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IDENTIFICATION OF DYNAMIC MODEL OF MULTI-SECTION FURNACE

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Abstract – This paper discusses a problem of non–unique parameter identification of a multi-section tubular furnace, intended for an accurate predictive temperature control and shows that the conformity of frequency characteristics of the model with those of the object in the neighborhood of the points having the phase shift -180° is sufficient for the synthesis of a suboptimal controller.

Keywords: identification, MIMO, control**.**

1. INTRODUCTION

A precise description of the thermal plant dynamics requires non-linear models with distributed parameters. The models of such plants used for the analysis of control are usually linear and employ lumped parameters, which are much better manageable. For a SISO plant („single input – single output") often a model that consists of a single first order lag and a delay is used.

In the case of a more complex thermal plants that have many heating sections, models MIMO („multi input – multi output") ought to be used. Such models can be made of several interlinked blocks of the type described above [1].

First, the plant model parameters must be found in order to assign the correct parameters of the controller. However, in the case of a MIMO model, a signal applied to anyone input brings about responses on many outputs, which makes any identification of the constituent blocks difficult and more complex than that for a SISO model.

This article deals with the problem of non-uniqueness of the parameter identification of the thermal object MIMO models used for the purpose of automatic control analysis. At the beginning, we show for a SISO plant an advantage of the frequency approach over that of the time response – in the parameter identification intended for the purposes of control. As it has been found out, the method of identification of the plant frequency characteristics segments in the vicinity of -180° phase shift is the best one in such a case, we used analogous measurement results of a 3 section tubular furnace (intended for diffusion), model SD-3/158 [1] for further work. The data was subjected to an optimizing processing to be adapted for the computation of model parameters. The performance indices in this case multimodal, so we obtain many sets of the furnace model parameters as a solution. Considering several sets of parameters, we have synthesized a predictive controller, optimal in the sense of minimal square error [1,2]. The results were then verified by simulation.

2. CHOICE OF THE PLANT IDENTIFICATION METHOD

For the purpose of identification, the step response of a SISO plant is often approximated by that of a model composed of a first order lag and a delay. The frequency methods of identification usually give different results. Only by increase of the model order, it is possible to obtain better convergence of the two results. For instance, the transmittance of a single-section chamber furnace can be well approximated in a wide frequency range by the following one:

$$G(jw) = \left(\sum_{n=1}^{4} \frac{G_{0n}}{1 + jwt_n} \right) e^{-jwT_D} \tag{1}$$

Fig. 1 shows a trajectory of such a function in the P–Q coordinates, while the corresponding step response has the following form:

$$h(t)\big|_{t>T_D} = \sum_{n=1}^{4} G_{0n} \left[1 - \exp\left(\frac{-(t - T_d)}{t_n} \right) \right] \tag{2}$$

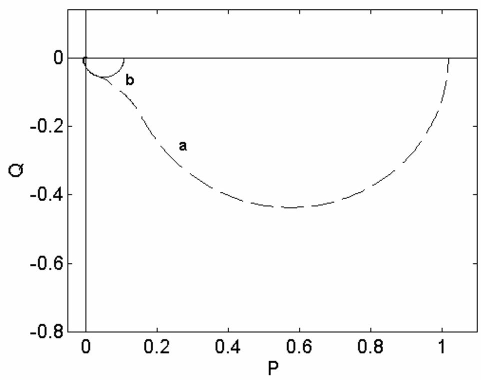

Fig. 1. P–Q plots for the POK–71 furnace models:
a) Forth order model with a delay, b) First order lag with a delay

Fig. 1 shows the comparison between the characteristics of the 4–th order model and the 1–st order model, whose transfer function is:

$$G(s) = \frac{G_0}{1 + st} \tag{3}$$

and is identified on the basis of the furnace characteristics measurements taken in the area where the phase shift is close to –180°, with the intention to assign the PID controller parameters. It can be seen that the two characteristics fundamentally differ for lower frequencies.

