12th IMEKO TC4 International Symposium
Electrical Measurements and Instrumentation
September 25–27, 2002, Zagreb, Croatia

A TEST SYSTEM FOR THE EVALUATION OF THE EXPANDED UNCERTAINTY IN DSP-BASED ANALYSIS OF BI-TONE SIGNALS

Carlo Muscas[(a)], *Roberto Tinarelli*[(b)], *Lorenzo Peretto*[(b)], *Renato Sasdelli*[(b)]

(a) Dipartimento di Ingegneria Elettrica ed Elettronica, Università degli Studi di Cagliari, Cagliari, Italy
(b) Dipartimento di Ingegneria Elettrica, Università degli Studi di Bologna, Bologna, Italy.

Abstract – A virtual instrument has been implemented to evaluate the uncertainty affecting the spectral component of bi-tone mixed signals when they are evaluated by means of the virtual time-domain approach. A numerical method relying on the Monte-Carlo method is used as an advantageous alternative to the analytical procedures suggested by the ISO "Guide to the expression of uncertainty in measurement". The fundamentals of the method are recalled and its effectiveness is proven by the results of some experimental tests reported in the paper.

Keywords: Uncertainty; Spectral analysis; Multi-tone signals; Digital signal processing; Statistical analysis.

1. INTRODUCTION

Programmable instruments based on Digital Signal Processing (DSP) techniques, Virtual Instrumentation (VI) technique and also the networking of personal computers are being widely used to perform measurements on electrical power systems for testing, monitoring, diagnostics, billing purposes. Determining the uncertainty affecting the instrument readings is a live issue, mainly when they refer to quantities different from those commonly measured. In the lack of reference meters or reliable calibration procedures, the traceability of such measurements is an unsolved problem.

Many efforts are being done in the scientific and technical community to provide indications about the possibility of evaluating the measurement quality in compliance with the general prescriptions of the ISO Standard "Guide to the expression of uncertainty in measurement" (GUM) [1].

The uncertainty affecting the measurement result is the effect of the uncertainty sources located in each one of the blocks (analog conditioning, sample and hold, analog-to-digital conversion, data processing) that constitute the modern measuring instruments. Therefore, the evaluation of their metrological performance requires the modelization of all the uncertainty sources and the estimate of the propagation of their effects through the measurement algorithm. This task can be tackled by using either an analytical approach or numerical simulations.

An interesting approach to the estimate of the propagation of the uncertainty sources effects through the measurement algorithms was proposed in [2]. It is based on three steps: theoretical analysis, numerical verification and experimental validation. The method seems to be very useful, but the proposed examples are limited to digital signals affected by totally uncorrelated contributions to uncertainty and do not consider correlated contributions. An approach to the estimate of the metrological performance of VIs was presented in [3]. It first divides the effects of the uncertainty sources into three classes (completely correlated with constant relative value, completely correlated with constant absolute value and uncorrelated) then evaluates their combination. Other contributions to the topic dealt with in the present paper can be found, for instance, in [4-7].

In this paper, a numerical method designed for the evaluation of the uncertainty affecting the readings of DSP-based instruments [8, 9] will be applied to the case of an instrument implementing the virtual time-domain approach for the analysis of bi-tone signals [10].

2. SIGNAL ANALYSIS IN THE VIRTUAL TIME DOMAIN

The function $x(t)$ representing a steady-state discrete-spectrum signal, no matter it is periodic or not, can be approximated by means of a trigonometric polynomial as follows:

$$x(t)=\sum_{n_1=-H_1}^{H_1}\cdots\sum_{n_j=-H_j}^{H_j}\cdots\sum_{n_m=-H_m}^{H_m}\underline{X}_{n_1\ldots n_j\ldots n_m}\cdot e^{j(n_1\omega_1+\ldots+n_j\omega_j+\ldots+n_m\omega_m)t}, \quad (1)$$

where $\underline{X}_{n_1\ldots n_j\ldots n_m}=\underline{X}^*_{-n_1\ldots -n_j\ldots -n_m}$ refers to the generic spectral component and ω_j is the generic angular frequency, i.e. $\omega_j = 2\pi/T_j = 2\pi f_j$, T_j and f_j being the relevant period and frequency, respectively.

