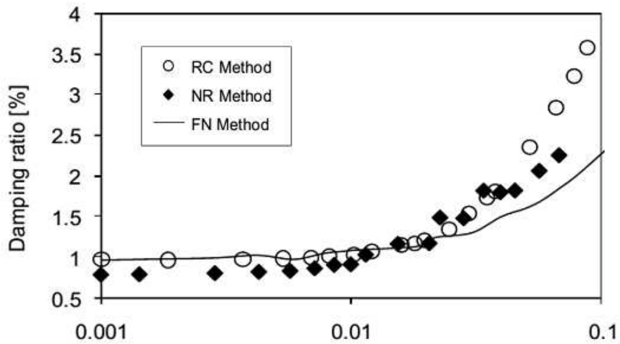

Fig. 10 Evaluation of damping ratio with different methods [20]

The measured changes in the shear modulus is very similar from RCS or RCC, NRC, and FN methods while damping ratio is better estimated by FN as by NRC at large strains.

X. PROPOSED HYBRID METHOD

Starting from these results, we added a software upgraded application to mix the benefit of RCC with those of FN method in our RCTS. This method called RCX consists in an excitation signals combining chirp with strain amplitude control and random noise.

In using this overlap of excitations lies a tradeoff between the length and breadth of chirp sweep and the duration and amplitude of the white noise. A chirp too fast and broad does not permit a sufficient quality of the system model reconstruction by the white noise. In general, the frequency range of both chirp and noise can overlap and should be limited to a variation between a decade under and a decade above the resonant frequency of the system.

This approach may be used to conduct in a relatively short time shear modulus and damping measurement versus frequency at constant strain level, ensuring greater investigation capacity on the soil non-linearity.

The aim is to improve the quality and to reduce the testing time for the investigation of the frequency effect on the dynamic properties of soils in the RC test.

The strain level induced in the specimen by the slow chirp acts on the resonant frequency and on the damping coefficient, whereas the random noise is used to detect the shape of the transfer function.

The implementation of an automatic system avoids accumulating loading cycles that may cause soil degradation especially at elevated strain level. The choice of the right combination of chirp and noise parameters is the key factor to accomplish the test.

This RC test uses equal strain method by feedback of its RMS value.

The first results are encouraging and to facilitate the testing we developed a dynamic straight-line motion simulator illustrated as follows.

XI. THE DYNAMIC SIMULATOR

For the execution of tests in simplified conditions while being able to simulate variations of damping, stiffness and further the inclusion of non-linearity (cubic stiffness, variable damping) we have developed a dynamic simulator with an equivalent axial mechanical SDOF system.

With such a system, it is possible to test identification algorithms and to change easily the excitation thanks to the use of a programming environment for the simulation of complex dynamic systems (Matlab/Simulink®).

The dynamic simulator uses a pair of accelerometers similar to the one used on the column and the resonant system includes a mass, a spring and a damper. The spring and the damper can be easily modified to introduce controlled non-linearities (softening stiffness and variable damping).

For example, the non-linear stiffness may be simulated with a magnet fixed in a position that exerts a symmetric attracting force when defined amplitudes of displacement are reached. The negative stiffness due to the magnetic interaction is exploited to cancel partially the positive stiffness of the spring resulting in a reduced natural frequency.

The test algorithm implemented, using chirp and noise excitation (RTX), identifies the dynamic system that transforms the input stress measured by one of the accelerometers into the output stress measured by the second accelerometer, thus viewing the system as a “transfer function” (or equivalently in a mechanical impedance or transmissibility).

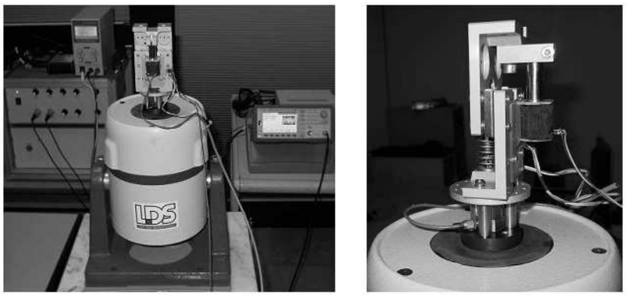

Fig. 11 The dynamic simulator in use

Thinking in terms of cause and effect, or input-output, we can identify the properties of the soil using the size and type of solicitation of input to highlight its non-linearity as described into the present work.

The identification procedure was carried out using Matlab/Simulink® script models. The function used for the first identifications is the PROCEST of the System Identification Toolbox, which estimates the model using a minimization algorithm of the prediction error [17].

XII. CONCLUSIONS

This paper presented a series of methods based on the RC ranging from the conventional method to the non-resonant method.

The force function generated by a sinusoidal sweep excitation induces different shear strain levels at different frequencies and the shape of the measured transfer function differs from the theoretical transfer function for an equivalent SDOF system.

The difference between the measured and assumed transfer functions becomes more apparent with the increase in shear strain levels.

The dynamic analysis of the soil in the laboratory with the RC system can benefit from the new testing techniques and data processing, giving greater flexibility, increased confidence in the quality of the results and the ability to perform parametric studies on the dynamic behavior of soils. The combined RCX method has a good potential for the evaluation of frequency effect on dynamic properties of soils in resonant column testing.

Regarding non-conventional methods, we can conclude that:

- RCC tests are an excellent alternative to perform RC tests. They can substitute RCS tests that generally are more time consuming and induce more loading cycles.
- RCN test allows to obtain dynamic properties at very small strain (<10-6) but is recommended just for small strain testing.
- NRC (non-resonance) method gives a wider point of view about the general behavior of the dynamic properties of the soil as a function of the number of cycles.
- FN method is a good starting point to perform more sophisticated test of the non-linearity of the soil.

The following table shows all the alternative methods considered and the relative limitations according to the literature [7].

Method	**Limitation**
RCS - controlled source (standard)	Too many cycle (soil disturbance)
RCC-sine sweep (chirp)	Some time consuming
RCN - random noise	Only harmonic excitation
NRC - non-resonant	Det. of G and D at resonant frequency
FN - sine and random noise	Shear strain > than 5 ppm - Time consuming
RCX - chirp and random noise	To be defined

Moreover, the use of a dynamic simulator allows a quick testing of new analysis software applications devoted to RC testing systems giving greater confidence in the quality of the work done to face this challenging test.

New RC systems, using different test methods, excitation signals and subsequent elaboration techniques, will ensure greater investigation capacity on the soil in frequency and at smaller strain levels.

To conclude this paper, even if one cannot foresee the arrival of a paradigm for RC test, it can be predicted that a “universal” technique capable of addressing the study of every possible type of soil will not be developed.

Therefore, it is likely that the soil model identification will retain its “toolbox of method” philosophy, with new and more powerful techniques.

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