3. IDENTIFICATION OF MIMO PLANT

This work describes the identification problem of a MIMO thermal process model, created for an SD-3/158 diffusion furnace, which is an electrical tubular unit with three heating sections outlined in Fig.2.

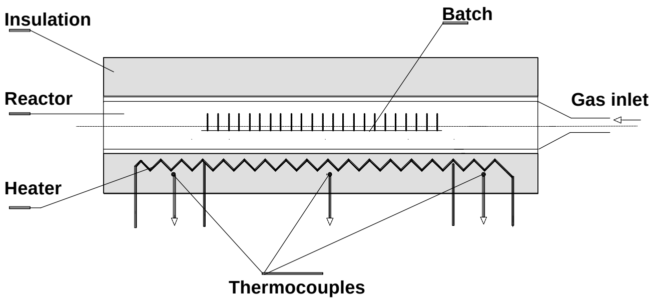

Fig. 2. Outline of a furnace for the diffusion of semiconductors

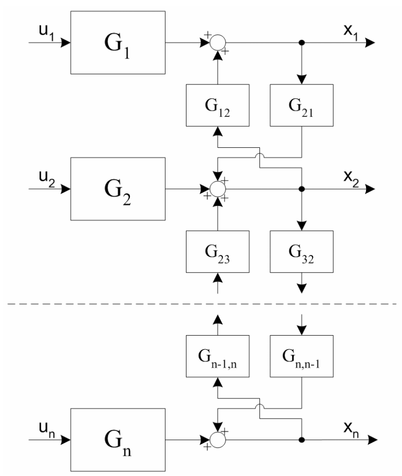

Fig. 3. Structural model of a tubular furnace with an n–section heater

As mentioned at the beginning, a MIMO thermal model can be presented in the form of several interconnected blocks of first order lag with delay G_{ij} [1]. A model of this kind is shown in Fig.3, where the following supplementary designations were used:

u_i - output power signal of i–th heating section,

x_i - temperature signal measured by the sensor of i–th heating section.

The parameters of the model shown in Fig. 3 can be found by means of identification, which can be, for example, spectral identification performed by means of MBS method [3]. Fig. 4 shows the results of such an identification for the SD-3/158 furnace.

The plots in Fig.4 comprise only a part of the whole characteristics, significant for our purposes. Many sets of parameters can be found as a result of identification, and they can be tuned to have the significant parts of their characteristics well convergent with those of the plant.

To limit the number of possible solutions we replaced the above model with another one, of the same character, but more associated with the physical parameters of the materials used, and with the furnace design. This is the Beuken's model, modified (among the others) by the addition of equivalent delays (Fig.5). The original Beuken's model is an electric analogue of the thermal process, in which thermal resistances and capacitances are substituted with R and C components, while the heating components are represented by current sources.

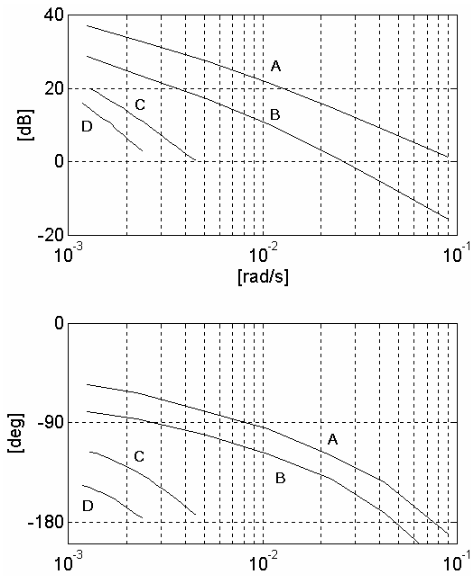

Fig.. 4. Frequency characteristics of blocks, or their combinations for the model of SD-3/158 furnace: A – G_1, B – G_2, C – G_2*G_{12}, D – G_1*G_{21}

We assumed in the model the following basic parameters, associated with the materials used and with the design of the furnace:

R_0 - relative radial thermal resistance of the insulation for a unitary pipe length,