In this paper, when the elements of the set $\{\omega_j\}$ of fundamental angular frequencies are not harmonically related, the *almost-periodic function*, in the sense of Bohr [11], (1) will be referred to as *multi-tone*; moreover, when the above elements are linearly independent, $x(t)$ is nonperiodic and will be referred to as *quasi-periodic*, see e.g. [12].

The spectral analysis of any periodic signal can easily be performed by applying the Discrete Fourier Transform (DFT) to a sequence of data acquired at regular time intervals over any observation interval T_o equal or multiple of the signal period T.

Several DSP techniques, which can be grouped into two main categories, windowing and statistical techniques, have been proposed in the literature to avoid the spectral leakage, which arises from the application of the DFT algorithm to a signal not periodic in T_o.

If the requirements of the sampling theorem are met, windowing allows to perform the spectral analysis by multiplying the digital signal with a function whose spectrum exhibits a low side-lobe level.

Discrete-spectrum signals that are nonperiodic in T_o can also be analyzed by using proper sampling strategies and implementing statistical techniques to process the acquired data. This also allows to overcome the sampling theorem limitations and, hence, to avoid aliasing.

The virtual time-domain approach relies on a method, first described in [13], which analyzes the multi-tone signals in a virtual time domain with coordinates $(\tau_1,\ldots,\tau_j,\ldots,\tau_m)$, i.e. with one coordinate for each tone. It allows to determine the spectral components of (1) by means of multiple applications of the Fourier Transform over a finite observation interval T_o that, if $x(t)$ is periodic, can be far lower than T.

Digital methods implementing the spectral analysis of multi-tone signals, no matter the number of tones, in the virtual time domain are not yet available. However, in many practical situations (such as in the case of power systems containing static converters), the electrical power systems can be modeled by means of nonlinear circuits forced by two tone generators; hence, $m = 2$ and (1) is bi-tone. A bi-tone signal can be analyzed by studying the virtual signal defined as follows:

$$x(\tau_1,\tau_2) \overset{\Delta}{=} \sum_{n_1=-H_1}^{H_1} \sum_{n_2=-H_2}^{H_2} \underline{X}_{n_1 n_2}\, e^{jn_1\omega_1\tau_1} \cdot e^{jn_2\omega_2\tau_2}, \quad (2)$$

which has the same spectrum of $x(t)$, is periodic in both τ_1 and τ_2 even when $x(t)$ is nonperiodic and finally is equivalent to the actual one if $\tau_1 = \tau_2 = t$. The spectral components of $x(\tau_1,\tau_2)$ can be determined by using the Fourier transform:

$$\underline{X}_{n_1 n_2} = \frac{1}{T_2}\int_{-T_2/2}^{T_2/2}\left\{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} x(\tau_1,\tau_2)\, e^{-jn_1\omega_1\tau_1} d\tau_1\right\} e^{-jn_2\omega_2\tau_2} d\tau_2 . \quad (3)$$

The digital technique proposed in [10] to implement (3) turns into a synchronous under-sampling in a virtual coordinate and an asynchronous sampling in the other one. Moreover, if $x(t)$ is quasi-periodic or is periodic with $T > T_o$, the asynchronous sampling turns into a random one. Finally, the recalled technique provides the estimate $\tilde{\underline{X}}_{n1n2}$ of the generic spectral component $\underline{X}_{n1n2}$ as follows:

$$\tilde{\underline{X}}_{n_1 n_2} = \frac{T_s}{T_2}\frac{1}{N}\sum_{k=0}^{T_2/T_S-1}\sum_{h=0}^{N-1} x(t_k + hT_2)\cdot e^{-jn_1\omega_1(t_k+hT_2)}\cdot e^{-jn_2\omega_2 t_k} \quad (4)$$

where: $t_k = t_0 + kT_S$ and t_0 is the starting instant of the acquisition process.

According to (4), f_1 and f_2 must be known; moreover, T_2/T_s, T_o/T_2 and NT_2/T_1 must be integer. Otherwise, the algorithm behaves as an error source and gives rise to a contribution to uncertainty that we refer to as *intrinsic contribution*. This contribution depends on a leakage effect (occurring when only the nominal, not the actual, value of either f_1 or f_2 is known, or at least one of the ratios: T_2/T_s , T_o/T_2, NT_2/T_1 is not integer) and a jitter effect occurring when f_s is not coherent with T_2, i.e. when T_2/T_s is not integer. Both the intrinsic contribution and the effect of the propagation through (4) of the error sources located in the measurement hardware give rise to the overall uncertainty.

