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APPLICATION OF ACTIVE COMPENSATED FOUR-COAXIAL CABLES TO AC RESISTANCE STANDARDS COMPARISON

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Abstract – The general features of active connection structures based on four-coaxial cables are summarised and their application to four-port resistance standards to allow them to be compared by means of a conventional transformer bridge is shown. As a test of the measurement method, a comparison between the measurements performed at IEN and at the CTU on a well characterised resistance standard with nominal value equal to the quantised Hall resistance at the i = 2 plateau (12 906,4 Ω) has been carried out. Some details about the reference resistor, the calculable resistors and the measuring systems are reported together with the results of the comparison.

Keywords: impedance measurement, multiconductor coaxial cable, ac standard resistor.

1. INTRODUCTION

The features of multiconductor coaxial cables had been systematically analysed from the point of view of their application for voltage and current transmission within high-accuracy measuring systems [1]. Starting from the usual two-conductor coaxial cable, the analysis is extended to the three-coaxial (triaxial) and four-coaxial cables. In particular, it was shown as the four-coaxial cable solves almost ideally the problem of avoiding the effect of the resistance of both the inner and outer conductors, while it does not necessarily reduce the effect of inductance. It was also proved that some compensation could be applied to the four-coaxial cable in order to eliminate the inductive effects, so reducing the voltage and current transmission errors to negligible levels. The analysis was performed both theoretically and experimentally, with results in good agreement. Furthermore, the advantages of using compensated four-coaxial cables to reduce the uncertainty in measurement of four-port impedance or admittance standards was pointed out.

More recently, new compensation techniques based on symmetry with respect to the centre cross section of the cable have been considered. This allows voltage and current transmission with very negligible errors even well beyond the low-frequency band [2].

As an additional advantage of the four coaxial cables, the possibility of obtaining compensation of any load effect on voltage or current sources has been investigated. Some solutions have been found combining active devices with the compensated cables.

Preliminary experimental tests of some theoretically identified solutions including feedback controlled generators were performed in view of application to measuring systems for comparison of admittance standards.

At last, actual application of the active connecting structures to a transformer bridge for capacitance or resistance comparison until the upper limit of about 10 nF or the lower limit of 10^4 Ω, have been tested.

In particular, resistors with values corresponding to the quantum Hall resistance on the i = 2 plateau were equipped with active connecting structures both at the voltage and the current defining port and comparison measurement were performed.

Because an alternative method of comparable accuracy was not available, the new technique was tested by comparing a quadrifilar resistor, with known AC-DC difference, with a resistance standard previously used as travelling standard in an international comparison of AC resistance already concluded.

2. THE ACTIVE CONNECTION STRUCTURES

The new connection structures [2] are substitutions for the normal coaxial cables connecting the four-port of an admittance standard to ratio transformers or to detectors within the measuring system.

2.1. The four-coaxial cable

As a transmission line, the four-coaxial cable is essentially a coaxial line with the addition of two guard conductors, for the inner and the outer respectively.

A schematic representation of such a line is given in Fig. 1, where the simplest connection is also shown in its two reciprocal uses: for voltage transmission to an open-circuit termination (a) and for current transmission to a short circuit termination (b).

In the voltage-transmission line, a voltage generator is applied both to the transmitting conductors (3 and 0) and the