C_0 - relative radial thermal capacitance of the insulation for a unitary pipe length,

W_0 - ratio of the relative longitudinal thermal resistance to the radial resistance for the inside of the pipe – taking into account the thermal resistances of insulation, of the internal layer, and of the inside of the reactor,

W_{SC}- coefficient of the face thermal losses – making allowance for the insulation of the pipe inlet and outlet,

A_{ij} - coefficient of coupling asymmetry of i–th to the j–th section. It relates the ratio of the thermal cross coupling in relation to the thermal cross coupling to the counterpart section in the opposite direction. The difference between the two results mainly from different distances from the temperature sensor of the given section to the boarders of the sections concerned (Fig.2).

Additionally, substitute delays were assumed. They are marked τ_i for the delays within a given section (G_i blocks in Fig.3), and τ_{ij} for cross–couplings between contiguous sections i–th and j–th (G_{ij} blocks in Fig.3).

The model shown in Fig.5 is compatible with the one shown in Fig.3. Though the first can be described by fewer parameters, because it is based on a physical entity.

Assuming L_i as the length of the i–th heating section, and using the earlier described designations, we can find the following substitute RC model parameters:

$$
\begin{aligned}
R_i &= R_0 / L_i \\
C_i &= C_0 L_i \\
r_{12} &= W_0 R_0 \frac{L_1 + L_2}{2} \\
r_{21} &= A_{12} r_{12} \\
R_{SCL} &= W_{sc} R_0
\end{aligned}
\tag{4}
$$

The symmetry of the analyzed 3–section furnace forces the following equalities:

$$
\begin{aligned}
R_3 &= R_1 \\
C_3 &= C_1 \\
r_{23} &= r_{21} \\
r_{32} &= r_{12} \\
R_{SCP} &= R_{SCL} \\
t_3 &= t_1 \\
t_{23} &= t_{21} \\
t_{32} &= t_{12}
\end{aligned}
\tag{5}
$$

The parameters of the model from Fig.5 were found in two stages. At first we found the transmittances of the model blocks – on the basis of RC model parameters. Then we minimized the performance index J_{id}, defined as:

$$
J_{id} = \sum_{m=1}^{k} \sum_{n=1}^{l} \left(100 \cdot \frac{\left| G_{mn} - G_{mnr} \right|}{\left| G_{mnr} \right|} \right) \tag{6}
$$

where:

J_{id} - minimized performance index,

m, k- consecutive number of a block and the quantity of blocks in the model of Fig.2,

n, l - consecutive number of a point and the quantity of measured points of characteristics from Fig.3,

G_{mn} - measured value of the object characteristics,

G_{mnr}- value of the model transmittance that approximates the measured characteristics.

Having limited the furnace model to the form shown in Fig.5 we obtained the number of the needed parameters equal to 9 – for a three section furnace, which is also the dimension of the variable space in which the performance index J_{id} has been minimized.

Because of the multimodality of the performance function, it was necessary to search for many local minima. We used the Nelder-Mead simplex method [4] to find the minimum of the performance index. Table 1 shows the extent of the obtained parameters. It is worthwhile to notice the wide range of the results.

TABLE I. Extent of the obtained parameters

	R_0	W_0	W_{SC}	C_0	A_{12}
min.	55.8	0.169	0.0826	31.8	1.95
max.	1757	5.43	450	38.7	2.50
max/min	31.5	32.2	5446	1.21	1.28
	τ_1	τ_2	τ_{12}	τ_{21}	J_{id}
min.	9.19	3.45	0.00	1.75	770
max.	37.0	38.6	43.0	360	1331
max/min	4.02	11.2	---	206	1.73

These results formed the basis for the synthesis of various versions of a predictive controller, optimal in the sense of minimization of a quality index defined as the integral of square error, summed, with an appropriate weight factor, with the square of driving signal local difference from the steady state value [1,2].