3. THE PROPOSED APPROACH FOR THE EXPANDED-UNCERTAINTY EVALUATION

The number of devices that constitute a complex measurement system requires a great number of different uncertainty sources to be dealt with. In the case of analog signal conditioning systems the accuracy specifications mainly concern gain, offset, harmonic distortion, slew rate, noise, bandwidth, etc. In the case of sampling and A/D devices, we have to consider, in addition to the above specifications, the effect of other uncertainty sources, such as time jitter, nonlinearity, quantization, etc. The information on the accuracy specifications given in the data sheets provided by the manufacturers are more or less complete and presented in very different ways. An effort should therefore be done to uniform as much as possible the treatment of the different situations, also by introducing suitable simplified assumptions.

The evaluation of uncertainty in the measurement of a quantity indirectly achieved as a function of acquired quantities involves the analysis of the effect combination of several random phenomena. The ISO Guide [1] proposes some practical rules to assess the combined uncertainty and, hence, the expanded uncertainty, in the presence of either statistically dependent or independent variables. In general cases these rules may be applied by means of analytical procedures, leading to theoretically correct results that can be easily generalized. However, they may show some limitations when particular situations occur, mainly in the presence of complex measurement systems (including both hardware and measurement algorithms).

As for the information characterizing the uncertainty sources associated to any device included in the system, it can be achieved by means of either statistical methods (type A evaluation, according to [1]) or by exploiting the data sheets provided by the device manufacturer (type B evaluation). Both categories are based on probability distributions. In the case of type B evaluation, if no information on the distribution is available, the random variables can be assumed to be as uniformly distributed in an interval centered in zero and with lower and higher limits given by the accuracy specifications provided by the manufacturer.

In the proposed numerical approach a general-purpose software package is used to generate populations of random variables that represent the uncertainties affecting the input data. Then, the Monte Carlo procedure is executed: a large number of simulated tests is performed by applying to the input data the known measurement function (which, in the present work, is given by equation (4)). In each test the input samples are corrupted by the different contributions given by the elements of the above populations.

As a consequence, a set of output values, representative of the possible values of the measured quantity, is obtained: whose probability distribution can be evaluated. This allows all the desired statistical parameters (combined uncertainty, expanded uncertainty, coverage factor, etc.) to be conveniently calculated.

4. EXPERIMENTAL WORK

4.1 The Test System

Fig. 1 shows the block diagram of the ad-hoc test system implemented, making use of virtual instrumentation, to investigate on the uncertainty affecting the spectral components of a bi-tone signal, when they are estimated by means of (4).

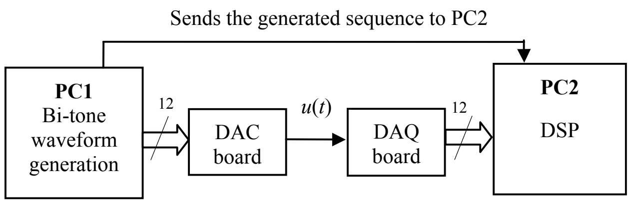

Fig. 1 Block diagram of the test system

A first processing unit (referred to as PC1) creates the numerical representation of a bi-tone signal. This data set is then sent to both a second processing unit (PC2) and a 12-bit Digital-to-Analog conversion (DAC) board, which converts the above digital signal into the analog voltage $u(t)$. $u(t)$ is then acquired by a 12-bit Analog-to-Digital conversion (DAQ) board and sent to PC2. The same unit processes, according to (4), both the acquired data and the data set generated by PC1. The results derived from the latter data set are taken as a *reference values*. They differ from the nominal values of the signal generated by PC1 because of the intrinsic contribution to uncertainty, whereas the *measured values*, i.e. those resulting from the acquired data, are affected by both the intrinsic contribution and the effect of the error sources located in the boards. The Monte-Carlo procedure recalled in Section 3 is then applied to evaluate the latter effect. To this purpose, the elements of the generated data set are corrupted by uncertainty contributions derived from the nominal specifications of DAC and DAQ boards. In this connection, it is worthwhile to highlight that those contributions show different statistical properties. Some of them, such as noise, quantization and nonlinearity, are considered to be statistically independent of each other, whereas others, such that offset and gain error, are assumed to be totally correlated variables [3]. This implies that in each test the random variables representing the latter contributions are considered as constants over the entire record of data: therefore, the offset is a constant term added to each element of the generated data set, whereas the gain error leads to an additive term proportional to the actual value of the sample, with constant proportionality factor. This simplifying assumption is valid for short observation intervals, like the ones used here. Finally, we have modeled the nonlinearity error as an additive term varying with sinusoidal law within the input range and whose amplitude is assumed to be a random variable with uniform distribution in the range defined by the nominal nonlinearity.