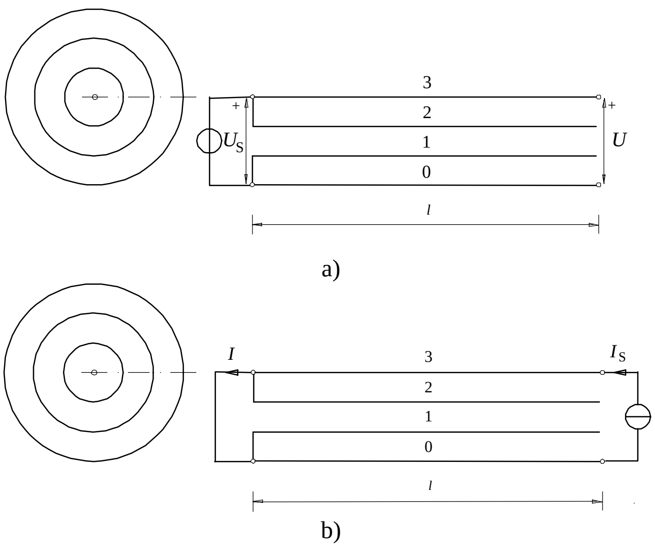


Fig. 1. Schematic representation of a four-coaxial line of length *l*, related to its cross section, showing the connections suitable to its two reciprocal uses for voltage and current transmission. In a) the voltage imposed by a voltage generator (U_S) is transmitted into the open-circuit voltage U; in b) the current imposed by a current generator (I_S) is transmitted into the short-circuit current I.

guard conductors (2 and 1). The capacitive current essentially flows only between and in the guard conductors. So the relative deviation $\varepsilon = (U - U_S) / U_S$ due to transmission is independent of the resistance of conductors 3 and 0, but depends on the mutual inductance between the loop formed by these conductors and that formed by conductors 1 and 2. Namely, neglecting second-order terms, it is

$$\varepsilon = \tfrac{1}{2}\omega^2 C_{12}\ L_{12} l^2 - \mathrm{j}\,\omega\, \tfrac{1}{2}\, G_{12}\ L_{12} l^2 \qquad (1)$$

where all parameters are intended per unit length and R_{03} is the series resistance of conductors 0 and 3, G_{12} and C_{12} the parallel conductance and the parallel capacitance between conductors 1 and 2, L_{12} the inductance of the loop formed by conductors 1 and 2, which is equal to the mutual inductance between the same loop and the one formed by 0 and 3, and ω the angular frequency.

In the current transmission line, a current generator is applied to conductors 3 and 0, while the guard conductors are maintained at the same potential by the short-circuit. So no capacitive current flows between conductors 3 and 0 due to the potential difference between those conductors produced as voltage drop on their resistance. Nevertheless, a voltage between conductors 1 and 2 is produced by effect of the mutual inductance between the loop formed by these guard conductors and the loop where the current flows. This produces a current leakage between the guards which makes I deviate from I_S by an amount $\varepsilon = (I - I_S) / I_S$ again given by (1).

Thus, compared with an ordinary coaxial cable, a four-coaxial cable produces an error with similar in-phase component but negligible quadrature component. The actual values of the relative deviations calculated with the measured values of the cable parameters, are plotted in Fig. 3, where the thick lines are referred to 1 m of a specifically constructed four-coaxial cable, with a 8 mm diameter, and the thin lines to the same length of an usual RG-58 coaxial cable.

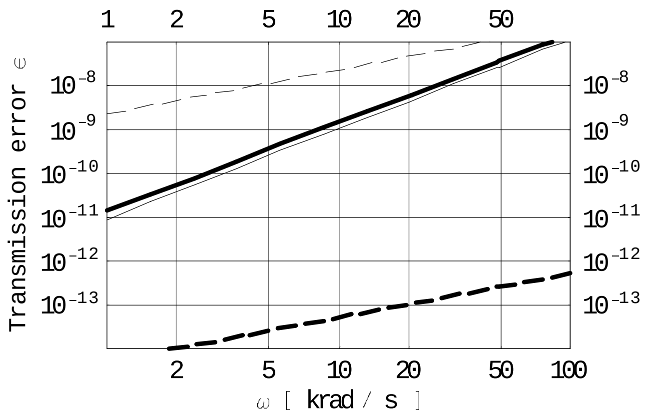


Fig. 2. Voltage or current deviation due to transmission by the structure of Fig. 1 for l = 1 m (thick lines) compared with those of a RG-58 coaxial cable (thin lines). The solid and dashed lines refer to the absolute values of the in-phase and quadrature components respectively.