Next, by means of simulation, we checked the control performance for various combinations of model parameter sets, with those of the controller. In the result, we have found that the results are much similar for all cases that had the value of J_{id} close to the global minimum. For example, table 2 shows two combinations of parameters, and farther on we present the corresponding results of simulation in Fig.6 and 7.

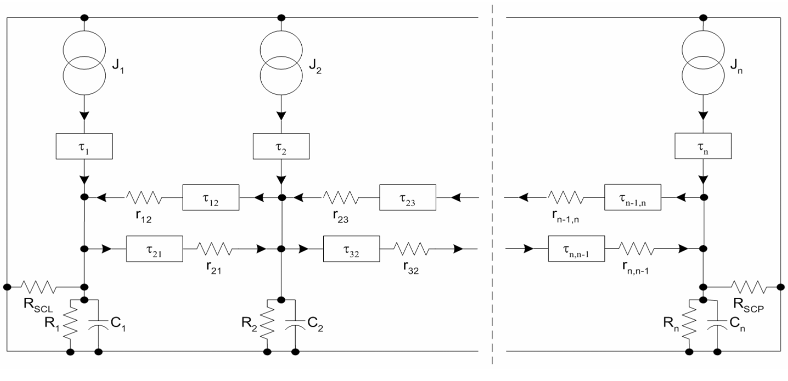

Fig. 5. Model of a multisection tubular furnace based on the physical parameters of the materials used and the design

TABLE II. Variants of parameters

	R_0	W_0	W_{SC}	C_0	A_{12}
1	83.2	3.55	4.75	33.5	2.13
2	188	1.51	0.82	33.9	2.03
	τ_1	τ_2	τ_{12}	τ_{21}	J_{id}
1	19.7	29.4	0.00	296	770
2	19.8	29.2	1.83	288	779

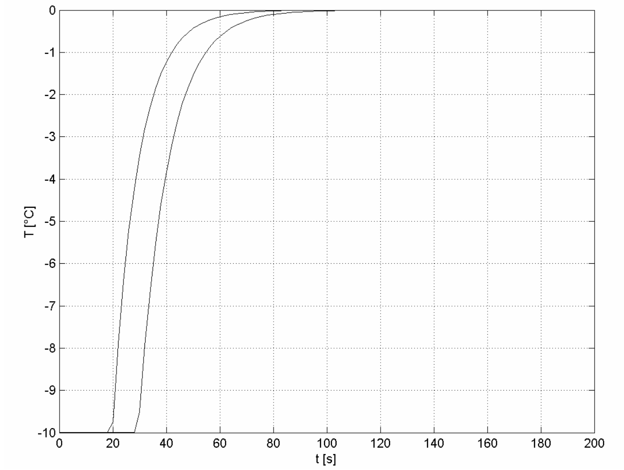

Fig. 6. Simulation of the process; parameters: variant 1

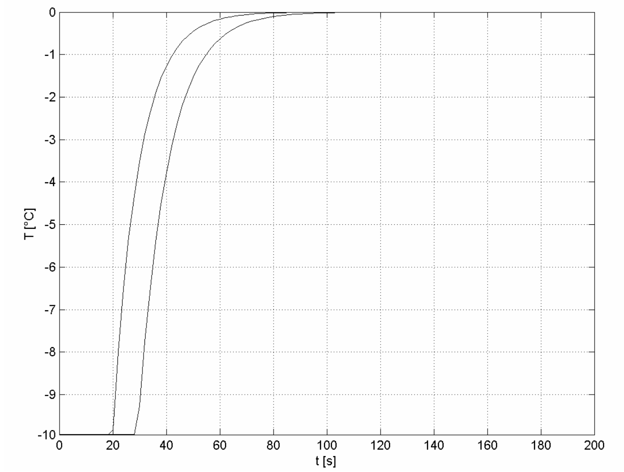

Fig. 7. Simulation of the process; parameters: variant 2

During farther investigations, we were repeatedly putting forward a hypothesis that one of the variants is close to the real plant. Then we simulated the variant in question, but using the controller gain matrices assigned for another variant that had a similar value of the performance index Jid, but with the parameters much different. Figures 8 and 9 show the results of simulation – after the controllers were mutually replaced between variants 1 and 2.

