12th IMEKO TC4 International Symposium
Electrical Measurements and Instrumentation
September 25–27, 2002, Zagreb, Croatia

A NEW ELECTRO-MECHANICAL OSCILLATOR SUITABLE TO METROLOGICAL APPLICATION

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Abstract – An experimental electrometric set-up, conceived as a physical model of a metrological system for electric-watt realisation or Planck constant determination, has been operated in a persistent oscillatory mode. In this condition, the behaviour of a moving element in levitation and in vacuum is simulated. An experimental test of the relation between the frequency of the oscillation and the bias voltage of the fixed electrodes confirms the possibility of including such a relation in the set of equations from which the geometrical constant can be eliminated. Metrological exploitation of the investigated operating principle, is prospected for absolute voltage measurement.

Keywords: harmonic oscillator, volt realisation.

1. INTRODUCTION

The measurement of electric quantities in terms of basic mechanical units has become of primary importance since the introduction of the quantum electrical standards. In particular, the comparison between an electric power or energy with the corresponding mechanical quantities, performed by means of electrodynamic power balances, has played a central role for the determination of the Planck constant [1] and has also been proposed for monitoring the stability of the mass unit, constituted by the prototype kilogram, or even for a realisation of the same unit in case of its possible redefinition.

Dual experiments have been performed with a voltage balance [2], while alternative solutions based on electrometric systems with a levitated moving element in oscillatory motion were proposed [3].

A physical model suitable to verify experimentally the feasibility of those alternative solutions and some other operating modes subsequently conceived was set-up [4-6]. After forced-oscillation operating modes, a free-oscillation mode was considered, where frequency is depending on a bias voltage applied to the fixed electrodes.

More recently, the experimental system has been improved with the introduction of an apparatus for the neutralisation of loss effects, which allows oscillation to be maintained in a steady regime. In this condition, some of the operating modes theoretically conceived for the nearly lossless situation of a moving element in levitation and in vacuum [6] can be experimentally verified.

The oscillation frequency is a parameter conveying quantitative information on the geometrical constant of the system. Therefore, an equation involving frequency can be significant as one of a set of equations upon which a metrological exploitation of the system could be based. Application to absolute voltage measurement is focused in particular, without excluding other outcomes already pointed out [3-6], such as the Josephson and Planck constants determination.

2. THE ELECTRO-MECHANICAL OSCILLATOR

The electro-mechanical resonance of the moving element was already considered within the several oscillatory solutions yielding equations suitable to metrological application [6]. Resonance implies constant-charge operation, for which either the fixed electrodes or the moving electrode must be practically isolated. The connection, already shown in [5], where the moving element is isolated and the fixed electrodes are applied to voltage generators has been further investigated.

2.1. The electro-mechanical resonator

The principle of the resonator is recalled in Fig. 1, which shows only the essential components of the system. The mechanical constraints of the moving element compensate for the effect of gravity and allow movement only along the x axis. Capacitance C_1, C_2 and C_G represent the distributed capacitance of the moving electrode toward each pair of fixed electrodes and ground.

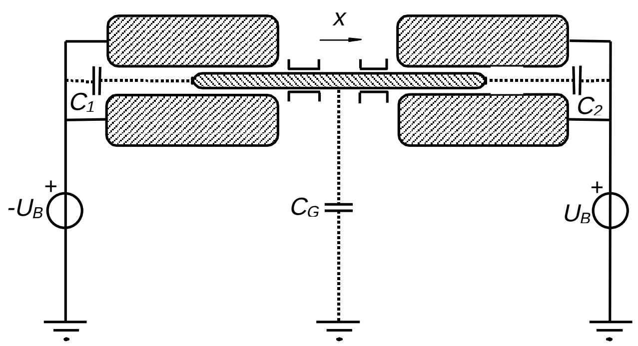

Fig. 1. Principle cross-section representation of the essential components of the electro-mechanical resonator.