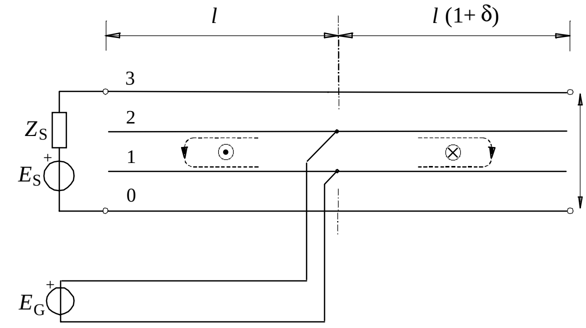
2.2. Active compensation of four-coaxial cables

The in-phase deviation (solid line in Fig. 2) of the four-coaxial cable can be reduced to negligible levels in some different ways. In [1] the inductive effect producing the in-phase deviation was corrected by a lumped compensating device. In [2] a compensation obtained by symmetry was considered and further developed in the subsequent applications. It consist in a combination of two equal pieces of cable, connected in such a way that inductive effects have opposite signs.

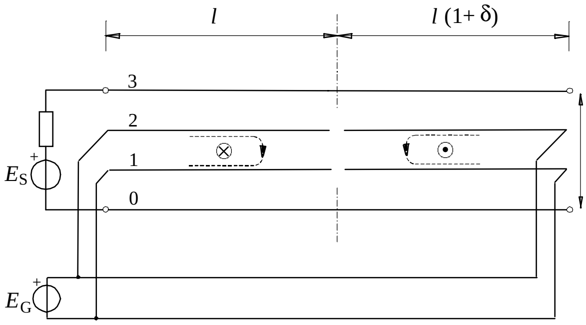
As an additional feature, compared to the two-conductor cable, the presence of guard conductors allow us to avoid loading effect at both ends of the connecting structure, which means no current in voltage transmission and no voltage in current transmission. Of course, this implies the introduction of active devices to supply the necessary power absorbed by the cables.

Two examples of active compensation of a voltage-transmission structure by symmetry are shown in Fig. 3, where an auxiliary voltage E_G is introduced to drive the guard conductors to potentials near those of the respective guarded conductors. The distributed currents (essentially due to capacitance), the symmetric paths of which are indicated in the figure, produce magnetic fluxes with opposite senses in the two halves of the structure, so that the equivalent mutual inductance between the total current and the loop formed by conductors 0 and 3, corresponding to L_{12} in (1), is close to zero. As a result, the in-phase component of ε is strongly reduced, if the relative deviation δ from symmetry is small, and the same is also for the voltage deviation due to loading effect on the source impedance Z_S.

Corresponding examples of active compensation of a symmetric current-transmission structure are outlined in Fig. 4, where the resistive and inductive voltage drop produced by the transmitted current is compensated by the auxiliary voltage source E_C. This produces a null on the detector at the end where the current source is applied and



a)



b)

Fig. 3. Examples of active compensation of a voltage-transmission structure by symmetry. The power absorbed by the line is supplied by an auxiliary voltage generator (E_G). The anti-symmetric configuration of the magnetic flux, produced by the distributed currents between the guard conductors, reduces close to zero the total induced voltage in the loop formed by the inner conductor (3) with the outer conductor (0).

prevents leakage current through Y_S. At the opposite end, the voltage is maintained to zero by the short circuit, which in practice could also be a virtual short circuit obtained by a null detector. In any case, it is assumed that conductors 1 and 2 are maintained at the same potential both in the configuration of Fig. 4 a) and in that of Fig. 4 b).

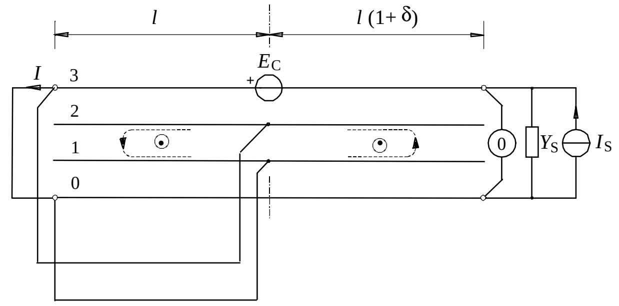
There would not be any leakage of current between conductors 1 and 2, unless for the voltage distribution induced by magnetic flux. Such distributed voltages increase linearly from the centre to both ends in configuration a) and from the ends to the centre in configuration b). In both configurations, the consequent distributed currents have opposite senses in the two halves of the structure, so that compensate for symmetry. Of course, any leakage of current between conductors 3 and 2 or between conductors 0 and 1 do not make I deviate from I_S.

