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A SAMPLING METHOD FOR HIGH ACCURACY AC MEASUREMENT

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Abstract − The method presented here is intended for high accuracy measurement of all parameters of a low frequency (method was tested up to 500 Hz) sine wave. It is relatively simple and, because of its exceptional resistivity to the random errors of samples, applicable for use similarly in calibration laboratories and less demanding measurements. Uncertainty of 10 ppm for all parameters of a sine wave is an ordinary result obtainable with this method, so it can be used for calibration of almost all AC calibrators and insurance of traceability. The only notable disadvantage of this method is relatively long duration of sampling (about 45 s), what makes this method suitable only for measurement of signals with time-invariant parameters.

Keywords: sampling, high accuracy, sine wave

1. INTRODUCTION

Development of fast and accurate digital voltmeters, i.e., A/D converters, such as HP3458A Digital Multimeter (later in text only HP), which is being used in our experiments, enabled implementation of (theoretically) old ideas about periodical alternating signal measurement, that is, it enabled sampling of signal. Soon, sampling became the far dominating manner of measurement of AC signals, mostly thank to very powerful theory of discrete-time signal processing and fast development of microelectronics, that is, digital computers. After the "conquest" of less demanding measurements, sampling methods started to "penetrate" into the last "fortress" of the analog methods, i.e., into the national calibration laboratories. The method presented here has already achieved full usability and is nowadays our standard method for the most accurate AC measurements in Croatian Primary Electromagnetic Laboratory (CPEL). Although it has been developed for measurement of real (sinusoid) signals, in this paper we shall present just its basic principles and consider a signal with no harmonics and DC component. Complexity of the theory that stands behind the whole method, including signals with non-neglectable harmonics, greatly exceeds the scope of this article and will be explained in our further papers (for now, the whole theory can be found in [1]).

2. THEORETICAL BACKGROUND OF THIS METHOD

Because accuracy is the highest priority in our work, it is essential to achieve the best possible accuracy of every single sample of the measured signal. This is why HP must be set to the DC mode (i.e., as if it was measured a DC signal), despite to the fact that it measures an alternating signal. Again because of accuracy, we implement rather inconvenient (in comparison with most of other sampling methods) manner of sampling, similar to the one implemented in digital osciloscopes. Instead of taking all samples of one period of the signal at once (in the same single period of the signal), we take only one sample per period, spreading the sampling along few hundreds of periods, beginning the integration of each sample at the point at which the integration of the precedent one ended. Denoting duration of one sample (in fact, integration time) as T_m, the time between two consecutive samples as T_s and the period of signal as T, Fig. 1 is clear illustration of the whole process. If we wish to have n samples per period, it follows that $T_s=T+T/n=T(1+1/n)$.

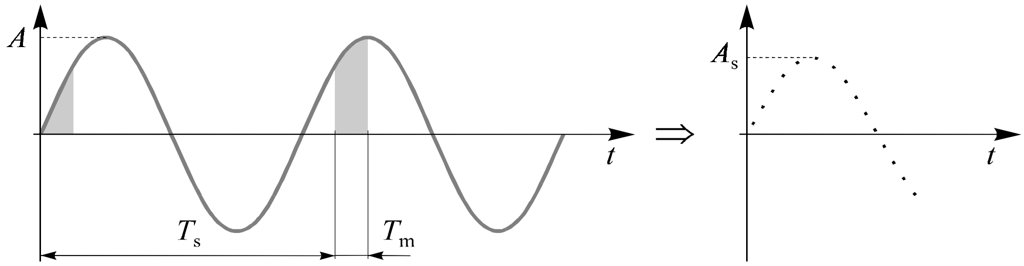

Fig. 1. Illustration of sampling. Left – shadowed areas are intervals of integration of a particular sample, right – samples obtained by sampling depicted at the sketch on the left.

If the general form of the measured signal is $U(t) = A\cdot sin(\omega t + \varphi)$ and we, for convenience, count the samples starting with zero, i^{th} sample is defined by integral [1,4]

$$U_i = \frac{1}{T_m}\int_{iT_s}^{iT_s+T_m} A\sin(\omega t+\varphi)\cdot dt \quad . \qquad (1)$$

Solving this integral, we obtain the mathematical form of a single sample.

