12th IMEKO TC4 International Symposium
Electrical Measurements and Instrumentation
September 25−27, 2002, Zagreb, Croatia

NOISE CHARACTERISTICS IN ELECTRICAL MEASUREMENTS: METROLOGICAL APPROACH

Valery A. Granovsky, Tatiana N. Siraya

Central Scientific Research Institute “Elektropribor”, 197046, St.-Petersburg, Russia

Abstract − Metrological approach to the analysis of noise is presented. It is based on the time-domain approach for random processes, which is valid both for stationary and non-stationary cases. The basic functional model is proposed as reproducing kernel (RK) Hilbert space H(R), produced by the correlation function R(s, t) of random process x (t). This space with the specific inner product provides an isomorphic representation of the random process x (t).

Several typical models of noise are suggested, including White noise, Brownian motion, Markovian, and 1/f – type processes. The corresponding RK-Hilbert spaces are studed. These RK-norms are recommended as the main characteristics for the noises, which are subordinated to the corresponding typical processes.

In particular, the set of the norms and estimates includes both well-known characteristics, such as sample variance and Allan variance, and some new characteristics, such as “Markovian” norms. The case is also considered when several norms (characteristics) from this set are valid. Thus one can employ several norms simultaneously.

Keywords: noise, random process, noise characteristic.

1. INTRODUCTION

Various kinds of noise are observed in different physical, technical and biological systems. But these are particularly important for electrical systems and devices [1-2].

An evidence of this field significance is that the noise problems have been one of major topics at numerous conferences and symposia, and from 1968 there were regularly organized symposia, which solely dealt with noise; for instance, see [3-5].

Moreover, there were special conferences on the problems of 1/f – noise; for instance, see [6].

The results of the noise properties investigations are presented, including:

- phenomenological descriptions of the fluctuations and noise, according to physical events and devices;
- various theoretical models of fluctuations and noise, which take into account more or less completely the real conditions and noise properties;
- analysis of the noise models agreement with experimental data and other theoretical models.

It is essential that electrical noises occupy a distinguished place on the topic, as they are the most general and universal ones according the sources, properties and consequences. Therefore, the main results obtained for electrical noises usually keep justified for the much wider range of applications.

Unfortunately, we have to admit, that metrological aspects of the topic are still less investigated. The metrological approach is concerned with the formulation and study of the quantitive characteristics of the noise, which are useful for the measurement errors estimation.

When dealing with the noise, there are two main aims; these are as follows:

- noise suppress or reduction;
- useful (valid) signal extraction from noise.

Certainly, these two tasks are closely interdependent, but there are some distinctions in methods and tools.

Nowadays, in practice noises are usually considered as stationary processes, which are investigated using spectral methods. Thus essential models are based on the spectral representations of the correlation functions [1]:

$$R(s-t) = \int exp\{ i(s-t)\lambda\}\, dF(\lambda). \tag{1}$$

So the spectral function $F(x)$, and spectral density $f(x) = F'(x)$ reveal as the most important characteristics of noise. Therefore, definitions and estimations of the noise characteristics are also based on the spectral models.

The noise reduction task, mentioned above, is mostly expressed in spectral terms. On the other hand, the task of the useful (valid) signal extraction from noise is not so closely linked with the spectral model. The term “extraction” is interpreted in wide sense; in particular, it includes the estimation of systematic errors in order to insert the corrections.

In practice, many noises are significant, which do not allow any stationary representation; hence non-stationary models are to be applied [1, 7]. Several kinds of generalized spectral representations are proposed for non-stationary random processes, but there are considerable limitations for their application.

For instance, there is a class of harmonizable processes, which have the generalized spectral representation:

$$R(s, t) = \int exp\{ i(s\lambda+t\nu)\}\, dF(\lambda, \nu). \tag{2}$$

But they are not very useful for practice, as in this case spectral mesure $dF(\lambda, \nu)$ proves to be two-dimensional. Therefore, Fourier transform lack its valuable properties, which are essential for use in stationary case.

The problem of non-stationary signals and noise characterisation and estimation is of great importance for the metrology in general, but it is of prime significance for the noise topic. Thus there is also an urgent necessity to extend the set of noise characteristics to represent the time-domain properties. The latter tendency is clearly confirmed by the expanded use of Allan variance as an estimate of data scatter [8, 9]. Consequently specific noise characteristics are to be investigated, with the development of the methods for their estimation.