A mathematical model has been set up on the basis of an equivalent circuit with lumped elements and the most important terms have been verified to be the same appearing in (1) but multiplied by the relative symmetry deviation. That is

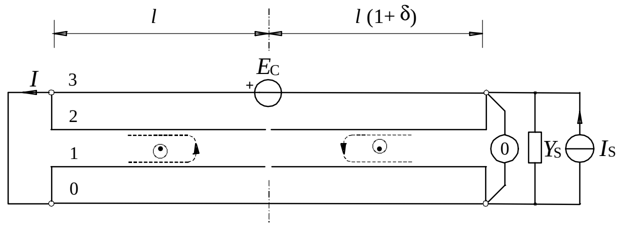
$$\varepsilon = \omega^2 C_{12}\ L_{12}\ l^2\delta - \mathrm{j}\,\omega\, G_{12}\ L_{12}\ l^2\delta \qquad (2)$$

for both the structures of Fig. 3 and Fig. 4.

Considering the graphics of Fig. 2, one can see that a symmetry corresponding to a δ of a few percent is sufficient to reduce ε to very negligible levels.



a)



b)

Fig. 4. Examples of active compensation of a current-transmission structure by symmetry. The power absorbed by the line is supplied by an auxiliary voltage generator (E_C). The anti-symmetric configuration of the distributed currents, produced between the guard conductors by the magnetic flux due to the transmitted current I, reduces close to zero the total leakage current between the inner conductor (3) and the outer conductor (0).

2.3. *Application to a four-port admittance standard*

In the application to the connection of a four-port admittance standard to a transformer bridge, a further feature was added to the voltage-connecting structure in view of using the auxiliary voltage source to supply both the cable guards and the admittance standard.

To this purpose, in addition to the auxiliary voltage source E_G, automatically controlled incremental voltage sources have been developed and located, with all the related electronic circuitry, in the middle of the connection structures. Special attention has been paid to the problem of making the electronic devices and circuits completely floating and guarded by means of multiple shielding enclosures and injection transformers.

The application of an experimental version of a voltage and a current active connection structure to a four-port admittance standard is shown by the simplified schematic diagram of Fig. 5, where he auxiliary voltage E_G is obtained from the first sage of the main two-stage transformer of the bridge

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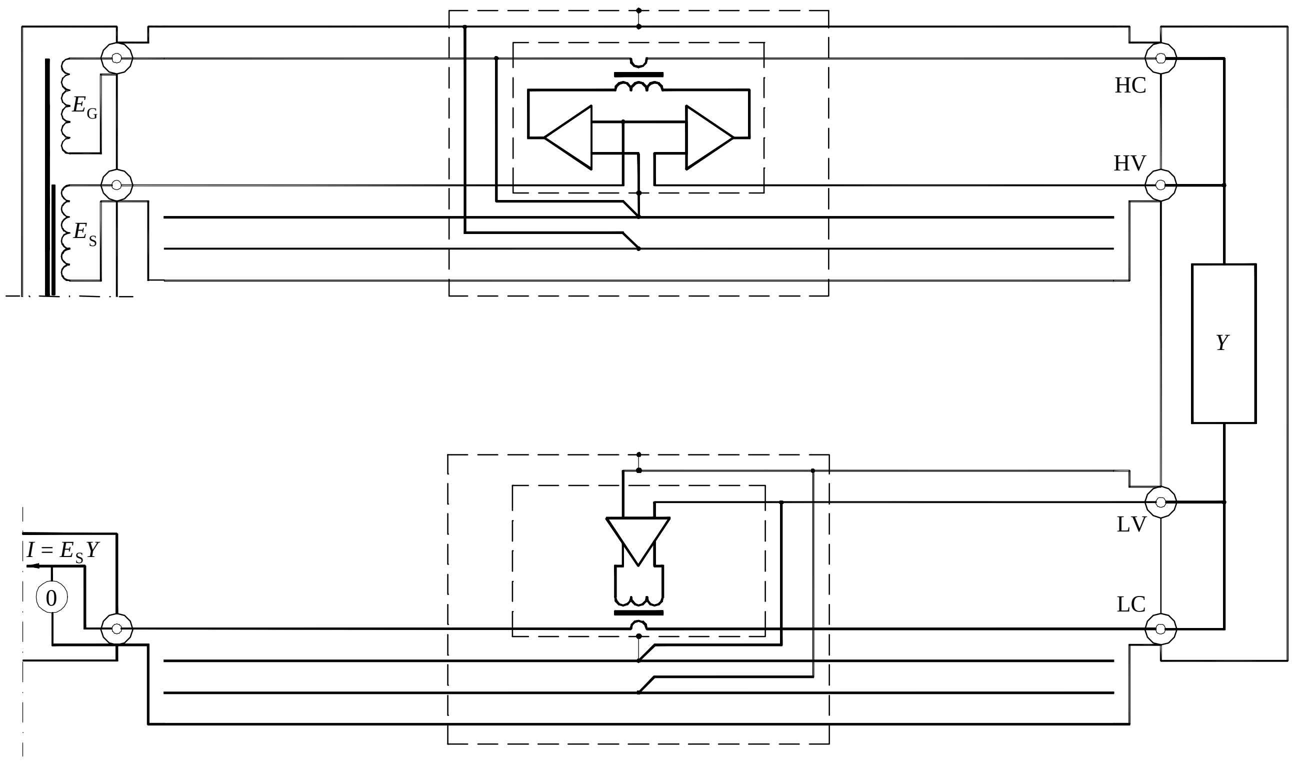