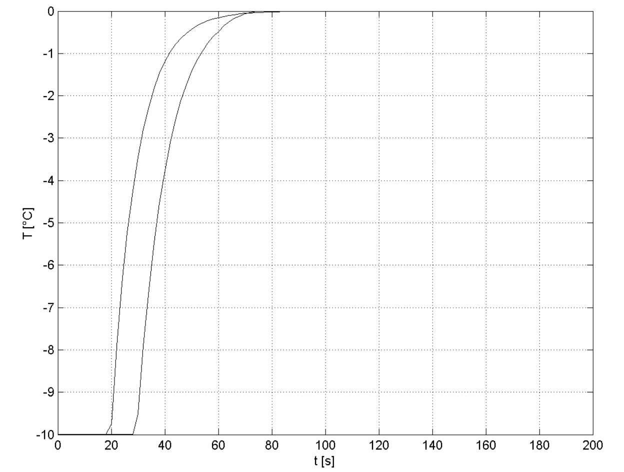

Fig. 8. Simulation of the process for the object with the parameters of variant 1, but with the controller designed for variant 2

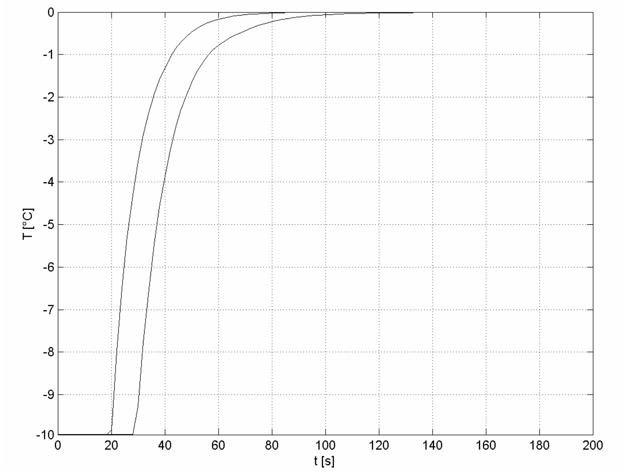

Fig. 9. Simulation of the process for the object with the parameters of variant 2, but with the controller designed for variant 1

To estimate the quality of control we used additional time criteria in the discussed cases. We added a criterion of the temperature set time with the accuracy 0.5°C, after the set value step change of 10°C, and the following integral indices:

ISE: $$I_2 = \int_0^\infty e^2(t)dt \tag{7}$$

ITAE: $$I_3 = \int_0^\infty t|e(t)|dt \tag{8}$$

ITSE: $$I_4 = \int_0^\infty te^2(t)dt \tag{9}$$

Where: ε – control error, t - time.

Table 3 shows the values of these indices:

TABLE III. Values of time and integral indices

Model	Controller	$\|e\|<0.5°C$	ISE	ITAE	ITSE
1	1	62s	4352	9174	62554
2	2	58s	4346	9020	62235
1	2	62s	4322	9202	62162
2	1	66s	4330	9570	62550

It can be seen from the table that the results are similar for all combinations of the parameter sets of the model and those of the controller.

In order to finalize the choice of the model parameters based on the material parameters and design of the furnace, we estimated creditable boundaries for the furnace parameters. Then we have chosen such sets of parameters that lay in the admitted area, and additionally were characterized by a small value of the performance index J_{id}. We found that both in the group containing the parameters of high credibility, as well as in the group containing little probable sets of parameters, there were variants of both high and low performance index. We can conclude that the material criterion and the criterion of minimization of the performance index are two different and independent criteria.

4. SUMMARY

The results presented here show that the identification of frequency characteristics of multi-section tubular furnaces in the vicinity of the phase shift of –180° proved to be effective and sufficient for the synthesis of a suboptimal predictive controller.

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