In this way, the noise investigations include the following topical questions:

- revealing the noise structure and development of typical noise models;
- definition of the parameters and functionals (norms) within the established models;
- development of the estimates for the selected parameters and functional;
- development of the recommendations for the practical use of estimates.

Studing these problems is just the primary aim of this paper. In particular, the time-domain approach is used for the investigation of the noise, irrespective to stationary property. The main noise characteristics are defined and studied, based on this approach.

2. METHODS AND RESULTS

2.1. Basic Model – Functional RK-Hibert Space

Originally, the noise is considered as the random process x(t) on the interval [0, T]; in general, it is non-stationary one. First of all, this process is characterized by the correlation function R(s, t), s, t $\in$ [0, T]. The time-domain approach for the investigation of the process is mainly based on its correlation function.

So the basic mathematical technique is based on the functional Hilbert space H(R) [10], or RK-space.

Hilbert space H(R) consists of the functions on the interval [0, T]. It is constructed with the use of the reproducing kernel R(s, t), which is the correlation function of random process x(t), t $\in$[0, T], and it contains all the functions of the form: $R_t = R(\cdot, t)$,

Functional space H(R) proves to be so useful for investigation of the random process x(t), owing to the fact, that there is an isomorphic representation

$$F: H(x) \rightarrow H(R). \tag{3}$$

Isomorphism (3) converts random value x(t) into the corresponding function $R_t = R(\cdot, t)$, belonging to the space H(R).

The inner product in RK-space H(R) is defined by the condition, that for any function g $\in$ H(R) and any point t$\in$[0, T] the product

$$(g, R_t)_R = g(t) \tag{4}$$

gives just the value of the function g in the point t.

So the given reproducing kernel R(s, t) just defines the specific metric in the space H(R), produced by the random process x (t):

$$(R_s, R_t)_R = (x(s), x(t)) = R(s, t). \tag{5}$$

Within the framework of RK-Hilbert space H(R) several specific parameters and functionals can be defined, which may be used as the noise characteristics. The most important ones are the norms in the spaces H (R) for various correlation functions (or kernels) R(s, t).

In practice, there are many tasks concerning random processes, which are formulated as the linear problems in Hilbert space H(x), generated by the random values according mean square norm. In particular, these are problems the following ones:

- linear filtering,
- extrapolation,
- deterministic signal extraction from noise.

Owing to the isomorphism (3), the RK-space provides a ready mathematical tool for solving all the mentioned problems in the well-defined functional space. The level of this tool efficiency depends on the expediency and ease of the major operations in the RK-space. So it is practically important to construct and use the RK-space with the simple rules of operation.

For many major classes of processes, which are to be studied in the first instance, the RK-spaces may be presented in direct form. Apart from the stationary random processes, it is valid for the non-correlated process (white noise), Brownian motion, Markovian and some other processes.

Firstly, for the stationary processes RK-representations are directly related with the spectral ones; in some sense, these two kinds of models are quite equivalent.

In particular, if x(t), t $\in(-\infty, \infty)$, is the stationary process with spectral density f(λ), then RK-Hilbert space H(R) consists of the Fourier transforms of $L^2(f)$ - square-integrable functions with the weight f:

$$H(R) = \{g: g(s) = \int exp(-is\lambda)\, h(\lambda)\, d\lambda,\ h \in L^2(f)\} \tag{6}$$

with

$$L^2(f) = \{h: \|h\|_f^2 = \int |\, h(\lambda)\,|^2 f(\lambda) d\lambda < \infty\}. \tag{7}$$

Thus, all the "spectral" results concerning stationary processes may be simply reformulated in terms of RK-space.

On the other hand, for some cases the RK-spaces have rather simple form. In particular, if $x_1, \ldots, x_n$ is non-correlated time series (or random sample), then RK-space is just l^2-vector space with norm

$$\|x\|^2 = \Sigma\, |x_k|^2. \tag{8}$$

In the case of generalized white noise process, Hilbert space H (R) is just the space of the square-integrable functions L^2 on the interval [0, T]:

$$H(R) = \{g: \|g\|^2 = \int |\, g(t)\,|^2 dt < \infty\}. \tag{9}$$

Properties of RK-representations, which are mentioned above, clearly demonstrate the advantages of these models, so the latter are worthy of note and wider use in practice. An important benefit of RK-representations over spectral ones is that the former are applicable for both stationary and non-stationary processes; and they are also valid for generalized random processes (such as white noise or flicker noise). They are extremely simple and useful for the white noise, Brownian motion and processes like those.