Fig. 5. Schematic diagram showing the connection of one of the two admittance standard under comparison to the relevant part of the transformer bridge by means of the active structures (only one half of the cable longitudinal sections are represented for clarity).

On the high side of the admittance standard (Y), the difference between the voltage at the potential port HV and the voltage E_S at the ratio-reference port of the main transformer is detected through the inner conductor of the four-coaxial cable and feedback injected into the connection between the current port HC of the standard and the rough-voltage port of the transformer, where a voltage $E_G \approx E_S$ is present.

In the low-side connection of Y, the two intermediate conductors of the four-coaxial cable are connected to the low voltage port LV of the standard and are maintained at the same potential by feedback. Therefore, and due to symmetry, the current leakage along the cable is eliminated and both the main detector and the low voltage port are maintained close to zero.

3. EXPERIMENTAL TEST

As a significant test of the actual performance of the new devices, particularly intended to verify the possible presence of unaccounted systematic source of uncertainty, a series of measurements was performed with well characterised standards. They were: a quadrifilar calculable resistor, with a value close to the quantised Hall resistance on the second plateau, to be adopted at IEN as primary reference standard for ac resistance, and a four terminal-pair thermostated reference standard of ac resistance, with the same nominal value of 12,906 kΩ, provided by the Czech Technical University (CTU).

3.1. The reference standard resistor

In the CTU standard resistor, a Vishay S 102 K resistor forming the resistive element has a 24-hour stability of the dc value better than 2 parts in 10^8. A 100 Ω platinum resistance thermometer, independent of the temperature control sensor, allows the inner temperature to be monitored by an external measuring system.

The standard resistor had served as one of the travelling standards in the EUROMET No. 432 comparison and its frequency dependence was measured by five laboratories in Europe and one laboratory in Canada. At each of these laboratories, a four-port transformer bridge was used to measure the frequency dependence by comparing the standard with resistors of calculable frequency performance. Calculable resistors of several different designs (monofilar, bifilar, quadrifilar and octofilar resistors) were available.

In order to make the measurement possible by means of a 1:1 bridge, a caclulable resistors of 12,906 kΩ had been prepared at the CTU. At first, a Gibbings's type quadrifilar resistor had been constructed, where the resistive element had the form of a double loop arranged to make the current flow in opposite directions in its two halves [3]. Then, to obtain an even shorter resistive element and also to make thermoregulation less difficult, an octofilar resistor had been fabricated [4]. Mathematical modelling of both resistors, based on a uniform transmission-line approach, confirmed the expected lower frequency dependence of the octofilar resistor [5].