The expected value and the standard deviation of the signal spectral components have been evaluated by means of one thousand test. The difference between the expected and reference values gives information on whether the propagation of the uncertainty contributions in the measurement algorithm introduces a bias or not. As for the standard deviation, it provides information on the expanded uncertainty arising from the error sources located in the hardware blocks.

4.2 Results and Discussion

Several test have been performed on bi-tone signals, generated by PC1 and represented by the following expression:

$$x(t) = \sqrt{2}X_1\sin(2\pi f_1 t + \varphi_1) + \sqrt{2}X_2\sin(2\pi f_2 t + \varphi_2) + \\ + \sqrt{2}X_{mix}\sin(2\pi f_{mix} t + \varphi_{mix}). \quad (5)$$

In (5), X_1, X_2 are the rms values of the tones having frequency f_1, f_2 respectively; X_{mix} is the rms value of the component having frequency $f_{mix} = n_1 f_1 + n_2 f_2$. During the tests, quasi-periodic signals have been generated and acquired with a sampling rate of 100 kSa/s, over an observation interval that could be varied in the range 0.5÷1 s.

The nominal specifications of the DAC and DAQ boards, provided by the manufacturer, and relevant to the selected range of ± 10 V, are summarized in Table 1. The quantization arising from the 12-bit converters is not explicitly mentioned in the table and must be dealt with differently for the two boards: in the case of the DAQ board its effects are included in the noise specification, whereas for the DAC board it must be considered as an additional 1/2 LSB uncertainty term. An auto-calibration routine has been executed for both boards before the tests.

Table 1 – Nominal specifications of the boards

Board	*Offset*	*Gain error*	*Nonlinearity*	*Noise*
DAC	5.9 mV	0.022 %	2.4 mV max	0.2 mVrms
DAQ	6.4 mV	0.072 %	7.4 mV max	6.1 mV

Fig. 2 shows the output section of the VI's front panel at the end of a test, with the results of the procedure.

Examples of the test results are shown in tables 2 and 3, where, for the sake of brevity, only the rms values are reported. The reference values are achieved by applying (4) to the data set generated by PC1. The measured values are the ones evaluated by applying (4) to the data acquired by the DAQ and processed by PC2. The expanded uncertainty U_{95} refers to a confidence level of 95 %, and has been evaluated by means of the Monte-Carlo procedure, as explained later.

As for the expected values, they are not reported in the tables since in the experiments the bias has always been found lower than $1.5 \cdot 10^{-5}$; hence, it is negligible.

Table 2 reports the results gauged in the case of a bi-tone signal given by the sum of two sinusoids, i.e. X_{mix}= 0, of frequency f_1= 28.9·√3 Hz and f_2= 280·√2 Hz. Keeping X_1= 1 V, three cases have been considered, that correspond to X_2=

0.1 V, 0.5 V, 1 V , respectively. It was always taken $\varphi_1 = \varphi_2 = 0$.

Table 3 shows the results relevant to the signal (5) in the case that is: f_1= 28.9·√3 Hz, f_2= 2800·√2 Hz, f_{mix}= $3f_1 + f_2$, X_1= 1 V, X_2= 1 V and X_{mix}= 0.5 V.

Table 2 – Tone rms values (f_1= 28.9·√3 Hz ; f_2= 280·√2 Hz)

Case	*Quantities*	*Reference value* [V]	*Measured value* [V]	U_{95} [V]
#1	X_1	0.9993	1.0006	$6.1\cdot10^{-3}$
	X_2	0.09999	0.10006	$4.5\cdot10^{-4}$
#2	X_1	0.9999	1.0007	$5.4\cdot10^{-3}$
	X_2	0.5000	0.5002	$2.2\cdot10^{-3}$
#3	X_1	0.9999	1.0004	$3.5\cdot10^{-3}$
	X_2	1.0000	1.0005	$3.5\cdot10^{-3}$