The voltage generators drive the two fixed-electrode pair to opposite bias potentials ($\pm U_B$), so that the potential of the moving electrode is zero in the centre position and becomes positive or negative for positive or negative x displacement.

The distributed capacitances C_1 and C_2 depend linearly on x, that is

$$C_1 = \frac{1}{2} C_0 - \frac{\partial C}{\partial x} x \ , \quad C_2 = \frac{1}{2} C_0 + \frac{\partial C}{\partial x} x \qquad (1)$$

where C_0 is the capacitance between the moving electrode in its centre position (x = 0) and all the fixed electrodes.

The potential U of the moving electrode is also linearly depending on x and results

$$U = \frac{k_U}{C_0 + C_G} \ x \ , \qquad k_U = 2 \ \frac{\partial C}{\partial x} \ U_B \qquad (2)$$

The electric force produced on the moving electrode is

$$F = -\frac{{k_U}^2}{C_0 + C_G} x \ . \qquad (3)$$

This means that the electric force performs like an elastic reaction with a pseudo-elastic constant

$$k_E = \frac{{k_U}^2}{C_0 + C_G} \qquad (4)$$

which determines, together with the mass m of the moving element and the damping factor d, a resonance angular frequency

$$\omega = \sqrt{\frac{k_E}{m} - \frac{d^2}{4\,m^2}} \ . \qquad (5)$$

2.2. From resonator to oscillator

In the resonator with a finite value of d, the time function of the displacement is

$$x(t) = \mathrm{e}^{-\frac{d}{2\,m} t} \ x_0 \cos \omega \, t \qquad (6)$$

where ω is given by (5). The (6) derives from the well known differential equation

$$m \ \ddot{x}(t) + d \ \dot{x}(t) + k_E \ x(t) = F(t) \qquad (7)$$

where the forcing term $F(t)$ is zero (free oscillation).

Introducing a forcing term proportional to velocity, that is $F(t) = k_F \ \dot{x}(t)$, one can change the damping factor by adjusting k_F. As a consequence, (6) becomes

$$x(t) = \mathrm{e}^{-\frac{d - k_F}{2\,m} t} \ x_0 \cos \omega \, t \ , \qquad \omega = \sqrt{\frac{k_E}{m} - \frac{(d - k_F)^2}{4\,m^2}} \ . \qquad (8)$$

Thus k_F can be adjusted close to d in order to obtain a steady oscillation at the angular frequency $\omega_0 = \sqrt{k_E / m}$, or even automatically controlled to drive and maintain the oscillation at an assigned amplitude.

2.3. Application to an experimental system

The above principle has been applied to an experimental system which had been developed as a physical model in the form of a balance suitable to oscillate around its centred position. The electrometric device reproduces the principle configuration of Fig. 1 in a cylindrical geometry and with a vertical x axis. In this way, the moving element is already maintained by gravity along its trajectory, to which it could be further constrained by automatic control (at present not yet effective) with capacitance sensors and actuators.

The experimental set-up is outlined in Fig. 2, where a cylindrical moving electrode (ME), of 30 cm in diameter, is suspended by a metal-glass tape to a pulley beam with a counterweight. ME is moving along the vertical axis between a double pair of coaxial fixed electrodes (FE), which are maintained to high-voltage bias potentials + U_B and – U_B for the upper and lower pair respectively. A metrologically traceable mass m, shared between ME and its counterweight, can be added or removed to increase or decrease by a known amount the equivalent inertial mass entering in the dynamics of the system. The vertical displacement of ME is measured by means of a laser interferometer, which also provides a digital output signal with a resolution of 1 nm.