But there are naturally some imperfections in the RK-models, which are clearly seen in comparison with spectral ones. The spectral representation has rather general and unified form, and all the operations are quite determined by the well-known properties of the Fourier transform. On the other hand, RK-space is too indivilual and specific. It is so closely linked with the particular kernel R(s, t), that the intrinsic metric in RK-space is usually of rather special form. In particular, the basic functions $R_t = R(\cdot, t)$ may be of complicated form.

So the merits of RK-space, such as individual character and direct presentation of the process values, turns to disadvantages of the complicated norms and unusual metrics.

2.2. The typical models of noises

The functional RK-model has an important property that it is possible to establish a partial order of the models. Firstly, the notions of subordinated and dominated kernels are introduced in the following way. The kernel $R_2(s, t)$ dominates $R_1(s, t)$ (and R_1 is subordinated to R_2):

$$R_1 \le R_2, \tag{10}$$

if the difference

$$R_0(s, t) = R_2(s, t) - R_1(s, t) \tag{11}$$

is the positively defined kernel.

As applied to the corresponding random processes, it means that the process $x_1(t)$ with the correlation function $R_1(s, t)$ may be obtained as a projection of the process $x_2(t)$ with the correlation function $R_2(s, t)$ onto a certain subspace H_1 in the space of random values.

The ordering relation for the kernels generates the corresponding ordering relation for the RK-spaces. If the kernel $R_2(s, t)$ dominates $R_1(s, t)$, then RK-space $H(R_1)$ is the subspace of the RK-space $H(R_2)$. In this case $H(R_1)$ can be also called as subordinated to the space $H(R_2)$.

There is a practically important case, then the actual correlation function of the process $R_1(s, t)$ is not known, but it is possible to construct or define the kernel $R_2(s, t)$, which dominates $R_1(s, t)$. Thus all the principal problems mentioned above, such as linear filtering, extrapolation, and the signal extraction from noise, may be solved at the dominating RK-space $H(R_2)$, instead of the actual RK-space $H(R_1)$.

Therefore, it is practically important to reveal a relatively limited set of such "typical" processes, that the corresponding RK-spaces would be easy to describe and handle. It is also preferable, that these RK-spaces would be rather extensive, in order to dominate a wide range of the actual processes RK-spaces. So a set of typical models should be established with the desired propertied stated.

In this paper a large variety of electrical noises has been analyzed, which are stationary or non-stationary processes [1, 2]. The most important typical models of noises are selected. The set of typical models includes:

1) stationary processes;
2) non-correlated process (white noise);
3) process with non-correlated increments;
4) Markovian and N-Markovian processes;
5) 1/f – type processes.

The latter type of noise is also included into this set of typical models, as it is very important for electrical measurements [4, 9].

For these typical processes the detailed structure of the corresponding RK-Hilbert spaces is studied. Firstly, the expressions for scalar products and norms are obtained; it is also necessary to derive and study the corresponding estimates based on experimental data.

In particular, if $x_1, \ldots, x_n$ is time series with non-correlated increments, the corresponding RK-space is the vector space with norm

$$\| x \|^2 = | x_0 |^2 + \Sigma | x_{k+1} - x_k |^2 \tag{12}$$

In the case of Brownian motion with the correlation function

$$B(s, t) = \sigma_0^2 + \sigma^2 \min(s, t), \tag{13}$$

RK-Hilbert space H (R) is just the space of the absolutely continuous functions with the square-integrable derivative:

$$H(B) = \{ f : \int | f'(u) |^2 du < \infty \}. \tag{14}$$

Likewise, the more general process with non-correlated increments has the correlation function of the form

$$B(s, t) = \sigma_0^2 + \sigma^2 \mu(0, \min(s, t)),\ 0<s, t<T, \tag{15}$$

with $\mu(0, s)$ being a certain measure of the interval. Then RK-Hilbert space H (R) is just the space of the functions, which are absolutely continuous relatively measure μ, with the square-integrable derivative:

$$H(B)=L_1^2\{\mu\}= \{f: \int(df(u)/d\mu(u))^2 d\mu(u) < \infty \} \tag{16}$$

So the scalar product in this RK-space is the following:

$$(f_1, f_2)_B = \int(df_1(u)/d\mu(u))\,(df_2(u))/d\mu(u))\,d\mu(u) \tag{17}$$

In the case of Markovian process the correlation function is represented in the form:

$$R(s, t) = \psi(s)\,\varphi(t),\ 0<s, t<T, \tag{18}$$

where $\psi(s)$ and $\varphi(t)$ are continuous functions, and the ratio $u(t)=\psi(t)/\varphi(t)$ is an increasing function.

Then the RK-space consists of the functions, which have the following repersentations:

$$f(t)= \varphi(t)\, g(t),\quad \int (g'(t))^2 / u'(t)\, dt < \infty \} \tag{19}$$

So the scalar product in this RK-space is the following:

$$(f_1, f_2)_R = \int (g_1'(t)\, g'_2(t) / u'(t)\, dt \,. \qquad (20)$$

As applied to these typical models, the corresponding classes of noises are formed as the sets of processes subordinated to the typical ones. It is important, that the necessary and sufficient conditions of subordination are also formulated in terms of RK-Hilbert spaces.

In conformity with these typical classes of noise, the main noise characteristics are defined in terms of RK-Hilbert spaces. They are constructed as the corresponding norms of related functionals.

The typical models of Brownian motion (or non-correlated increments) are extremely significant, as they produce the RK-norm, which is closely related with Allan variance. The latter is an important characteristis of data scatter, which is useful both for time (frequency) and electrical measurements. As Allan variance is produced by the process with non-correlated increments, or innovation process, it may be also called as "innovation variance".

It is to be noted, that the noise classes, formed in a way stated above, do not produce a proper classification system. The classes are not disjoint, but they are complementary in some sense.

So there are many processes, which are simultaneously subordinated to several typical processes. Thus there are several characteristics, which may be used to describe noise properties. These properties are more or less distinct, so the characteristics are complementary. Therefore, by the use of several characteristics concurrently, one can study and evaluate the noise in greater details.

In particular, for a wide range of processes it seems very appropriate to use sample variance in combination with Allan variance. The practical use of these characteristic pointed at the preferential domains of either characteristic.

The time-domain approach is primarily directed at the investigation of the non-stationary noise. But it is valid for the stationary case as well.

It is proposed to include the stationary noise into the set of typical noises. Then the class of subordinated processes will include both stationary and non-stationary noises. On the other hand, any class mentioned above contains some stationary processes, although typical process is non-stationary. So this approach is independent of the stationary property, and it provides an essential linking between stationary and non-stationary range.

The special case of 1/f noise is to be particularly mentioned. It is well known, that 1/f – noises, or flicker-noises are observed in various physical and technical systems, and they are particularly important for electrical systems and devices [1, 2]. There are many papers devoted to the expertimental and theoretical aspects of 1/f – noise [4-6].

However, the 1/f – noise behavious is not yet well understood. The model of stationary increment random function has been proposed [11], and there are further investigations in this field.

The time-domain approach seems to be promising for 1/f – type noise problem. The characteristics based on RK-Hilbert space in this case are close to the proper modifications of Allan variance.

2.3. Signal extraction from noise

An example of the realization of the metrological approach for noise processing is the estimation of the useful (valid) signal of the noise. Here the signal is treated in wide sense; in particular, it may be the systematic errors of the device. Then it is estimated in order to insert the corrections.

It is supposed that the signal is a deterministic function, and it is represented as the sum of given functions $f_1, \ldots, f_m$ with unknown parameters $a_1, \ldots, a_m$:

$$f(t) = \Sigma\, a_i f_i\,(t) \qquad (21)$$

The deterministic assumption is not very strict; in practice, it is often just an irregular function, which is considered as quasi-deterministic on the observation interval.

This function is observed in presence of random noise x(t), in the discrete points within the interval [0, T]:

$$y_k = y\,(t_k) = \Sigma\, a_i\, f_i\,(t_k) + x\,(t_k), \qquad 0 \le t_1 < t_2 < \ldots < t_n \le T\,. \qquad (22)$$

The statistical characteristics of the noise x (t), in particular, its correlation function R(s, t), are not known completely. But it is known, that correlation function B(s, t) dominates R(s, t). In this case one cannot construct the optimal linear estimates of parameters $a_1, \ldots, a_m$, but it is possible to find the pseudo-best B-estimates, which are found according correlation function B(s, t).