Table 3 – Signal rms values (f_1= 28.9·√3 Hz, f_2= 2800·√2 Hz, f_{mix}= $3f_1 + f_2$)

Quantities	*Reference value* [V]	*Measured value* [V]	U_{95} [V]
X_1	1.0000	1.0005	$3.1\cdot10^{-3}$
X_2	1.0001	0.9993	$3.1\cdot10^{-3}$
X_{mix}	0.4987	0.4979	$1.2\cdot10^{-3}$

The results reported in the tables show that the rms value measured by the VI falls in the confidence interval (with a 95%-confidence level) evaluated by means of the proposed approach. This demonstrates that the adopted numerical technique can be successfully employed to evaluate the uncertainty, arising from both the DAC and the DAQ boards, that affects the spectral component estimates.

The numerical method allows determining the distribution of each output quantity, thus providing further information. The expanded uncertainties reported in the above tables have been directly calculated on the basis of the relevant distributions, by simply determining the value U_{95} for which 95 % of the output values fall in the range $X \pm U_{95}$, being X the expected value of the estimated quantity. Assuming a given probability distribution and choosing a coverage factor, as suggested by [1], was therefore not necessary.

As for the mixed signal having tone frequencies f_1= 28.9·√3 Hz, f_2= 2800·√2 Hz the front panel of the VI in fig. 2 shows: the waveform, the histogram relevant to the values of X_1, along with the measured and the reference value of X_1, the standard deviation, the expanded uncertainty and the bias .

Finally, the Monte-Carlo procedure has been applied by considering only one source of uncertainty each time. It has been observed that the gain error and the nonlinearity of the boards are the prevailing sources of uncertainty, whereas the effects of offset, noise and quantization are strongly limited by the very large number of samples processed.

CONCLUSIONS

Nowadays DSP-based instruments are widely used to measure electrical quantities. The determination of the measurement uncertainty is a crucial issue, mainly in non-conventional measurements for which reference meters are not available. In such cases, the traceability of DSP-based instruments is an unsolved problem.

As far as the above instruments are considered, the main sources of uncertainty are located in the transducers and in the data acquisition (DAQ) boards; their effects propagate through the measurement algorithm. As for the DAQs, the contributions to uncertainty that must be taken into account arise from the analog conditioning, the sample-hold, the analog-to-digital conversion blocks.

According to the GUM, the uncertainty affecting the measurements must be evaluated by applying the error propagation law to the measurement algorithms. This can turn into a highly complex task involving an huge computational effort.

A numerical method has recently been proposed for the evaluation of the uncertainty affecting the readings of DSP-based instruments. It relies on the assumption that all the error contributions due to a given DAQ can be assumed to be random variables with uniform distributions taking values inside an interval centered in zero and delimited by values

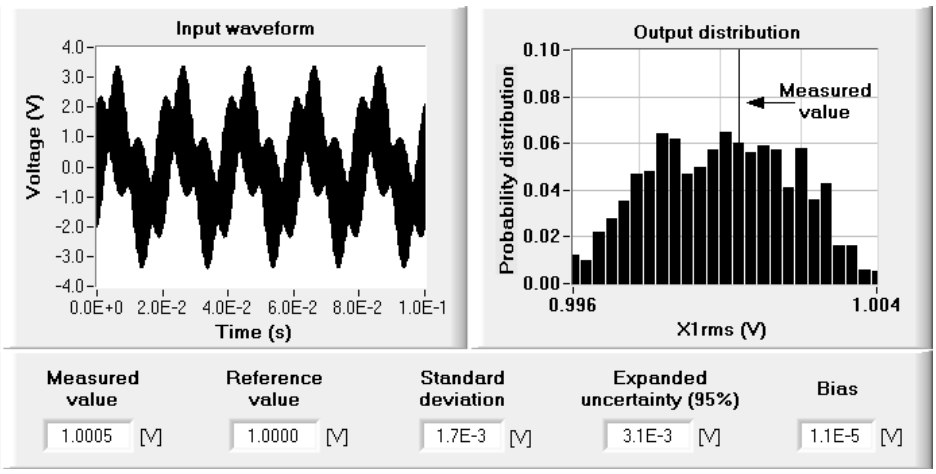

Fig. 2 Front panel of the VI

equal to those derived from the nominal specifications provided by the manufacturer. The method also involves a Monte-Carlo procedure, which provides statistical information on the expanded uncertainty affecting the measurement results.