Equations from (1) to (4) still hold, while (5) must be completed with the introduction of a mechanical pseudo-elastic constant k_M. In fact, even if it was possible to reduce k_M to zero by raising the centre of gravity up to the fulcrum, it was found preferable to maintain a finite distance between them, in view of the advantage of operating with a mechanically stable system. Thus (5) becomes

$$\omega = \sqrt{\frac{k_E + k_M}{m_{eq}} - \frac{d^2}{4\,{m_{eq}}^2}} \ . \qquad (9)$$

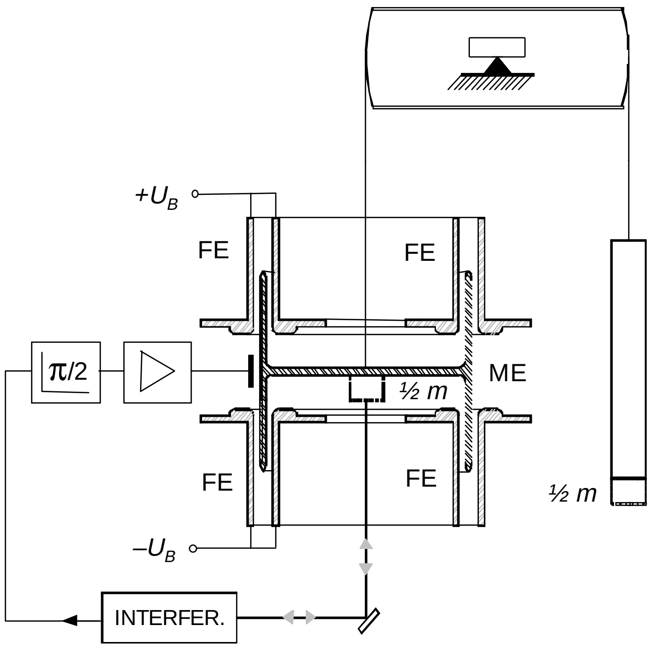

Fig. 1. Simplified cross section of the cylindrical electrodes with the other essential parts of the experimental set-up.

A total equivalent mass m_{eq} has been introduced to account for all the moving parts of the system. It includes, in addition to the mass of the moving electrode with its counterweight and the calibrated mass m, also the effect of the moment of inertia of the pulley-beam.

The oscillatory signal proportional to displacement is rotated by $\pi/2$ rad and applied, after amplification, to the moving electrode by capacitive coupling. The voltage so induced is very smaller than U, as expressed by (2), and in quadrature with it.

3. EXPERIMENTAL TESTS

A first experimental test has been performed to verify the possibility of a damping compensation not affecting the free oscillation frequency. Starting from a merit factor $Q \cong 100$ of the system in free oscillation around the angular frequency $\omega \cong 0{,}5$ rad/s, it was possible to reduce the damping by adjusting the gain applied to the displacement signal until a nearly steady oscillation was reached. This produced only very little frequency variation, as a quadrature voltage does not contribute to the pseudo-elastic constant.

A second test concerned the validation of the model assumed to represent the operation of the system in the particular oscillating mode considered. The model was assumed as consisting essentially of (5), rewritten to put in evidence the effect of U_B and taking into account the compensation of d. That is

$$\omega \cong \sqrt{\frac{4\left(\partial C/\partial x\right)^2}{\left(C_0+C_G\right)m_{eq}}U_B^2+\frac{k_M}{m_{eq}}} \tag{10}$$

A series of frequency measurements were performed with different values of U_B and oscillations of the same amplitude (8,5 mm). The experimental data are reported in Fig. 3 together with the fitting curve derived from (10) with convenient values of the parameters.