$$\begin{aligned} U_i &= \frac{A}{\pi}\frac{T}{T_m}\sin\frac{\pi T_m}{T}\cdot\sin\left(2\pi\frac{T_s}{T}i+\pi\frac{T_m}{T}+\varphi\right) \\ &= A_s\cdot\sin(xi+y) \end{aligned} \qquad (2)$$

$$A_s = \frac{A}{\pi}\frac{T}{T_m}\sin\frac{\pi T_m}{T} \quad , \quad x = 2\pi\frac{T_s}{T} = 2\pi\frac{n+1}{n} \quad ,$$

$$y = \pi\frac{T_m}{T}+\varphi$$

Obviously, samples are also points of pure sinusoid (we shall name it "mirror sinusoid") which is firmly mathematically related to the original (measured) sinusoid according to (2), where symbols have the following meanings: U_i is the value of i^{th} sample, n is the number of samples per period of the signal, A is the amplitude of the measured signal, φ is phase of the measured signal, A_s is the amplitude of the mirror sinusoid, parameter x represents the frequency of mirror sinusoid, parameter y represents the phase of mirror sinusoid and the other symbols have the same meaning as before.

Measurement of the effective (rms) value (or amplitude) of the original sine wave requires only one more formula, i.e., we must be able to calculate amplitude A_s of mirror sinusoid in order to calculate amplitude A of the measured signal, because T_m and T (their ratio T_m/T) are known quantities. Here we use well known [1] formula (3),

$$U_{rms}^2 = \frac{1}{T}\int_a^{a+T} U^2(t)\cdot dt \quad \leftrightarrow \quad U_{rms}^2 = \frac{1}{n}\sum_{i=0}^{n-1}U_i^2 \qquad (3)$$

$$n \geq 3$$

which generally relates (equidistant) samples of any sine wave and its rms value. So, it follows that measurement of amplitude of a sine wave requires only three steps: 1) taking samples of the signal, 2) calculation of A_s (U_{rms}) according to (3) and finally 3) calculation of A according to (2). Calculation of the frequency x and phase y of mirror sinusoid can be accomplished by the following [1,4] simple, but very effective formulae,

$$x = \arccos\frac{U_{i-1}+U_{i+1}}{2\cdot U_i} \quad ;$$

$$y = arcctg\frac{\rho\cdot ctg(xi)+1}{ctg(xi)-\rho} \quad ,$$

$$\rho = \frac{U_{i+1}-U_{i-1}}{2\cdot U_i\sin(x)} \qquad (4)$$

which relate every three equidistant samples with frequency and phase of the signal. Please note that it is possible to obtain the phase φ of the original sinusoid, because we are able to calculate parameter y and T_m/T is known quantity. Formulae (2), (3) and (4) are enough for calculation of all parameters of a sine wave from its samples and are therefore the core of this method. However, the whole precedent discussion is valid only in an ideal measurement, that is, under condition that we perfectly sample a perfect sinusoid. Of course, the measured signal will never be completely time-invariant and spectrally clean, sampling device will not have ideal characteristics and the whole measurement, along with environmental influences, will introduce a number of additional errors in samples. As we stated in the introduction of this article, we shall presuppose the perfect sinusoid as the measured signal, but now we are going to take into account imperfections of the sampling device.

There are two main problems with a real sampling device. Firstly, its response to any (external or internal) excitation will never be instant, and secondly, the device will never be able to reproduce exactly the time intervals we want it to obey. An illustration (Fig. 2) will clarify this further. Part a) of the Fig. 2 shows a perfect sampling. Abscissa represents time and the shadowed areas represent the intervals of integration of a particular sample. As depicted, sampling starts immediately in the moment ($t = 0$) we send a signal to the instrument to start measurement. After integration of the first sample, which lasts exactly T_m as we assigned, instrument waits until the period of sampling T_s elapses and then starts the integration of the next sample. Such process is being repeated until all samples are taken. Because of non-instant response to our signal for starting measurement, real instrument will not start integration of the first sample exactly in $t = 0$, but in some moment after the time needed for reaction, which we named "offset time" and denoted as T_o (Fig. 2, part *b*). Consequently, all samples will be shifted by the same time interval and this leads to the additional phase shift between mirror and original sinusoid, what makes calculation of phase φ of the original sinusoid by formula (2) impossible. In difference to T_o, which equally appears in all samples, inaccuracy of reproduction of assigned sampling period has the cumulative effect (Fig. 2, part *c*). If sampling period is not exactly T_s, but $T_s+\Delta T_s$, contribution of error ΔT_s in the first sample will be ΔT_s , in the second one $2\Delta T_s$ and so on. This is very unpleasant error, because its consequence is the difference of frequencies of mirror and original sinusoid, that is, the number of samples per period is not n as we wanted it to be, but some unknown (and generally non-integer) number $n+\Delta n$, what makes the formula (3) useless. Inability of sampling device to respond instantly to the internal excitations for starting and stopping integration of a single sample introduces another time shift in every sample, equal for all of them (Fig. 2, part *d*). We named it "delay time" and denoted as T_d. Its effect is the same as of offset time T_o. The last imperfection in sampling we succeeded to take into account is inaccuracy of integration interval ΔT_m (Fig. 2, part *e*). This is probably the most destructive error in sampling process, because it is the only error that appears in two parameters of mirror signal simultaneously. This can be shown as follows. Taking into account all mentioned imperfections changes the boundaries of the integral (1) that defines samples. It becomes