In this paper, the approach has been successfully applied to investigate on the propagation of the effects of the above error sources through an algorithm proposed for the spectral analysis of bi-tone mixed signals. The results show that the confidence interval includes, with a confidence level of 95%, the uncertainty affecting the measurement results.

ACKNOWLEDGMENT

This work is a part of a research program supported by MIUR (Italian Ministry for University and Research).

REFERENCES

[1] "Guide to the expression of uncertainty in measurement", ISO, *International Standardization Organization*, Geneva, Switzerland, 1995.

[2] G. Betta, C. Liguori, A. Pietrosanto, "A structured approach to estimate the measurement uncertainty in digital signal elaboration algorithms", *IEE Proc. Sci. Meas. Technol.*, vol. 146, no. 1, pp. 21-26, January 1999.

[3] S. Caldara, S. Nuccio, C. Spataro, "Measurement uncertainty estimation of a virtual instrument", *Proc. IEEE IMTC/2000*, Baltimore, USA, vol. 3, pp. 1506-1511, May 2000.

[4] K. Chrzanowski, R. Matyszkiel, M. Szulim, "Calculations of uncertainty in measurements", *Proc. XVI IMEKO World Congress*, Wien, Austria, vol. IX, pp. 53-57, September 2000.

[5] G. Betta, C. Liguori, A. Pietrosanto, "Propagation of uncertainty in a discrete Fourier transform algorithm", *Measurement*, vol. 27, no. 4, pp. 231-239, June 2000.

[6] C. De Capua, C. Landi, G.C Malafronte, "Measurement uncertainty in evaluating total harmonic distortion factor", *Proc. IEEE IMTC/2001*, Budapest, Hungary, vol. 1, pp. 675-679, May 2001.

[7] C. Liguori, "Uncertainty on signal parameter estimation in frequency domain", *Proc. 11th IMEKO TC-4 Symposium*, Lisbon, Portugal, vol. 1, pp. 276-280, September 2001.

[8] N. Locci, C. Muscas, L. Peretto, R. Sasdelli, "A numerical approach for the evaluation of uncertainty in nonconventional measurements in power systems", *IEEE Trans. Instr. Meas.*, vol. 51, 2002, *in press*.

[9] N. Locci, C. Muscas, E. Ghiani, "Evaluation of uncertainty in digital processing of quantized data", *Proc. 11th IMEKO TC-4 Symposium*, Lisbon, Portugal, vol. 1, pp. 102-106, September 2001.

[10] L. Peretto, R. Sasdelli, "Measurements for the characterization of quasi-periodic waveforms", *ETEP, European Trans. on Electr. Power*, vol. 12, no. 1, pp. 11-16, 2002.

[11] H. Bohr, "Fastperiodische Funktionen", Erg. Math., *Springer*, Berlin, Germany, 1932.

[12] Kiyoshi Itô (Editor), "Encyclopedic dictionary of mathematics", *The Mathematical Society of Japan, MIT Press*, Cambridge, USA, 1987.

[13] F. Filicori; V.A. Monaco: "Computer-aided design of non-linear microwave circuits", *Alta Frequenza*, vol. 57, no. 7, pp. 355-378, 1988.

Prof. C. Muscas: Dipartimento di Ingegneria Elettrica ed Elettronica, Università degli Studi di Cagliari, Piazza d'Armi, I-09123 Cagliari, Italy. Phone: +39 070 6755860; e-mail: carlo@diee.unica.it

Dr. R. Tinarelli: Dipartimento di Ingegneria Elettrica, Università degli Studi di Bologna, Viale Risorgimento 2, I-40136 Bologna, Italy. Phone: +39 051 2093488; e-mail: roberto.tinarelli@mail.ing.unibo.it

Dr. L. Peretto: Dipartimento di Ingegneria Elettrica, Università degli Studi di Bologna, Viale Risorgimento 2, I-40136 Bologna, Italy. Phone: +39 051 2093483; e-mail: lorenzo.peretto@mail.ing.unibo.it

Prof. R. Sasdelli: Dipartimento di Ingegneria Elettrica, Università degli Studi di Bologna, Viale Risorgimento 2, I-40136 Bologna, Italy. Phone:+39 051 2093482; e-mail: renato.sasdelli@mail.ing.unibo.it