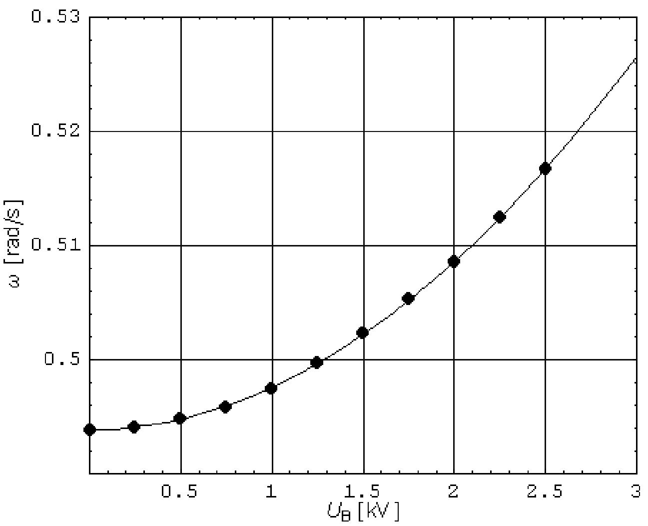

Fig. 3. Experimental results of frequency measurements at different bias voltages. The data are fitted using (10).

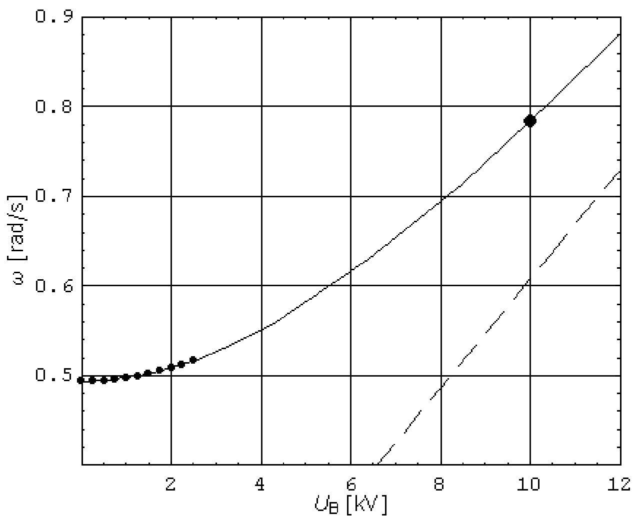

Fig. 4. Extension of the fitting function beyond the limit imposed to the measurements by the stability condition. The target operating point made possible by the trajectory and attitude control is also shown together with the characteristic be obtaineble with a purely electric pseudo-elastic constant (dashed line).

The bias voltage U_B was limited because the automatic control of the trajectory and attitude was not yet active and a higher voltage would have made the moving electrode unstable. Fig. 4 reports an extension of the fitting function beyond the limit imposed to the measurements with an indication of the target operating point, corresponding to 10 kV. The dashed line indicates the behaviour of the same system where only the electric pseudo-elastic constant would be active ($k_M = 0$). One can evaluate that the same electrode configuration, when operated at 10 kV develops a pseudo-elastic constant $k_E \cong 4{,}2$ N/m.

4. METROLOGICAL EXPLOITATION

Different from the experimental model described above, a system designed for high-accuracy metrological application should have all parasitic forces of mechanical nature reduced to negligible levels. This is intended to be obtained replacing the balance mechanism with a system in vacuum and in electric levitation, where the mechanical constraints are avoided in favour of virtual constraints obtained by a contactless automatic control [6]. A conceptual drawing of such a system, again of cylindrical geometry but with horizontal axis, is reported in Fig. 5, which gives a top view of its essential parts.

In such a system, all forces of mechanical origin in the direction of the movement (x axis) should be reduced to negligible levels and the damping-compensation apparatus could be conveniently used to control the oscillation amplitude.

In the situation outlined above, (10) reduces to

$$\omega = \frac{2\left(\partial C/\partial x\right)}{\sqrt{\left(C_0+C_G\right)m_{eq}}}\ U_B = \frac{k_U}{\sqrt{\left(C_0+C_G\right)m_{eq}}} \tag{11}$$

On the other side, the potential U of ME oscillates in phase with x and, for (2) and (6), its time function is