$$U_i = \frac{1}{T_m + \Delta T_m} \cdot \int_{To+Td+i(Ts+\Delta Ts)}^{To+Td+i(Ts+\Delta Ts)+Tm+\Delta Tm} A \sin(\omega t + \varphi) \cdot dt \qquad (5)$$

$$= \frac{A}{\pi} \frac{T}{T_m + \Delta T_m} \sin \frac{\pi (T_m + \Delta T_m)}{T}$$

$$\cdot \sin\left(2\pi \frac{T_s + \Delta T_s}{T} i + 2\pi \frac{T_o + T_d}{T} + \pi \frac{T_m + \Delta T_m}{T} + \varphi \right)$$

and we see how every mentioned error affects every single sample. These imperfections impose a modification of the whole measurement method. The first one is need for improving of formula (3), so that we can calculate the effective value of mirror sinusoid when the number of samples per period is not exactly n. Although the problem is rather unpleasant, the solution we found is very simple and effective. The rms value of mirror sinusoid can be calculated by the following formula

$$U_{rms}^2 = \frac{\sum_{i=0}^{N-1} U_i^2}{N - \sum_{i=0}^{N-1} \cos[2(x' i + y')]} \quad ; \qquad (6)$$

$$x' = 2\pi \frac{T_s + \Delta T_s}{T} = x + \Delta x \quad ,$$

$$y' = 2\pi \frac{T_o + T_d}{T} + \pi \frac{T_m + \Delta T_m}{T} + \varphi = y + \Delta y$$

where N is the overall number of samples, not necessarily in only one (and whole) period, while parameters x' and y' can still be calculated by formulae (4), i.e., as parameters x and y. Unfortunately, amplitude of mirror sinusoid A_s is not enough for calculation of amplitude of the original signal any more, because ΔT_m , as we can read out from (5), embedded itself into the relation between A and A_s. The solution of this problem lies in doubling of measurement (sampling), but with two different integration intervals. This repetition of sampling gives us two different values of A_s from which we are able to calculate ΔT_m , and once we determine ΔT_m, from (5) we can easily calculate amplitude A of the measured sinusoid. It would be too extensive to explain calculation of ΔT_m more detailed here, so we direct the reader to the references [1,2].