The results of the comparison are summarised in Table I, where r_{mean} is the arithmetic mean of the relative ac-dc differences of the parallel equivalent resistance of the travelling standard obtained at the listed frequencies (f) by the laboratories participating in the comparison; r_{CTU} is the value of the same quantity obtained at the CTU; $U(r_{mean})$ and $U(r_{CTU})$ are the corresponding expanded uncertainties for a coverage factor of 2. All the quantities, excepet frequency, are expressed in parts in 10^8.

The variation range of ac-dc differences of the parallel equivalent resistance measured by the different laboratories, is better than 9×10^{-8} at 500 Hz and 58×10^{-8} at 5 kHz. The results of 8 measurement runs performed at the CTU over an extended period of 2,5 years did not indicate any systematic change in the frequency characteristic of the travelling resistor.

All comparisons have been made by means of coaxial transformer bridges equipped with sets of current equalizers. Effectiveness of the current equalizers has been tested and results of these tests have been taken into account in uncertainty calculations. In view of the fact that the travelling resistor has a relatively large time constant, attention has been paid to the calibration of the quadrature circuits used in the balancing injection and to the evaluation of corrections and uncertainties corresponding to possible deviations from quadrature.

TABLE I. Frequency dependence of the parallel-equivalent resistance of the travelling resistor. All relative ac-dc differences and their uncertainties are in parts in 10^8.

f / Hz	r_{mean}	$U(r_{mean})$	r_{CTU}	$U(r_{CTU})$
500	- 42,4	4,2	- 39,3	14,8
1 000	- 83,0	4,4	- 78,4	15,9
1 500	- 124,4	4,6	-117,5	17,6
2 000	- 164,8	5,0	-156,6	19,4
3 000	- 245,4	5,9	- 234,4	23,6
4 000	- 324,1	7,4	- 311,9	28,0
5 000	- 401,8	9,4	- 389,1	32,7

3.2. Measurement results

The reference resistor that served as a travelling standard in the international comparison was measured at the seven frequencies indicated in Table I by 1:1 comparison with the IEN quadrifilar calculable resistor. The results are reported in Fig. 6 together with the results obtained at the CTU and the mean of all results.

The results are within the range of those obtained by the laboratories participating in the intercomparison and particularly in good agreement with one of them (NRC, Canada). Furthermore, the residues from the second order fitting curve appearing in the figure are all less than 2×10^{-8}.

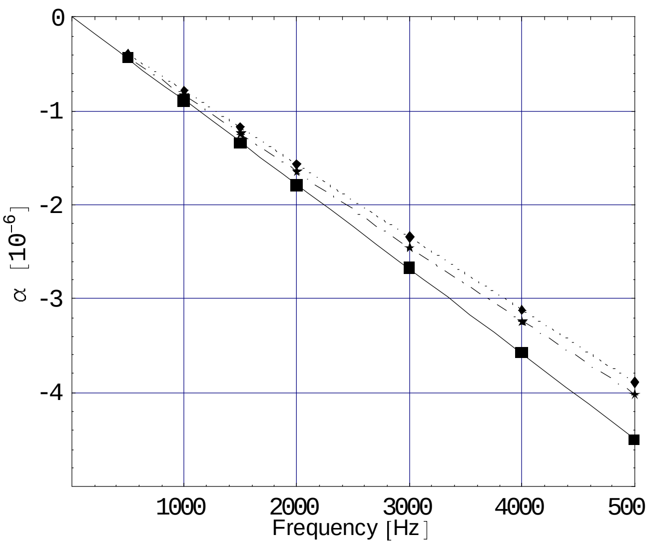

Fig. 6. Diagram vs. frequency of the relative ac-dc difference of the parallel-equivalent resistance of the travelling resistor evaluated with respect to the IEN quadrifilar calculable resistor (box marks). The results of the CTU (dash-dotted curve) and the mean of all results (dotted curve) are also reported.

5. CONCLUSIONS

The adoption of connecting structures of new conception allows the optimal voltage and current transmission features of four-coaxial cables to be exploited and a classical transformer bridge to be applied also to four-port defined resistors. Measurements on a temperature controlled ac resistor, used as travelling standard in an international comparison, has proved that the accuracy of the measuring system and of the resistor with calculable ac-dc characteristic is comparable with those obtained by the other laboratories with different systems.

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