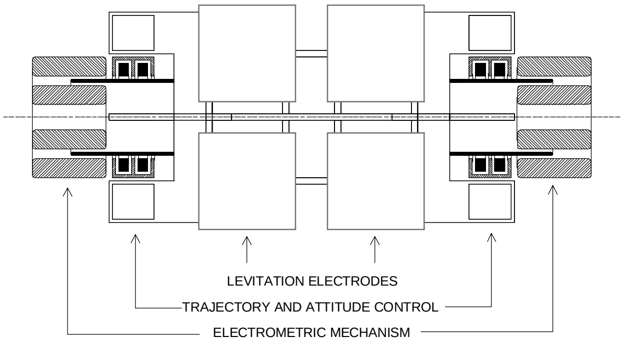

Fig. 5. Conceptual drawing (top view) of an electrometric system conceived for high-accuracy metrological use. The cylindrical geometry of the main electrodes has a horizontal axis, which is also the axis of movement (*x* axis).

$$U(t) = \frac{k_U}{C_0 + C_G} x_0 \cos \omega t \qquad (12)$$

From (11) and (12), one obtains

$$U(t) = \omega \sqrt{\frac{m_{eq}}{C_0 + C_g}} x_0 \cos \omega t \ . \qquad (13)$$

That is, the electro-mechanical oscillator produces a voltage only related to a mass, a displacement, a frequency and the total capacitance of ME toward its environment.

More conveniently, instead of the whole moving mass of the system, the measurement would be referred to a metrologically qualified mass *m* to be added to the moving element, as indicated in the system of Fig. 2. In the system of Fig. 5, *m* does not need to be equally shared between ME and its counterweight, but can be simply loaded on the moving element or removed from it.

If T_1 is the period of the oscillation with *m* loaded on ME and T_0 is the period with *m* removed, one obtains

$$U(t) = \frac{2\pi}{\sqrt{T_1^2 - T_0^2}} \sqrt{\frac{m}{C_0 + C_G}} x_0 \cos \omega t \ . \qquad (14)$$

Thus, from two measurements, a voltage is related to mechanical quantities through a capacitance. In practice, the oscillation frequencies can be evaluated from a sufficient number of data obtained by interferometer sampling; from the same data, also x_0 can be derived by interpolation around the maximum and minimum displacement; *m* can be related to a reference mass standard with proper mass-comparison techniques; and $C_0 + C_G$ can be measured as steady capacitences by reference to fixed standards. It must be noted that such capacitances are depending on the trajectory and attitude control, but due to their cylindrical geometry the sensitivity to eccentricity is small around the centred position and a definition even at the 10^{-9} level should be well within the possibility of the control.

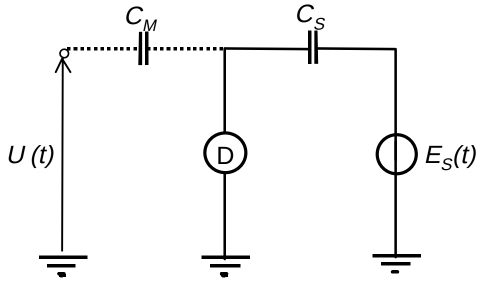

Fig. 6. A convenient measurement technique suitable to access to the voltage of ME without any loading effect when the null detector D is zeroed. E_S is a reference generator synchronous with $U(t)$, C_S is a standard capacitor and C_M takes the place of C_G or part of it in all the relations above.

A special technique is required by the measurement of $U(t)$, the access to which is not trivial. A convenient solution is believed to be the one outlined in [5] an recalled in Fig. 6. The current trough capacitance C_G or a part of it is conveyed to a null detector (D), where is balanced by the current produced by a reference generator (E_S), synchronous with $U(t)$ but with opposite phase, and a reference capacitance (C_S). It results

$$U(t) = E_S(t) \frac{C_S}{C_M} \qquad (15)$$

Where C_M can be coincident with the whole capacitance C_G, or a part of it, in all the relations above and the ratio C_S / C_M must be measured in steady condition.

Some difficulty could arise for the reference generator to be synchronised with the oscillation of the moving element. However, if it is a digital generator, once adjusted for a null on D it could also contribute to the frequency measurement.

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