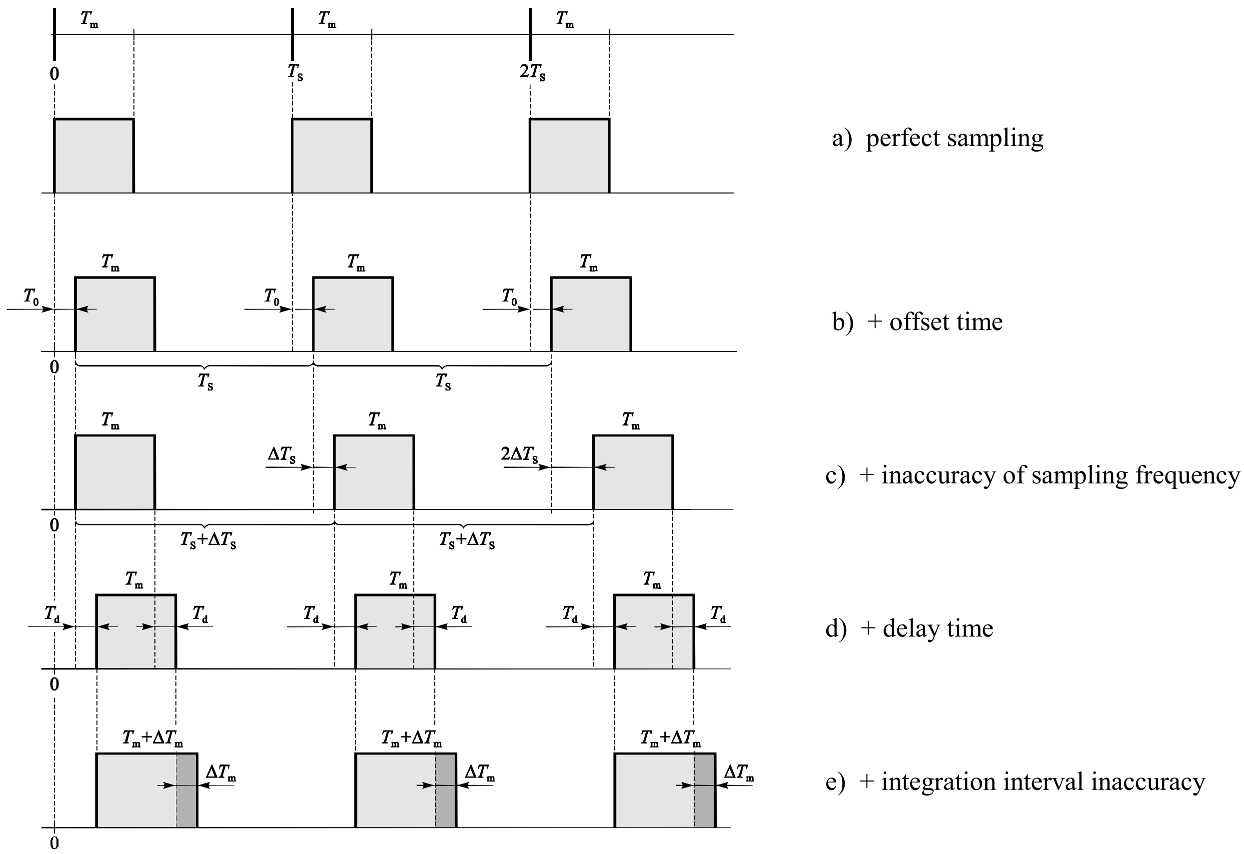

Fig. 2. Illustration of sampling inaccuracies.

Till now, we have explained only the measurement of amplitude of the sinusoid signal. The method is however fully completed in sense that it can be applied in measurements of all parameters of the sine wave (with some constraints related to the phase, as discussed later in the text). Of course, measurement of frequency (i.e., parameter x, that is x') does not have too much sense, since we assume that the period T is a known quantity. But, parameter x can be very useful in evaluation of characteristics of the sampling device, because it contains information on inaccuracy of sampling frequency ΔT_s . If we know the signal frequency (period T), from parameter x (i.e. samples) we can (applying relations (6)) calculate ΔT_s and find out how accurate our sampling device reproduces (and obeys) the assigned sampling frequency (which is determined by T and n).

Measurement of phase is something that can be only conditionally done by this method. Why is that so tells us formula (5), from which we can read out that phase of mirror sinusoid contains T_o and T_d. These two quantities can not be calculated from samples (at least we did not find the way to do that), and therefore the absolute phase φ of the original signal can not be determined. However, absolute phase is not something we frequently need. The more common needs are related to the phase difference between two sine waves, and that is something we are able to calculate with this method, in spite to the fact that we can not determine T_o and T_d. Solution for phase difference measurement is based on one assumption that greatly depends on hardware of the sampling device and dictates its quality. Namely, if we use quality digital instruments (in our work we use HP3458A Digital Multimeter as the sampling device), it is justified to assume that they have very repeatable[1] response in two or more sampling cycles. The benefit of such assumption is that additional phase shift of mirror signal due to T_o and T_d will always be the same. In that case, by sampling two sine waves consecutively[2], we obtain two parameters y_1 and y_2.

$$y_1 = 2\pi \frac{T_o + T_d}{T} + \pi \frac{T_{m1}(1+\Delta t_m)}{T} + \varphi_1 \qquad (7)$$
$$y_2 = 2\pi \frac{T_o + T_d}{T} + \pi \frac{T_{m2}(1+\Delta t_m)}{T} + \varphi_2$$

Because we know T_{m1} and T_{m2}, and are able to calculate Δt_m as explained in [1,2], the second term in formulas 0 is known and can be subtracted. We shall name the remainder as *reduced* parameter y and denote it with Y. So, after subtracting of known quantities we have

$$Y_1 = 2\pi \frac{T_o + T_d}{T} + \varphi_1 \quad \text{and} \quad Y_2 = 2\pi \frac{T_o + T_d}{T} + \varphi_2 \ . \qquad (8)$$

It is obvious that the difference of reduced parameters Y exactly matches the phase shift between two sine waves.

$$Y_1 - Y_2 = \varphi_1 - \varphi_2 \qquad (9)$$

There is still one significant problem in calculation of phase difference due to accuracy of calculation of parameter y. However, all difficulties have been successfully solved [1,3] and phase difference can be determined with uncertainty of about 10 ppm without rigorous demands on measurement system. Final limits of accuracy, under controlled laboratory conditions, are of order of magnitude of few ppm for all parameters of the signal.