These estimates are constructed like the classic least squares estimates, with using RK-space H(B) metric (instead the sum of squares). So the system of equations is of the form:

$$\Sigma\,(f_i, f_j)_B\, a_j = (y, f_i)_B\,,\ i = 1 \ldots m, \qquad (23)$$

B-estimates are the direct generalization of the classic least squares estimates; the latter correspond to the white noise RK-space H(B). Apart from this case, the B-estimates formed according to Brownian motion, and Markov processes are of practical interest.

For the applications of B-estimates it is necessary to study the basic properties of these estimates, in particular:

1) conditions of the statistical consistency of B-estimates (when the estimates are converging to the true values of parameters);
2) conditions of the asymptotic efficiency of B-estimates (when B-estimates are almost as accurate as the optimal estimates);
3) estimates of the relative efficiency of B-estimates (which reflect the loss of accuracy caused by using B-estimates instead of the optimal estimates).

All the conditions and estimates mentioned above can be formulated in terms of the corresponding RK-spaces. They depend on the interrelations of the spaces H(B) and H(R), which is a subspace of H(B), and also of the properties of the functions $f_1, \ldots, f_m$. These conditions may be given in the explicit form for the cases of Brownian motion, and Markov processes.

The study of B-estimates shows that B-estimates are rather simple and convenient. They also give an acceptable accuracy, so the loss of accuracy is not significant.

3. CONCLUSIONS

The time-domain approach, proposed in this paper, forms a methodological basis for expansion of the range of noise characteristics, which are useful for metrological practice. It relies on the general physical properties of the processes and devices, which are often at the disposal of the researcher.

Several typical noise classes are formed on the basis of the ordering relation for RK-Hilbert spaces. These classes do not produce a proper classification system, but they are complementary.

It provides the opportunities for the better investigation of the noise properties based on the set of distinct and harmonizing characteristics. The well-known characteristics, such as variance and Allan variance, could be better studied; as well as new ones could be introduced.

Various characteristics are intended to represent distinct properties of noises; so the joint employing of several characteristics will serve for more full and objective notion of the noise properties.

REFERENCES

[1] Van der Ziel A. Noise in measurements. – John Wiley and Sons, 1976.
[2] Ott H. W. Noise reduction technoques in electronic systems. – Wiley Publ., N. Y., 1976.
[3] Neidenoff A. “General equivalent noise” of electric networks. – In: “Noise in physical systems, including 1/f noise, biological systems and membranes”. - Proc. 10th International conference, 1989, Hungary. – Budapest, 1990.
[4] Wolf D. 1/f noise. – In: “Noise in physical systems”. – Proc. 5th International conference on noise, 1978, Germany. – Berlin, 1978.
[5] Van der Ziel A. Theory and experiments on quantum 1/f noise. – In: “Noise in physical systems and 1/f noise”. – Proc. 8th International conference, 1985, Rome. – North-Holland, 1986.
[6] Agu M., Kinoshita T. 1/f noise. – In: “Proceedings of International Symposium on 1/f – Fluctuations”. – Tokyo, 1977.
[7] Max J. Methodes et techniques de traitement du signal et applications aux mesures physiques. T. 1, 2. – Masson, Paris, 1981.
[8] Allan D.W., Ashby N., Hodge C.C. The Science of Timekeeping. – Application Note 1289, Hewlett-Packard Company, 1997.
[9] Witt T. J. Testing for correlations in measurements – Advanced Mathematical and Computational Tools in Metrology, IV, 2000.
[10] Kallianpur G. The role of reproducing kernel Hilbert spaces in the study of Gaussian processes. – “Advances in probability and related topics”, v. 2, 1970.
[11] Solo V. Intrinsic random functions and the paradox of 1/f noise. – SIAM J. Appl. Math., v. 52, N 1, 1992.

Authors: D. Sc., Prof. Valery A. Granovsky, D. Sc. Tatiana N. Siraya.
Central Scientific Research Institute “Elektropribor”, 197046, St.-Petersburg, Russia.
Fax: +7 (812) 232 33 76   E-mail: elprib@online.ru