3. TYPICAL MEASUREMENT RESULTS

Described method, i.e., its functionality and accuracy, can be best proven by experiments. We do all our research in CPEL. When it is about measurement of amplitude, very simple measurement system can be used. All needed is one AC source, HP3458A and an ordinary PC computer equipped with GPIB interface for driving the whole measurement and performing calculations. In our experiments we use FLUKE 5200A calibrator as the signal source to ensure short time-invariance of signal's parameters. Typical result we obtain, without rigorous demands on environmental conditions and without almost any protection from outer disturbances is shown at Fig. 3 (in that experiment we measured voltage of 7 V_{rms}, frequency f = 50 Hz).

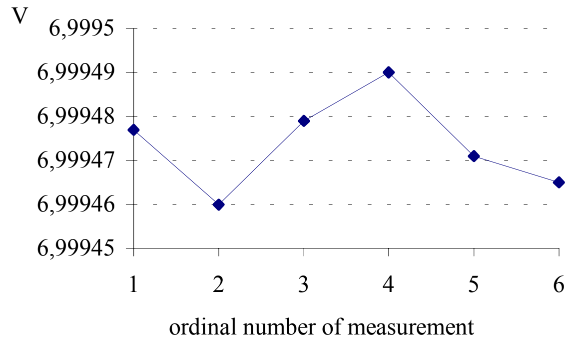

Fig. 3. Typical result of six successive measurements of the output voltage (in this experiment 7 Vrms) of FLUKE 5200A calibrator.

The mean voltage value was 6,999474 V and standard deviation 10,7 µV or only 1,5 ppm relatively. This result (relative standard deviation) is deeply in the range of calibration accuracy, which usually imposes very rigorous demands on measurement system, environment and personality. That is why we stress again that in our experiment no one of these precautions was necessary.

In the next example we demonstrate measurement of phase difference. Measurement system was very simple again, as the method itself does not impose any too hard requirements. The source was HP3245A Universal Source, which applies Discrete Digital Synthesis (DDS Synthesis). That enabled us to set the desired phase difference of its two channels and to have it relatively stable and accurate. The sampling device was HP3458A Digital Multimeter. We sampled both channels with the same instrument, commutating its inputs. Such procedure ensured maximal possible invariance of instrument's delays T_o and T_d. Start moment of sampling was synchronised with the same channel for both samplings through the TTL output of the source. This preserved the phase relationships during two separate samplings. In experiment we show here (Fig. 4), the preset phase difference was zero (f = 50 Hz again), i.e., both signals were phase matched, so every particular point

[1] According to the factory specifications, HP's response delay is less than 175 ns, with jitter less than 50 ps.

[2] It is understood that we must ensure synchronization between signals and sampling device in order to maintain relative phase relationships.

at the Fig. 4 represents the absolute error of phase difference measurement. With the same flexible laboratory conditions and inexpensive measurement system as before, we obtained relatively good results. Maximal error in 30 successive measurements was less than 60 µrad, and there were only two extreme values that are to be ascribed to the DDS synthesis, rather than to the measurement method itself. All other values were within ±40 µrad.

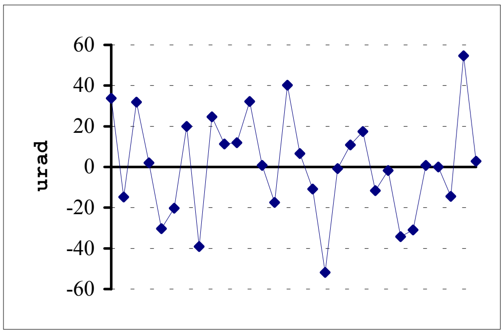

Fig. 4. Absolute error of phase difference in 30 successive measurements, expressed in µrad. On the abscissa are ordinal numbers of measurements (omitted as irrelevant).

4. CONCLUSIONS

Simplicity and efficiency of the method described here make it very suitable for wide range of applications, and its high accuracy enables it to be used in calibration measurements and insurance of traceability. It was tested only at low frequencies of up to about 500 Hz, but its theoretical background itself does not impose any limitations regarding the frequency of the signal, and we believe that it could be used also in the kHz range without significant problems. Possibility of measurement of all parameters of a sine wave, along with the background mathematics that can be extended to the signals that have non-neglectable harmonics, makes this method very applicable in more complex measurements such as measurement of power [1].

5. REFERENCES

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